International Journal in Management and Social Science
Volume 09 Issue 11, November 2021 ISSN: 2321-1784 Impact Factor: 7.088
Journal Homepage: http://ijmr.net.in, Email: irjmss@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal

SOME FRACTIONAL DIFFERENTIAL EQUATIONS INVOLVING GENERALIZED HYPERGEOMETRIC FUNCTIONS& CASES

SYEDA FARRA FATIMA NAHRI
RESEARCH SCHOLAR SUNRISE UNIVERSITY ALWAR
DR. MANISH KUMAR SINGH
ASSISTANT PROFESSOR SUNRISE UNIVERSITY ALWAR

ABSTRACT

In the paper, using the generalized Marichev–Saigo–Maeda fractional operator, the authors establish some fractional differential equations associated with generalized hypergeometric functions and, by employing integral transforms, present some image formulas of the resulting equations. In this paper, we establish some (presumably new) differential equation formulas for the extended Mittag-Leffler-type function by using the Saigo-Maeda fractional differential operators involving the Appell function F3(·) and results in terms of the Wright generalized hypergeometric-type function m+1ψ({κl}l∈N0) n+1 (z; p) recently established by Agarwal. Some interesting special cases are also pointed out.

Keywords: -Saigo, Fractional, Hypergeometric, Equation, Formula

I. INTRODUCTION

Fractional calculus (derivative and integrals) is very old as the conventional calculus and has bern recently applied in various areas of engineering, science, finance, applied mathematics, and bio engineering (see, e.g,).

Many differential equations involving various special functions have found significant importance and applications in various subfields of mathematical analysis.

During the last few decades, a number of workers have studied, in depth, the properties, applications, and different extensions of various hypergeometric operators of fractional derivatives.

A detailed account of such operators along with their properties and applications have been considered by several authors and.

The fractional calculus operators, involving various special function, has been found highly significant. It has gained popularity due to diverse application in fields, like physical sciences and engineering.

During the past four decades, a number of workers have studied, in depth, the properties, applications, and different extensions of various operators of fractional calculus. A detailed account of such operators along with their properties and applications have been considered by several authors.

For more details and information, please refer to and the closely related references therein.

As long ago as 1974, Marichev introduced a family of fractional integral operators as Mellin type convolution operators with the Appell function F3 in their kernel.

With a view to presenting solutions of various boundary value problems involving the celebrated Euler-Darboux equation, Srivastava and Saigo (see, for details, [23]) consider the multiplication of certain classes of operators of fractional calculus defined in terms of the Gaussian hypergeometric function.Subsequently, in the middle of the 1990s, these same fractional integral operators were rediscovered and studied by Saigo and others as generalizations of the celebrated Saigo fractional integral operators.

II. SPECIAL CASES

The results in Theorems 1 and 2 can be easily specialized to yield the corresponding formulas involving simpler functions like Mittag-Leffler-type functions and extended confluent hypergeometric functions given by (1.8)-(1.12) after appropriate selection of the sequence {κl}l∈N0 .

Setting $\kappa_l = \frac{(\phi)_l}{(\varphi)_l} \ (l \in \mathbb{N}_0),$, we obtain the following results from Theorems 1 and 2, respectively.

Corollary 1 Let $x > 0, \tau, \tau', \upsilon, \upsilon', \sigma, \mu, \xi, \zeta, \lambda, \phi, \varphi, t, p \in \mathbb{C}\ (\Re(\xi) > 0, \Re(\zeta) > 0, \Re(p) \geq 0)$ be such that

$$\Re(\mu) > \max\{0, \Re(-\tau + \upsilon), \Re(-\tau - \tau' - \upsilon' - \sigma)\}.$$

Then the following formula holds:

$$\left(D_{0+}^{\tau,\tau',\upsilon,\upsilon',\sigma} t^{\mu-1} E_{\xi,\zeta}^{(\phi,\varphi);\lambda}(t;p)\right)(x)$$
$$= \frac{x^{\mu+\tau+\tau'-\sigma-1}\Gamma(\varphi)}{\Gamma(\lambda)\Gamma(\phi)}$$
$$\times {}_5\psi_5\left[\begin{matrix} (\mu,1), & (\mu-\upsilon+\tau,1), & (\mu+\tau+\tau'+\upsilon'-\sigma,1), & (\gamma,1), & (\phi,1) \\ (\mu-\upsilon,1), & (\mu+\tau+\tau'-\sigma,1), & (\mu-\sigma+\tau+\upsilon',1), & (\zeta,\xi), & (\varphi,1) \end{matrix}\middle| (x;p)\right].$$

Corollary 2 Let $x > 0,\ \tau, \tau', \upsilon, \upsilon', \sigma, \mu, \xi, \zeta, \lambda, \phi, \varphi t, p \in \mathbb{C}\ (\Re(\xi) > 0, \Re(\zeta) > 0, \Re(\lambda) >$

$1, \Re(p) \geq 0)$ be such that

$$\Re(\varrho) > \max\{\Re(-\upsilon'), \Re(\tau+\tau'-\sigma), \Re(\tau'+\upsilon-\sigma)+\Re(\sigma)\}.$$

Then the following formula holds:

$$\left(D_{-}^{\tau,\tau',\upsilon,\upsilon',\sigma} t^{-\mu} E_{\xi,\zeta}^{(\phi,\varphi);\lambda}(1/t;p)\right)(x)$$
$$= \frac{x^{-\mu+\tau+\tau'-\sigma}\Gamma(\varphi)}{\Gamma(\lambda)\Gamma(\phi)}$$
$$\times {}_5\psi_5\left[\begin{matrix}(\mu+\upsilon',1), & (\mu-\upsilon-\tau'+\sigma,1), & (\mu-\tau-\tau'+\sigma,1), & (\gamma,1), & (\phi,1) \\ (\mu,1), & (\mu-\tau-\tau'-\upsilon+\sigma,1), & (\mu-\tau'+\upsilon,1), & (\zeta,\xi), & (\varphi,1)\end{matrix}\middle|(x^{-1};p)\right].$$

For κl = 1, Theorems 1 and 2 become as follows.

III. HYPERGEOMETRIC FUNCTION

The study of hypergeometric functions in one variable dates back over 200 years. There are references to them in the writings of Euler, Gauss, Riemann, and Kummer. Barnes and Mellin examined their integral representations, while Schwarz and Goursat investigated their unique features. Many well-known partial differential equations may be simplified to Gauss' equation by separating variables, making it omnipresent throughout mathematical physics.

The Mellin-Barnes integral is an example of an integral that defines a hypergeometric function. Another way to characterize hypergeometric functions is as solutions to a system of differential equations that is, in the right sense, holonomic and has mild singularities. For hypergeometric functions in a single variable, our understanding of this relationship dates back decades. However, in the case of several variables, one may expand any of these methods with the possibility of somewhat different outcomes. As a result, there isn't a universally accepted definition of a hypergeometric function over several variables. For instance, Horn introduced a way of thinking about multivariate hypergeometric series in terms of their coefficients. They generate a set of partial differential equations by satisfying a recursive structure. For more than two variables, the space of local solutions may be unlimited in dimension, hence the system is not always holonomic. However, there is an easy technique to transform this PDE system into a holonomic one. In the two-variable instance, the connection between these two systems is well recognized. The Lie algebra of differential equations satisfied by the classical Horn, Appell, Pochhammer, and Lauricella, multivariate hypergeometric functions, and their relationship to differential equations arising in mathematical physics was not studied until the 1970s and 1980s, thanks to the efforts of W. Miller Jr. and his coworkers.

Sometimes the generalized hypergeometric function is meant when the phrase "hypergeometric function" is used.

The hypergeometric series is a class of functions that contains the Gaussian or ordinary hypergeometric function 2F1(a,b;c;z) as a particular instance. It's the answer to an ODE of the second kind, linear, and of the second order.

This equation may be used as a transformation for any linear ODE of the second order with three regular singular points.

Many thousands of identities using the hypergeometric function have been published; Erdélyi et al. (1953) and OldeDaalhuis (2010) provide systematic catalogues of some of these identities. No known method exists for cataloging all of the identities, and no one algorithm is known to produce every possible identity, however several algorithms are known to produce certain sequences of identities. The notion of algorithmic identity discovery is still a hotspot for academic inquiry.

John Wallis used the phrase "hypergeometric series" in his work ArithmeticaInfinitorum, published in 1655. Leonhard Euler investigated hypergeometric series, but Carl Friedrich Gauss provided the first comprehensive, systematic study of the topic.

Bernhard Riemann (1857) provided a key characterisation of the hypergeometric function in terms of the differential equation it satisfies, building on the work of Ernst Kummer (1836).

Riemann demonstrated that the three regular singularities of the second-order differential equation for 2F1(z), studied on the complex plane, could be described (on the Riemann sphere).

The cases where the solutions are algebraic functions were found by Hermann Schwarz (Schwarz's list).

IV. THE HYPERGEOMETRIC SERIES

The hypergeometric function is defined for |z| < 1 by the power series

$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} = 1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} + \cdots. $$

It is undefined (or infinite) if c equals a non-positive integer. Here (q)n is the (rising) Pochhammer symbol, which is defined by:

$$ (q)_n = \begin{cases} 1 & n = 0 \\ q(q+1)\cdots(q+n-1) & n > 0 \end{cases} $$

The series terminates if either a or b is a non-positive integer, in which case the function reduces to a polynomial:

$$ {}_2F_1(-m,b;c;z)=\sum_{n=0}^{m}(-1)^n\binom{m}{n}\frac{(b)_n}{(c)_n}z^n. $$

For complex arguments z with |z| ≥ 1 it can be analytically continued along any path in the complex plane that avoids the branch points 1 and infinity.

As c → −m, where m is a non-negative integer, one has 2F1 (z) → ∞. Dividing by the value Γ(c) of the gamma function, we have the limit:

$$ \lim_{c\to -m}\frac{{}_2F_1(a,b;c;z)}{\Gamma(c)}=\frac{(a)_{m+1}(b)_{m+1}}{(m+1)!}z^{m+1}{}_2F_1(a+m+1,b+m+1;m+2;z) $$

2F1 (z) is the most common type of generalized hypergeometric series pFq, and is often designated simply F (z).

V. CONCLUSION

In this study, we established some fractional differential formulas involving a family of Mittag-Leffler functions. Due to practical importance of the Mittag-Leffler functions, our results are of general character and hence encompass several cases of interest.

In this section, we consider some consequences of the main results derived in the preceding sections. If we set α = β in and respectively, then by the known formula due to Chadudhary et al. (see, e.g., the yields to the following corollaries

The generalized Gauss hypergeometric type functions defined by (3), possess the advantage that most of the known and widely-investigated special functions are expressible also in terms of the generalized Gauss hypergeometric functions F (α ,β) p (for some interesting examples and applications.Therefore, we conclude this paper with the remark that, the results deduced above are significant and can lead to yield numerous other integral transforms and fractional integral formulas involving various special functions by the suitable specializations of arbitrary parameters in the theorems.

REFERENCES:-

1. Al-Mdallal, QM, Hajji, MA: A convergent algorithm for solving higher-order nonlinear fractional boundary value problems. Fract. Calc. Appl. Anal. 18(6), 1423-1440 (2015)

2. Al-Mdallal, QM, Omer, AS: Fractional-order Legendre-collocation method for solving fractional initial value problems. Appl. Math. Comput. 321, 74-84 (2018)

3. Agarwal, P: Some inequalities involving Hadamard-type k-fractional integral operators. Math. Methods Appl. Sci. 40(11), 3882-3891 (2017)

4. Agarwal, P: Certain properties of the generalized Gauss hypergeometric functions. Appl. Math. Inf. Sci. 8(5), 2315-2320 (2014)

5. Agarwal, P: Fractional integration of the product of two multivariables H-function and a general class of polynomials. In: Advances in Applied Mathematics and Approximation Theory, Springer Proceedings in Mathematics & Statistics, vol. 41, pp. 359-374 (2013)

6. Agarwal, P: Further results on fractional calculus of Saigo operators. Appl. Appl. Math. 7(2), 585-594 (2012)

7. Agarwal, P: Generalized fractional integration of the H-function. Le Matematiche LXVII, 107-118 (2012)

8. Agarwal, P, Jain, S: Further results on fractional calculus of Srivastava polynomials. Bull. Math. Anal. Appl. 3(2), 167-174 (2011)

9. E. Özergin, Some properties of hypergeometric functions, Ph.D. thesis, Eastern Mediterranean University, North Cyprus, 2011.

10. E. Özergin, M. A. Özarslan, and A. Altın, Extension of gamma, beta and hypergeometric functions, J. Comput. Appl. Math. 235 (2011), no. 16, 4601–4610. http://dx.doi.org/10.1016/j.cam.2010.04.019

11. R. K. Parmar, A new generalization of gamma, beta, hypergeometric and confluent hypergeometric functions, Matematiche (Catania), 68 (2013), no. 2, 33–52.

12. T. Pohlen, TheHadamard product and universal power series (dissertation). Universitat Trier, 2009.

13. H. M. Srivastava and Ž. Tomovski, Fractional calculus with an integral operator containing a generalized Mittag–Leffler function in the kernel, Appl. Math. Comput. 211 (2009), no. 1, 198–210. http://dx.doi.org/10.1016/j.amc.2009. 01.055

14. E. ¨Ozergin, Some properties of hypergeometricfunctions,Ph.D. Thesis, Eastern Mediterranean University,North Cyprus, February (2011).