International Journal in Management and Social Science
Volume 09 Issue 11, November 2021 ISSN: 2321-1784 Impact Factor: 7.088
Journal Homepage: http://ijmr.net.in, Email: irjmss@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal

EXAMINE THE APPLICATION AND PERSPECTIVE OF AXIOMS FOR THE REAL NUMBERS

V RAMANGOUD
RESEARCH SCHOLAR, SUNRISE UNIVERSITY, ALWAR, RAJASTHAN

DR. VIBHA GUPTA
RESEARCH SUPERVISOR, SUNRISE UNIVERSITY, ALWAR, RAJASTHAN

ABSTRACT

This Paper shows that the performance, with a reduced user involvement or, in some situations, without user participation, of a class of numerical techniques to solve partially differential equations is improvable by increasing the abstraction level. The use of high-level languages to describe issues using mathematics allows optimization of the domain by compilers. These improvements across a range of realistic applications and computer kernels have proved to be successful. The emphasis is on numerical techniques, such as the finite element method, based on unstructured meshes. Unstructured mesh crossing nests are frequently irregular, making it basically unable to re-order transformations for low-level compilers in the location of data. In addition, the computer kernels are frequently characterized by complicated mathematical expressions and it is not possible to do manual optimization.

Keywords: - Real, FTA, Number, Computer, Axioms.

I. INTRODUCTION

The field of Real Analysis had just been formalized in other theorem proverbs when this work was started. This section starts with an outline of these various formalizations, along with a concise analysis of their upsides and downsides and a legitimization of why it was as yet reasonable to accomplish this work in another unique situation. The earth where this formalization was to be created was the FTA-library. As its name proposes, this library contained a formalization of the Fundamental Theorem of Algebra (the FTA) deliberately grew in order to be usable in future work. In the notation and shows utilized in the FTA-library are presented and clarified, just as the hidden way of thinking and the more conceptual objectives at the core of the FTA-venture. Since Real Analysis can barely be named a piece of the Fundamental Theorem of Algebra, the library was renamed during the advancement of this work, and is by and by known as C-Corn.

As indicated by the objectives of the FTA-venture, which are additionally the objectives of C-CORN, the formalization ought to be as general and broadly appropriate as could be expected under the circumstances. Therefore, rather than focusing on a particular theorem, it was chosen to attempt to formalize a section of a reference book as intently as could be expected under the circumstances, with the focal points and weaknesses of such an alternative. Since C-Corn is a formalization of productive science, the normal bibliographic reference was Bishop's "Establishments of Constructive Analysis".

II. AXIOMS FOR THE REAL NUMBERS

Field axioms

Definition: A field is a set F together with two operations (functions)

f: $F \times F \to F$, $f(x, y) = x + y$ andg : $F \times F \to F$, $g(x, y) = xy$,called addition and multiplication, respectively, which satisfy the following axioms:

• F1: Addition is commutative: $x + y = y + x$, for all $x, y \in F$.

• F2: addition is associative: $(x + y) + z = x + (y + z)$, for all $x, y, z \in F$.

• F3: existence of additive identity: there is a unique element $0 \in F$ such that $x + 0 = x$, for all $x \in F$.

• F4: existence of additive inverses: if $x \in F$, there is a unique element

$-x \in F$ such that $x + (-x) = 0$.

• F5: multiplication is commutative: $xy = yx$, for all $x, y \in F$.

• F6: multiplication is associative: $(xy)z = x(yz)$, for all $x, y, z \in F$.

• F7: existence of multiplicative identity: there is a unique element $1 \in F$

Such that $1 \neq 0$ and $x1 = x$, for all $x \in F$.

• F8: existence of multiplicative inverses: if $x \in F$ and $x \neq 0$, there is a unique element $(1/x) \in F$ such that $x .(1/x) = 1$.

• F9: distributivity: $x(y + z) = xy + xz$, for all $x, y, z \in F$.

the similarity between axioms F1-F4 and axioms F5-F8. In the language of algebra, axioms F1-F4 state that F with the addition operation f is an abelian group. (The group axioms are studied further in the first part of abstract algebra, which is devoted to group theory.) Axioms F5-F8 state that $F - \{0\}$ with the multiplication operation g is also an abelian group. Axiom F9 ties the two field operations together

Proposition: If F is a field and $x, y, z \in F$, then

$$x + z = y + z \Rightarrow x = y.$$

Proof: Suppose that $x + z = y + z$. Let $(-z)$ be an additive inverse to z, which exists by Axiom F4. Then $(x + z) + (-z) = (y + z) + (-z)$.

By associatively of addition (Axiom F2),

$$x + (z + (-z)) = y + (z + (-z)).$$

Then by Axiom F4, $x + 0 = y + 0$ and by Axiom F3, $x = y$

Proposition: If F is a field and $x \in F$, then $x \cdot 0 = 0$

Proof: By Axiom F3, $x\cdot 0 = x\cdot (0+0)$. By distributive (Axiom F9), $x\cdot(0+0) = x \cdot 0 + x \cdot 0$. By Axiom F3 again,

$$0 + x \cdot 0 = x \cdot 0 + x \cdot 0,$$

And by Axiom F1,

$X \cdot 0 + 0 = x \cdot 0 + x \cdot 0$. Hence $0 = x \cdot 0$ by the preceding proposition.

Several similar propositions can be found in §11 of the text [2]. You should learn how to prove the easiest of these directly from the axioms.

Complete ordered fields

Note that the field F of rational functions contains a subfield of constant functions, which we can identify with R. Thus we have inclusions

$$Q \subset R \subset F$$

In some sense, Qhas too few elements to use as a foundation for calculus, while Fhastoomany.Thereisanadditionalaxiomwhichtogetherwithearlieraxioms completely characterizes the real numbers.

Classical Real Analysis

Math Comp-Analysis provides a theory of sets, limits, continuity, compact sets, and comparison of functions, differentials and derivatives. Most of the results about limits and continuity are ported from Coquelicot, through the adaptation of a significant part of its hierarchy to make it compatible with Mathematical Components (Fig. 1). We extended this hierarchy with structures that, combined with a new set of tactics, ease the handling of filters

Our library contains in particular the proofs of various standard theorems: Zorn's Lemma, Tychonoff's Theorem, Heine-Borel's Theorem, the Intermediate Value Theorem, Rolle's Theorem and the Mean Value Theorem. Our proofs of Zorn's Lemma and Tychonoff's Theorem are inspired by D. Schepler's work

Infrastructure formalizing analysis is a substantial endeavor and we are now in the process of sharpening adequate tools to facilitate the writing of scripts and their maintenance. For example, since Bachmann-Landau notations are pervasive, we chose to carefully craft an infrastructure for educational reasoning using little-o and big-O. It was in particular instrumental in producing a generic theory of differentiation. Similarly, we provide an infrastructure to facilitate ε/reasoning. Its main ingredients are a set of tactics, based on the idea of the big enough tactic from the Mathematical Components library, to delay the proof of witness properties in existential proofs and small-scale automation to rule out proofs of positivity for ε's.

III. APPLICATIONS AND PERSPECTIVES

We briefly presented MathComp-Analysis, a Coq library about elementary real analysis. The library lies in a firmly classical setting and is integrated into the Mathematical Components algebraic hierarchy. This development has been used for the formalization of the soundness of probabilistic relational logic. This logic is at the core of EasyCrypt2, a toolset for reasoning about probabilistic computations, and whose main application is the construction and verification of game-based cryptographic proofs.

Notably, it required the development of a discrete probability distributions theory. We are also in the process of applying MathComp-Analysis to the formalization of motion and control theory in robotics, another topic that requires results on functional analysis. Our long-term goal is to provide a comprehensive Coq library, that can serve the formal study of topics relying both on results in algebra and in analysis. Examples of such topics include: computer-algebra algorithms, cyber-physical systems, post-quantum cryptographic primitives, information theory, etc

IV. FORMALIZATIONS OF REAL NUMBERS

Before even tackling the topic of real analysis, one has to understand how real numbers are represented in the various systems and libraries. They have chosen vastly different approaches and we will classify them into three categories. Whenever relevant, we also mention whether analysis is built directly on those sets or on higher-level concepts like topological spaces.

In mathematics, a real number or real is a value of a continuous quantity that can represent a distance along a line. The adjective real in this context was introduced in the 17th century by René Descartes, who distinguished between real and imaginary roots of polynomials. The real numbers include all the rational numbers, such as the integer −5 and the fraction 4/3, and all the irrational numbers, such as √2 (1.41421356..., the square root of 2, an irrational algebraic number). Included within the irrationals are the transcendental numbers, such as π (3.14159265...). In addition to measuring distance, real numbers can be used to measure quantities such as time, mass, energy, velocity, and many more. The set of real numbers is denoted using the symbol $\mathbf{R}$ or $\mathbb{R}$ and is sometimes called "the reals".

Real numbers can be thought of as points on an infinitely long line called the number line or real line, where the points corresponding to integers are equally spaced. Any real number can be determined by a possibly infinite decimal representation, such as that of 8.632, where each consecutive digit is measured in units one-tenth the size of the previous one. The real line can be thought of as a part of the complex plane, and the real numbers can be thought of as a part of the complex numbers.

Real numbers can be thought of as points on an infinitely long number line. These descriptions of the real numbers are not sufficiently rigorous by the modern standards of pure mathematics. The discovery of a suitably rigorous definition of the real numbers—indeed, the realization that a better definition was needed—was one of the most important developments of 19th-century mathematics. The current standard axiomatic definition is that real numbers form the unique Dedekind-complete ordered field {R} ; + ; · ; <), up to an isomorphism,[a] whereas popular constructive definitions of real numbers include declaring them as equivalence classes of Cauchy sequences (of rational numbers), Dedekind cuts, or infinite decimal representations, together with precise interpretations for the arithmetic operations and the order relation. All these definitions satisfy the axiomatic definition and are thus equivalent.

V. AXIOMATIZED REAL NUMBERS

The PVS system and the standard library of Coq both postulate real numbers as a complete Archimedean field. This leads to short and intuitive formalizations, but there is always a slight doubt on whether the additional axioms might have introduced an inconsistency with the native axioms of the system.

PVS Thanks to its pervasive support for subtyping, PVS takes a top-down approach. Instead of starting from natural numbers and building more and more complicated types on top of them (as is done in other systems), PVS starts from a number type that is a superset of all numerals. These numerals encompass all the integers available in the underlying Lisp system which runs PVS. Indeed, there are no abstract integers, only these concrete integers. In addition to the integers, this number type contains anything related to numbers, in particular real numbers. PVS then defines a subtype number field of number that provides the field operators and axiomatizes their properties. Real numbers, complex numbers, hyper-real numbers, etc, will all be subtypes of number field. Below is a selection of some theory declarations that start this hierarchy. Of particular interest is that the division is an operation between a number and a nonzero number.

VI. CONCLUSION

Ifwearetoreasonaboutprogramsspecifiedbycodewritteninanygeneral-purpose(thatistosay,Turing-complete) language, then we are certainly going to need to be able to express some kind of un-bounded iteration operation. There are negative results in this area, which we reported, but a worthwhile state of affairs has not yet been completely ruled out.Regarding validities, the important paper is the Goldblatt and Jackson paper. They rule out decidability for a whole family ofpropositional dynamic logics, and this immediately translates into undecidability of equational theoriesfor partial functions over a range of signatures.However, obtaining results by translating in this waynecessarily requires antidomain in the signature.If we have in mind to model partial recursive functionswithoutanyrestrictions,thenitisdifficulttojustifyincludingantidomain,asidentifyingthepointswherepartialrecursivefunctionsareundefinedisnotingeneralandeffectivelycomputableoperation andso not expressible in any programming language. We therefore pose the following informally specifiedproblem.Thishassimilaritiesto[58,Problem 5.6]byJacksonandStokes.

REFERENCES:-

1. Boldo, S., Lelay, C. and Melquiond, G. (2012), Improving real analysis in Coq: a userfriendly approach to integrals and derivatives, in C. Hawblitzel and D. Miller, eds, 'Proceedings of the 2nd International Conference on Certified Programs and Proofs', Vol. 7679 of Lecture Notes in Computer Science, Kyoto, Japan, pp. 289–304.

2. Sylvie Boldo, Catherine Lelay, and Guillaume Melquiond. Improving real analysis in Coq: a user-friendly approach to integrals and derivatives. In Chris Hawblitzel and Dale Miller, editors, Proceedings of the 2nd International Conference on Certified

Programs and Proofs (CPP), volume 7679 of Lecture Notes in Computer Science, pages 289–304, Kyoto, Japan, 2012.

3. Sylvie Boldo, Catherine Lelay, and Guillaume Melquiond. Formalization of real analysis: A survey of proof assistants and libraries. 2014. To be published in Mathematical Structures in Computer Science. http://hal.inria.fr/hal-00806920.

4. RobbertKrebbers and Bas Spitters. Type classes for efficient exact real arithmetic in Coq. Logical Methods in Computer Science, 9(1:1):1–27, 2013.

5. Condon, M., Deano, A., Iserles, A. and Kropielnicka, K. “Efficient computation of delay differential equations with highly oscillatory terms”, Math. Model. Numer. Anal., Vol. 46, No. 6, pp. 1407–1420, 2012.