International Journal in Management and Social Science
Volume 09 Issue 10, October 2021 ISSN: 2321-1784 Impact Factor: 6.319
Journal Homepage: http://ijmr.net.in, Email: irjmss@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal

STUDYING THE CONTROLLABILITY OF A NONLINEAR ORDINARY DIFFERENTIAL EQUATION

Sri Ganesh Narayan Praharaj

Research Scholar, Capital University, Koderma, Jharkhand

Dr. Anand

Professor in Mathematics, Capital University, Koderma, Jharkhand

ABSTRACT

The theory of difference equations, together with the methods utilized and their numerous applications, has matured past its juvenile phase and now stands at the forefront of practical analysis. Indeed, in the last 15 years, the subject has exploded in popularity, as evidenced by the publication of hundreds of research papers, several books, multiple international conferences, and dozens of special sessions. Differential and difference equations are two extreme models of real-world issues that may be found in mathematical theory. As many dynamical systems undergo sudden transitions at critical junctures, their dynamical behavior is often described as impulsive. These occurrences are mathematically described by means of impulsive differential equations. The precise controllability of a nonlinear ordinary differential equation subject to impulses is investigated further, and some novel conclusions are presented here.

Keywords: Nonlinear, Impulse, Controllability, Fractional, Differential

I. INTRODUCTION

Physical processes like anomalous diffusion can be difficult to represent with integer-order differential equations. Such events motivate the use of differential equations of the fractional order. Having more model flexibility is the primary benefit of researching fractional order systems. Image processing; signal processing, bio-engineering, viscoelasticity, fluid flow, and control theory are just few of the many branches of research and engineering that regularly use fractional-order differential equations. To learn the basics of fraction calculus and the numerical techniques that accompany it.

Yet, certain phenomena are characterized by frequent transitions. The first type of transformation occurs during a brief period of time when compared to the total lifetime of the operation. Specifically, impulsive differential equations are used to create the corresponding mathematical models. It's important to note that the second type of shift is not instantaneous but rather begins at specific times and persists for longer stretches of time. A differential equation with non-instantaneous impulses arises from the mathematical modeling of such events.

Several fields can benefit greatly from the study of non-instantaneous impulsive differential equations, such as hemodynamic equilibrium and the theory of rocket combustion. The administration of insulin is a great example of a non-instantaneous impulse in action. There is an immediate effect on the circulatory system. Consequent absorption is a slow process that continues for a limited amount of time.

Several scholars have recently demonstrated an interest in studying impulsive issues with non-instantaneous impulses, namely their existence, uniqueness of solutions, stability, and controllability. For the impulsive differential equation with non-instantaneous impulses, Hernández and Regan investigated mild and classical solutions. Solutions to this broad category of impulsive first-order differential equations are proved to exist, to be unique, and to be stable by Wang and Fekan. Second-order differential equations with non-instantaneous impulses were further studied by Muslim et al. in terms of their existence, uniqueness of solutions, and stability. The existence and uniqueness of solutions are more commonly discussed than the controllability of the non-instantaneous impulsive control system.

If anything can be manipulated in some way, then the control system is the network of parts that comes together to make such manipulation possible. As a structural feature, dynamical systems may be controlled. In this way, a system may be manipulated over its full configuration space using a small set of possible operations. The question of whether or not an input may influence the output state of a dynamic system in state space is at the heart of this topic.

II. GENERAL RESULTS ON EXACT CONTROLLABILITY

Take the following linear system in finite dimensions into account:

$x'(t) = Ax(t) + bu(t),$

where A is an n × n matrix, and b ∈ Rn.

If for every x0, x1 Rn there exists a T > 0 and a control function u(t) defined for t [0, T] such that the solution to with starting condition x(0) = x0 satisfies x(T) = x1 then this linear system is fully controllable. It is generally accepted that one may exert full command over a linear system if and only if

$rank[b \; Ab \; A^2 \; b \ldots A^{n-1} \; b] \; = \; n \; .$

If the system has an unlimited number of dimensions,

$$x^{'}(t) = Ax(t) + bu(t)$$

If the semigroup is compact, then the linear system is not quite controllable, where A is the infinitesimal generator of a strongly continuous semigroup in a Hilbert space X, and b: U → X is a linear bounded operator from a Hilbert space U into X.

III. EXACT CONTROLLABILITY WITH IMPULSES

The Banach space is taken into account, as is customary when dealing with impulsive differential equations.

$$PC([0,T],\mathbb{R}^n) = \Big\{x : [0,T] \longrightarrow \mathbb{R}^n : x \in C(J',\mathbb{R}^n),$$

x is left continuous at tj and the right − hand limits $x(t_j^+)$ exist}with the norm

$$\|x\|_{PC} = \sup_{t\in[0,T]} \|x(t)\|.$$

Let $J_j = [t_{j-1}, t_j], j = 1,2, \ldots p, and \; x_j$ the limiting of x to the range J_j. Location: in the void

$$PC^1([0,T],\mathbb{R}^n) = \Big\{x \in PC([0,T],\mathbb{R}^n) : x_j \in C^1(J_j,\mathbb{R}^n), \text{ and the limits } x'\left(t_j^-\right), \; x'\left(t_j^+\right) \text{ exist}\Big\}$$

with the norm

$$\|u\|_{PC^1} = \max\{\|x\|_{PC}, \|x'\|_{PC}\}$$

is a Banach space

Now think about the differential equation for impulsive control.

$$x'(t) + \lambda x(t) = f(t, x(t)) + u(t), \quad t \in [0,T], \; t \neq t_j, \; j = 1,2,\ldots,m,$$

$$x\left(t_j^+\right) = x\left(t_j^-\right) + I_j\left(x\left(t_j\right)\right), \quad j = 1, 2, \ldots, m,$$

where f : J' × Rn → Rn is without beginning or end, and there are boundaries

$$f\left(t_j^+, x\right) = \lim_{t \to t_1^+} f(t, x), \qquad f\left(t_j^-, x\right) = \lim_{t \to t_j^-} f(t, x),$$

$$f(t_j, x) = f\left(t_j^-, x\right),$$

u ∈ PC [0,T] and Ij : Rn → Rn are continuous

By a solution of above equationwe mean a function x ∈ PC1([0, T], Rn) satisfying the equation for any t ∈ J' and the impulses.

IV. FRACTIONAL CONTROL SYSTEMS WITH NONLOCAL CONDITIONS

Nonlocality research is driven by the need to understand fundamental physical phenomena. In some inverse heat conduction situations, for instance, it is utilized to calculate the values of the physical parameters that are otherwise unknown. Byszewski initially stated and verified the conclusion that there exist moderate solutions to abstract Cauchy problems with nonlocal beginning conditions. In the years after the publication of this article, other studies have investigated the problem of proving existence and uniqueness for specific classes of nonlinear differential equations. For various fractional differential equations under nonlocal circumstances, Mophou and Guérékata addressed the presence of mild solutions. The presence of weak solutions to nonlinear fractional differential equations under nonlocal circumstances has been investigated in recent years. By combining the noncompactness approach with the Sadovskii fixed point theorem, Chang et al. constructed a new set of adequate criteria for the controllability of a class of first-order semi linear differential systems with nonlocal starting conditions in Banach spaces. Contrarily, Mahmudov proved adequate criteria for the approximable controllability of a subclass of abstract evolution equations with nonlocal beginning conditions. Unfortunately, no reports of work on the approximate controllability of fractional systems under nonlocal circumstances can be found in the current literature. In light of this, we analyze the nonlocal condition of the form that allows for approximate controllability of the fractional system.

x(0) + g(x) = x0,

where g : C([0, b], X) → X is a given function which satisfies the following condition:

There exists a constant L > 0 such that

$$\|g(x) - g(y)\| \leq L\|x - y\|, \text{ for } x, y \in C([0, b], X)$$

The conventional beginning condition x(0) = x0 can be improved upon by applying the nonlocal condition in physics. As an illustration, g(x) may be expressed as

$$g(x) = \sum_{i=1}^{m} c_i x(t_i),$$

where ci(i = 1, 2, . . . , n) are given constants and 0 < t1 < · · · < tn ≤ b. Certain physical occurrences may be better described using nonlocal conditions than the usual beginning ones, as explained in.

V. CONCLUSION

Due to its greater depth compared to traditional differential equation theory, impulsive differential equation theory is quickly becoming a focal point of research. Oftentimes, systems in physics and biology display impulsive dynamical behavior because of abrupt transitions at critical junctures. Existing literature mostly ignores differential equations of fractional order and impulses. In addition, the theory of impulsive differential equations, on which impulsive control is based, has lately seen a resurgence of attention due to its potential for use in taming systems with chaotic behavior. One may demonstrate the approximability of controlling fractional control systems using impulses by adopting the methods and ideas introduced in this study.

REFERENCES:-

1. Liu, R.; Wang, J.; O'Regan, D. On the solutions of first-order linear impulsive fuzzy differential equations. Fuzzy Sets Syst. 2020, 400, 1–33.
2. You, Z.; Wang, J.; O'Regan, D.; Zhou, Y. Relative controllability of delay differential systems with impulses and linear parts defined by permutable matrices. Math. Methods Appl. Sci. 2019, 42, 954–968.
3. Shen, Y. On the Ulam stability of first order linear fuzzy differential equations under generalized differentiability. Fuzzy Sets Syst. 2015, 280, 27–57.

4. Ji, S.; Li, G.; Wang, M. Controllability of impulsive differential systems with nonlocal conditions. Appl. Math. Comput. 2011, 217, 6981–6989.

5. G.M. Mophou, Existence and uniqueness of mild solutions to impulsive fractional differential equations, Nonlinear Anal. 72 (2010) 1604–1615.

6. Z. Tai, X. Wang, Controllability of fractional-order impulsive neutral functional infinite delay integrodifferential systems in Banach spaces, Appl. Math. Lett. 22 (2009) 1760–1765.

7. Y.-K. Chang, J.J. Nieto, W.S. Li, Controllability of semilinear differential systems with nonlocal initial conditions in Banach spaces, J. Optim. Theory Appl. 142 (2009) 267–273.

8. R.P. Agarwal, M. Benchohra, B.A. Slimani, Existence results for differential equations with fractional order and impulses, Mem. Differential Equations Math. Phys. 44 (2008) 1–21.

9. R. Sakthivel, Juan J. Nieto, N.I. Mahmudov, Approximate controllability of nonlinear deterministic and stochastic systems with unbounded delay, Taiwanese J. Math. 14 (2010) 1777–1797.