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A STUDY OF MULTIPLE OBJECTIVE LINEAR TRANSPORTATION PROBLEMS AS AN APPLICATION OF MULTIPLE RESOURCE ALLOCATION PROBLEMS

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Abstract :

Allocating limited resources to various competing activities is known as simple resource allocation problem. A resource is defined by the physical variables such as men (labour), money (cost), equipment, material etc available to the management to achieve objective of the task. The sum of the resources allocated to various activities does not exceed the total available resource. Whereas in multiple objectives resource allocation problem, resources are allocated to different activities with different objectives. In the present paper we have studied a multiple objective linear transportation problem with two objective and we attempt to solve it by Fuzzy compromise method and compare it with earlier methods.

1. Introduction

In resource allocation problem, a single resource is allocated among various activities with a single objective. The objective is maximized in case of profit or minimize in case of pay off or cost. Whereas in multiple objectives resource allocation problem resources are allocated to different activities with different objectives.

Lai and Li (1999) presented a dynamic approach to solve a multiple objective resource allocation (MORA) problem. Dynamic programming is a powerful optimization technique for dealing with a large spectrum of complex problems which involves sequential or multistage decision making in many areas. A multistage decision problem can be separated into a number of sequential steps and stages, which can be accomplished in one or more ways. The solution of each stage is a decision and the sequence of decision for all the stages makes or constitutes a decision policy. Each decision is associated with some return in the form of costs or profits. The objectives in dynamic programming are to select a decision policy, which is a sequence of decision, so as to optimize the return.

Lai and Li. (1999) also discussed the two key issues for solving multiple objectives resources allocation problem using dynamic approach. First they developed the methodology of fuzzy evaluation and fuzzy optimization. And the other is to design a dynamic optimization algorithm by using the method of fuzzy evaluation and fuzzy optimization and discussed a fuzzy approach to the multi objective transportation problem as an application of MORA problem.

2. Multiple Objective Resource Allocation Problem

A general resource allocation problem is defined as the allocation of limited resources among activities so that the return from the activities is optimized. In multiple objective resource allocation problem the objectives may be both maximized and minimized. The multiple objective resource allocation problem is described as follows :

Let b - is the units of resources to be allocated to activities

n - is the number of activities

K - is the number of objectives

In this case there may be some profit type objectives which are expected to be maximized and some pay type objectives which are expected to be minimized simultaneously. The negative pay type objectives can be maximized instead of minimizing the pay type objectives themselves. Thus the situation of maximization can be considered without loss of generality. Suppose $f_{ik}(x)$, (i = 1,2,…n; k = 1,2,…,K) representing the return by the k^{th} objectives from an allocation of x units of resource to the i^{th} activity (x = 0,1,2,…,n). The problem is to allocate some units or all of the 'b' units of resources to the 'n' activities so that the return of K objectives from the activities are maximized simultaneously. Mathematically the multiple objective resource allocation (MORA) problems is stated as follows.

$$Max \quad Z_i(x_i, x_2, ..., x_n) = \sum_{i=1}^{n} f_{ik}(x_i), \quad k = 1,2,...,K \qquad ...(2.1)$$

$$s.t. \quad \sum_{i=1}^{n} (x_i) \leq b \qquad ...(2.2)$$

$$x_1, x_2, ..., x_n \geq 0$$

Where x_i is the quantity of resources allocated to the i^{th} activity (i =1,2,…,n). Dynamic programming is a multi stage decision problem. For solving it, the problem is divided into number of stages. If the stage i is defined as the process of allocating the remaining s_i units of the resource among the activities from the i^{th} activity through the n^{th} activity (i=1,2,…,n; s_1=b), then the MORA problem can be viewed as a multiple objective dynamic programming (MODP) problem with n stages. Each stage has a decision process, which is called its state. These states represents various condition of the decision process at that stage. The variable which specify the condition of the decision process is known as state variable. In above problem, in stage i, the state variable $s_i \in \{0,1,\ldots,b\}$ represents the quantity of remaining resources to be allocated among the activities from i^{th} activity through n^{th} activity and the decision variable $x_i \in \{0,1,\ldots,s_i\}$ represent the quantity of resources to be allocated to the i^{th} activity (i = 1,2…n). Thus for this deterministic system the state transition equation is

$$s_{i+1} = s_i - x_i \qquad (i = 1,2, \ldots,n) \qquad \ldots (2.3)$$

3. Multiple Objective Linear Transportation Problem

A dynamic approach to solve a MORA problem given by Lai and Li (1999). They optimize the objective vector $Z(x) = [Z_1(x), Z_2(x),\ldots,Z_K(x)]$ dynamically at each stage, instead of transforming the multi objective resource allocation problem into single objective resource allocation problem. Here we have considered a multi objective linear transportation problem (MOLTP) as MORA problem.

Lushu and Lai (2000) presented a fuzzy compromise approach to multi objective linear transportation problem. The characteristic feature of their approach is that various objectives are synthetically considered with the marginal evaluation for individual objective and the global evaluation for all objectives.

A transportation problem is concerned with the distribution of goods (products) from several destinations (demand point) at minimum total transportation cost. The problem is originally developed by Hitchcock. Generally in actual TP, multi objective functions are considered like average delivery time of commodities, minimum cost, maximum profit, more efficiency etc. Thus the multi objective linear transportation problem deals with number of objectives such as transportation cost, profit, delivery time, quantity of goods delivered, deterioration etc simultaneously.

Aneja and Nair (1979) presented a bicriteria transportation problem, where they considered two relevant objectives. For example, the objective may be minimization of the total cost and minimization of the total deterioration. The degree of deterioration may be depending on the route, mode and time of transportation.

For this purpose, two objective functions are defined for each objective. The problem is formulated as follows:

$$\text{Min} \quad Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad i=1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j=1,2,\ldots,n$$

$$x_{ij} \geq 0 \text{ for all } i,j$$

Where C_{ij} = Cost of transporting a unit from source i to destination j.

d_{ij} = Deterioration of a unit while transporting from source i to destination j.

a_i = Availability of i^{th} source.

b_j = Demand at j^{th} destination

x_{ij} = Amount transported from source i to destination j.

3.1 Algorithm for bi-criteria transportation problem

The algorithm for solving bicriteria transportation problem is proposed by Aneja and Nair (1979) is as follows:

Step 1: Find $Z_1^{(1)} = \min(Z_1 | x \in X)$ and $Z_2^{(1)} = \min(Z_2 | Z_1 = Z_1^{(1)}$ and $x \in X)$.

Record $(Z_1^{(1)}, Z_2^{(1)})$ and set k =1.

Step 2: Find $Z_2^{(2)} = \min(Z_2 | x \in X)$ and $Z_1^{(2)} = \min(Z_1 | Z_2 = Z_2^{(2)}$ and $x \in X)$.

If $(Z_1^{(2)}, Z_2^{(2)} = Z_1^{(1)}, Z_2^{(1)})$ then stop. Otherwise record $(Z_1^{(2)}, Z_2^{(2)})$ and set k = k+1.

Step 3: Define the sets L = {(1,2)} and E = ϕ and go to step 4.

Step 4: Choose an element $(r, s) \in L$ and set $a_1^{(r,s)} = | Z_2^{(s)} - Z_2^{(r)}|$ and $a_2^{(r,s)} = |Z_1^{(s)} - Z_1^{(r)}|$. Let $\bar{x}$ be an optimal solution to the T.P.

$$\min \sum_i \sum_j \left(a_1^{(r,s)} c_{ij} + a_2^{(r,s)} d_{ij}\right) x_{ij}$$
$$\text{s.t. } x \in X$$

Let $\bar{Z}_1 = \sum\sum c_{ij}\bar{x}_{ij}$ and $\bar{Z}_2 = \sum\sum d_{ij}\bar{x}_{ij}$. If (Z_1, Z_2) is either equal to $(Z_1^{(r)}, Z_2^{(r)})$ or $(Z_1^{(s)}, Z_2^{(s)})$, set E=E U {(r,s)}, goto step 5. Otherwise record $(Z_1^{(k)}, Z_2^{(k)})$ such that $Z_1^{(k)} = \overline{Z_1}$ and $Z_2^{(k)} = \overline{Z_2}$ and set k=k+1, L=L U {(r, k), (k, s)} and goto step 5.

Step 5: Set L = L – {(r, s)}. If L= ϕ, stop. Otherwise goto step 4.

Now we discuss the multi objective linear transportation problem, suggested by Li and Lai (2000). They considered a transportation problem having K objectives, which may have different types of goals. It is as follows:

Let there are m sources $S_1, S_2, \ldots, S_m$ and n destination $D_1, D_2, \ldots, D_n$ and K objectives $Z_1, Z_2, \ldots, Z_K$. Without loss of generality, it is assumed that all K objectives are to be minimized. Let a_i (i =1,2,…,m) be the availability at source S_i and the destination D_j has required the demand b_j (j=1,2,…,n). With these assumptions, the multi objective linear transportation problem (MOLTP) can be formulated as following multi objective linear programming problem.

$$\min \quad Z_K(\{x_{ij}\}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}, \quad k = 1,2,\ldots,K \qquad \ldots(3.1)$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{m} x_{ij} = b_j \quad \text{for all } j=1,2,\ldots,n \\ \sum_{j=1}^{n} x_{ij} = a_i \quad \text{for all } i=1,2,\ldots,m \\ x_{ij} \geq 0 \text{ for all } i = 1,2,\ldots,m\ ;\ j = 1,2,\ldots,n \end{cases} \qquad \ldots(3.2)$$

$C_{ij}^{(k)}$ is the cost of transporting a unit of goods from source S_i to destination D_j for each objective Z_K.

x_{ij} is the quantity of goods to be transported from source S_i to destinations D_j (i=1,2,…,m; j=1,2,…,n). It is assumed that the balance condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ holds. In case of imbalance, a fictitious source or destination can be introduced.

Let X denotes the set of all feasible solutions of MOLTP formulated in (3.1)-(3.2). Then following two concepts are suggested

1) A feasible solution $x^*=\{x_{ij}^*\} \in X$ is said to be a non dominated solution of the MOLTP (3.14) - (3.15) if there exists no other feasible solution $x=\{x_{ij}\} \in X$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}^* \quad \text{for all } k = 1,2,\ldots K$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}^* \quad \text{for atleast one } k = 1,2,\ldots,K$$

The set of all non dominated solution is generally called the complete solution.

2) An optimal compromise solution of the MOLTP (3.1)-(3.2) is a feasible solution $x=\{x_{ij}\} \in X$ at which decision maker's preferences value (decision maker's global subjective evaluation value), taking into consideration the various respective objective, is maximum.

3.2. Fuzzy compromise programming for Multi Objective Linear Transportation Problem (MOLTP)

The multi objective linear transportation problem given in (3.1) and (3.2) is also a multi objective linear programming problem. Thus it can be solved by fuzzy compromise programming approach. The steps are as follows:

Step 1 : Solve the single transportation problem and its anti problem independently for each objective Z_k (k = 1,2,…,K) to obtain the optimal solution by ignoring all other objectives each time.

$$Min \quad Z_K(\{x_{ij}\}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}, \quad k = 1,2,\dots,K \qquad \dots(3.3)$$

$$Max \quad Z_K(\{x_{ij}\}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}, \quad k = 1,2,\dots,K \qquad \dots(3.4)$$

By solving (3.3) and (3.4), an optimal solution $x^{(k)+} = \{x_{ij}^{(k)+}\} \in X$ for (3.3) and $x^{(k)-} = \{x_{ij}^{(k)-}\} \in X$ for (3.4) were obtained. X is the set of all feasible solution.

Step 2 : After solving step 1, the values of U_k and L_k can be obtained as below

$$L_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}^{(k)+}$$

$$U_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(k)} x_{ij}^{(k)-}$$

k = 1,2,…,K

Step 3 : Obtain the marginal evaluation corresponding to k^{th} objective

$$\phi_k(\{x_{ij}\}) = \begin{cases} 1 & \text{If } Z_k(\{x_{ij}\}) \le L_k \\ \dfrac{Z_k(\{x_{ij}\}) - U_k}{L_k - U_k} & \text{If } L_k < Z_k(\{x_{ij}\}) < U_k \\ 0 & \text{If } Z_k(\{x_{ij}\}) \ge U_k \end{cases}$$

Step 4 : Determine the weight **w** = $(w_1, w_2, \dots, w_K)$ and apply a suitable weight to formulate a fuzzy compromise programming problem as follows :

$$\min_{\{x_{ij}\}\in X} \mu(\{x_{ij}\}) = M_w^{(\alpha)}(\phi_1(\{x_{ij}\}), \phi_2(\{x_{ij}\}), \dots, \phi_k(\{x_{ij}\})) \qquad \dots(3.5)$$

Where for $\alpha = 1$

$$\mu(\{x_{ij}\}) = \sum_{k=1}^{K} w_k \frac{Z_k(\{x_{ij}\}) - U_k}{L_k - U_k}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\sum_{k=1}^{K}\frac{w_k\, c_{ij}^{(k)}}{L_k - U_k}\right)x_{ij} - \sum_{k=1}^{K}\frac{w_k\, U_k}{L_k - U_k}$$

Step 5 : Solve the fuzzy compromise programming problem (3.5) by using the ordinary optimization technique to obtain the optimal compromise solution of MOLTP.

3.2.1 Numerical Example

For solving the transportation problem with two objectives, a numerical example is considered and we solve it by fuzzy compromise programming method. The same example was considered by Aneja and Nair (1979) and they solved it by determining the extreme points of the non dominated set in the objective space.

The two objective transportation problem is as follows:

Min $Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$

Min $Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}$

$$\text{s.t.} \quad \sum_{j=1}^{4} x_{ij} = 8, \quad \sum_{j=1}^{4} x_{2j} = 19, \quad \sum_{j=1}^{4} x_{3j} = 17,$$

$$\sum_{i=1}^{4} x_{i1} = 11, \quad \sum_{i=1}^{3} x_{i2} = 3, \quad \sum_{i=1}^{3} x_{i3} = 14, \quad \sum_{i=1}^{3} x_{i4} = 16,$$

$$x_{ij} \ge 0, \quad i=1,2,3; \quad j=1,2,3,4$$

For solving this problem, by fuzzy compromise programming method, first assigning various weights and determined the corresponding fuzzy compromise objective values for $\alpha = 1$. These optimal compromise objective values are shown in table 1

Table 1 : Optimal compromise objective values

Weights (W_1, W_2)	$\alpha = 1$
(0.0, 1.0)	(208, 167)
(0.1, 0.9)	(208, 167)
(0.2, 0.8)	(186, 171)
(0.3, 0.7)	(176, 175)
(0.4, 0.6)	(176, 175)
(0.5, 0.5)	(176, 175)
(0.6, 0.4)	(156, 200)
(0.7, 0.3)	(156, 200)
(0.8, 0.2)	(156, 200)
(0.9, 0.1)	(143, 256)
(1.0, 0.0)	(143, 256)

Where $(Z_1^+, Z_2^+) = (143, 167)$ are ideal values and $(Z_1^-, Z_2^-) = (265, 310)$ are anti ideal values. From the table, we have observed that, for $\alpha = 1$, as the weight increases for first objective, the objective value is decreases. And for the second objective, as the weight decreases, the objective value increases.

The ideal objective values i.e. min $Z\{x_{ij}\}$ obtained are 143 and 167. From the table 1, we observed that the ideal objective value 143 is obtained when the weights are 1.0 and 0.9. Similarly for second objective, the ideal objective value 167 is obtained when the weights are 1.0 and 0.9. Thus, we can conclude that when the weights are closer to 1, the objective values are ideal.

The same transportation problem of two objectives was solved by Aneja and Nair (1979) by determining the extreme points of non dominated set. The set of points recorded by the algorithm is the set of extreme points of the non dominated set in the objective function.

In the following table, the status of set L, E and the recorded extreme points, at each iteration are given :

Table 2: Bicriteria solution in objective space

Iteration	L	E	Extreme points
1	{(1,2)}	ϕ	$Z^{(1)} = (143,265)$ $Z^{(2)} = (208,167)$
2	{(1,3),(3,2)}	ϕ	$Z^{(3)} = (156,200)$
3	{(3,2)}	{(1,3)}	-
4	{(3,4),(4,2)}	{(1,3)}	$Z^{(4)} = (176,175)$
5	{(4,2)}	{(1,3),(3,4)}	-
6	{(4,5),(5,2)	{(1,3),(3,4)}	$Z^{(5)} = (186,171)$
7	{(5,2)}	{(1,3),(3,4),(4,5)}	-
8	ϕ	{(1,3),(3,4),(4,5),(5,2)}	-

It is observed that the method proposed by Aneja and Nair (1979), for solving two objective transportation problem is also applicable to bicriteria linear programming problems in general.

4 Conclusion

For multiple objective Transportation problem, we have observed the extreme points in the table 2. We have noted that in each iteration the objective values are increases or decreases randomly. The objective value of last iteration is (186,171) which is the optimal solution by Aneja's method, whereas the optimal solution obtained in fuzzy compromise method is (143,167), when the weights are 1.0 or 0.9. Since the problem is of minimization type, so we conclude that the solution obtained by the fuzzy compromise method is an optimal solution and which is better than the method presented by Aneja and Nair (1979).

Thus, we conclude that

Fuzzy compromise method performs better than the extreme points algorithm for solving multi objective linear transportation problem.

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