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EXAMINING HOMOTOPY ANALYSIS METHODS FOR SOLVING THE DIFFERENTIAL EQUATIONS

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ABSTRAT

A hybrid computational method 'Generalized Homotopy Analysis Method (GHAM)' based on G-transform and HAM is proposed and applied to solve various categories of linear and nonlinear differential equations. Using GHAM, the closed form series solutions are obtained for the higher-order nonlinear ordinary and partial differential equations and the computational efficiency of the method is studied. Compared to HAM, GHAM completely avoids multiple differentiation and integration in solving higher-order nonlinear differential equations.

Keywords: - Homotopy, Topology, Function, HAM, GHAM.

I. INTRODUCTION

Topology, an important branch of mathematics, deals with the study of the properties of the geometric figures that are preserved through deformations, twisting and stretching regardless of the size and the absolute position. A topological space is a set endowed with a structure called as topology, which allows continuous deformation of sub-spaces and all kinds of continuity.

The following are the classifications of the studies in Topology:

i. General Topology
ii. Combinatorial Topology
iii. Algebraic Topology
iv. Differential Topology

The notion of homotopy in algebraic topology which deals with deformation of continuous functions is very much useful for solving differential equations. This dissertation introduces the various types of hybrid methods to solve fractional differential equations.

II. SOLUTION OF DIFFERENTIAL EQUATION

In real time, modeling the dynamical systems, is mostly modeled either as linear or nonlinear differential equations. Hence, solving the differential equations and studying the properties of the solution obtained from the given differential equation is the most important and an interesting part of the study of mathematics. Many researchers have solved linear differential equations and have either obtained the closed form series solution for the given linear differential equations or obtained the numerical solution using some numerical techniques. On the other hand, due to the chaotic behavior of the nonlinear differential equation in the extended time intervals, it is very difficult to obtain the exact solution of the nonlinear differential equation. The approximate solution for the nonlinear differential equation are obtained by using the following two mathematical techniques: perturbation and non-perturbation methods.

III. FRACTIONAL DIFFERENTIAL EQUATION

Fractional calculus is a branch of mathematical analysis that studies the several different possibilities of defining real number powers or complex number powers of the differentiation operator $D = \frac{d}{dx}$.

Many researchers have modeled natural phenomena as a system of differential equations and have solved them by using various integral transform techniques. Though the methods are useful, the system of differential equations fails to investigate the pre-local properties of the governing system. In order to study the pre-local properties of the system, the fractional differential equations have been developed.

1. Riemann-Liouville Integral and Derivative

The Riemann–Liouville integral is named for Bernhard Riemann and Joseph Liouville, the latter of whom was the first to consider the possibility of fractional calculus in 1832. The operator agrees with the Euler transform, after Leonhard Euler, when applied to analytic functions.

2. Caputo Fractional Derivative

Michele Caputo (1967) introduced an another option for computing fractional derivatives, which is called as the Caputo fractional derivative. In contrast to the Riemann-Liouville fractional derivative, when Caputo’s definition is used for solving differential equations it is not necessary to define the fractional order initial conditions.

3. Modified Riemann Liouville Derivative

Many researchers developed fractional transform techniques via the Caputo derivative and the Riemann Liouville derivative. Identified the disadvantages of the above fractional derivatives in solving fractional differential equations. These derivatives have been used to obtain the first derivative of a function with the help of the second derivative, and the troublesome feature of this definition is that the α^{th}derivative, $0 < \alpha < 1$, would be defined only for differentiable functions and cannot be applied in fractional calculus with non-differentiable functions.

IV. REVIEW OF LITERATURE

Liao (1992) introduced an analytical method, namely the Homotopy Analysis Method (HAM) using the homotopy initiated by Sen (1983) to obtain the solution in the form of a series of various types of linear and nonlinear differential equations. Liao (1997) proposed a strong solution methodology using the HAM to solve nonlinear problems and applied the method to solve a nonlinear heat transfer problem that arises in the microwave heating of a unit plate. Unlike all the other analytic techniques, the HAM provides a convenient way to establish the convergence of the series solution for nonlinear problems and the rate of approximation of the series solution by selecting the suitable auxiliary parameter, which is known as the convergence-control parameter.

Liao (2004) studied the fundamental concepts related to HAM and applied it to solve nonlinear differential equations. The effectiveness of the solution obtained by HAM depends on defining the suitable base functions and the initial conditions. Abbasbandy (2007) applied HAM method to solve generalized Hirota-Satsuma coupled KdV equation and he showed that the obtained solution is more accurate than the solution obtained by HPM and ADM (Adomian Decomposition Method). Zou*et al.* (2007) applied HAM method to solve discrete KdV differential-difference equations. Nasabzadeh and Toutounian (2013), Lu and Liu (2014), Brociek*et al.* (2016), Hariharan (2017), applied the HAM to solve linear and nonlinear differential and integral equations.

Recently, Morales-Delgado *et al.* (2016) proposed a hybrid method that yields new analytical solutions for fractional partial differential equations subject to Liouville-Caputo and Caputo-Fabrizio conditions. The initiated method is based on the notions integration of homotopy and the Laplace transform. Khan *et al.* (2017) proposed a new method known as HANTM (Homotopy Analysis Natural Transform Method) to carry out the study of the different types of the linear and the nonlinear Fokker-Plank equations. Abbasbandy (2006) proposed a Homotopy Perturbation Method (HPM) to solve quadratic Riccati differential equation and compared the obtained solution with ADM. Odibat and Momani (2008) applied modified homotopy perturbation method to solve the quadratic Riccati differential equation of fractional order.

V. PRELIMINARIES

Homotopy Analysis Method

Liao (1992, 1997) proposed, the Homotopy Analysis Method (HAM) based on the notion of homotopy initiated by Sen (1983) to solve the nonlinear differential equations which is independent of small or large physical parameters. More importantly, unlike all the other analytic techniques, HAM provides a convenient way to describe the convergence of the obtained series solution of the nonlinear differential equations by means of introducing an auxiliary parameter known as the convergence-control parameter. In 2003, the basic ideas of the HAM and some applications mostly related to nonlinear ODEs were described systematically by Liao (2003).

The HAM which attracted the attention of the many researchers and it was applied to solve several nonlinear differential equations that arise in science, finance and engineering field. For example, the solution obtained by Liao (2005), Liao and Magyari (2006) using HAM is more accurate than the solution obtained by the other existing methods. The analytic approximate solutions obtained by Zhu (2006), Cheng (2008) and Cheng *et al.* (2010) for the optimal exercise boundary of American put option using HAM is more applicable than the solution obtained by Bunch and Johnson (2000), Knessl (2001) and Kuske and Keller (1998) using perturbation methods. Furthermore, Liao (2011) succesfully applied HAM to solve partial differential equation.

VI. CONCLUSION

This study primarily focused on homotopy analysis methods forsolving the differential equations and fractional differential equations that arise while modeling dynamical systems. The introductory chapter discussed the necessary definitions of homotopy, fractional calculus, G-transform and provided the important properties of the Homotopy Analysis Method. Moreover, this thesis studied the various kinds of hybrid techniques to solve the differential equations and fractional differential equations. The proposed new hybrid techniques were applied to the various types of differential equations and the obtained solutions showed an excellent agreement with the already existing methods.

REFERENCES:-

1. Abbasbandy, S 2006, 'Homotopy perturbation method for quadratic Riccati differential equation and comparison with Adomian's decomposition method', Applied Mathematics and Computation, vol. 172, pp. 485–490.

2. Abbasbandy, S 2007, 'The application of homotopy analysis method to solve a generalized Hirota-Satsuma coupled KdV equation', Physics Letters A, vol. 361, no. 6, pp. 478–483.

3. Abassy, TA 2010, 'Improved Adomian decomposition method', Computers and Mathematics with Applications, vol. 59, no. 1, pp. 42–54.

4. Agnew, RP 1944, 'Euler transformations', Journal of Mathematics, vol. 66, pp. 313–338.

5. Al-Jawary, MA 2014, 'A reliable iterative method for solving the epidemic model and the prey and predator problems', International Journal of Basic and Applied Sciences, vol. 3, pp. 441–450.

6. Arqub, OA & El-Ajou, A 2013, 'Solution of the fractional epidemic model by homotopy analysis method', Journal of King Saud University – Science, vol. 25, pp. 73–81.

7. Awawdeh, F, Adawi, A & Mustafa, Z 2009, 'Solutions of the SIR models of epidemics using HAM', Chaos, Solitons and Fractals, vol. 42, no. 5, pp. 3047–3052.

8. Bagyalakshmi, M, SaiSundara Krishnan, G &Madhu Ganesh 2017, 'On chaotic behavior of temperature distribution in a heat exchanger', International Journal of Bifurcation and Chaos, vol. 27, no. 11, 1750168 (19 pages).

9. Bagyalakshmi, M &SaiSundara Krishnan, G 2020, 'Tarig projected differential transform method to solve fractional non linear partial differential equations', Boletim da SociedadeParanaense de Matematica, vol. 38, no. 3, pp. 23–46.

10. Batiha, AM &Batiha, B 2011, 'A new method for solving epidemic model', Australian Journal of Basic and Applied Sciences, vol. 5, pp. 3122–3126.

11. Biazar, J 2006, 'Solution of the epidemic model by Adomian decomposition method', Applied Mathematics and Computation, vol. 173, pp. 1101–1106.

12. Brociek, R, Hetmaniok, E, Matlak, J &Slota, D 2016, 'Application of the homotopy analysis method for solving the systems of linear and nonlinear integral equations', Mathematical Modelling and Analysis, vol. 21, no. 3, pp. 350–370.

13. Brychkov, Yu.A&Prudnikov, AP 2001, Euler transformation, Encyclopedia of Mathematics, Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4.

14. Bunch, DS & Johnson, H 2000, 'The American put option and its critical stock price', The Journal of Finance, vol. 5, pp. 2333–2356.