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Certain Problems on Viscous Heating Effect of Non-Newtonian Fluid (Poiseille Flow)

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Abstract:
The present paper provides solution for the problems concerning the viscous heating effects of Bingham plastic flow in a circular tube when the walls of the pipe is thermally isolated. Here we have taken the rheological equation of the Bingham plastic as defined by Oldroyed to be a material obeying Von-Mises yield condion.

Key Words : Viscous, heating effect, circular tube, fluid, Non-Newtonian.

1. INTRODUCTION

As a matter of fact the essence of mathematics lie in its freedom. This freedom from the bondage of linearity of relationship between stress and rate of stress tensors led to a great deal of creative work in Non-Newtoniaon flow theory. The classical theory of hydrodynamics which deals with the fluids whose rheological equation is given by the Newtonian hypothesis viz. that the stress depends linearly on the rate of strain and is independent of the strain, successfully explains the phenomena of life, skin friction, separation, secondary flow etc., but it is inadequate in explaining the rheological behaviour of the materials like paints, plastics, ceramics, colloids, high polymers, synthetic fibers etc., which are of increasing industrial interest. The fluids whose behaviour shows deviation from Newtonian hypothesis are called non-Nowtonian fluids [1-3, 7].

Since they possess complicated structures and exhibit new physical phenomena, like Weisserberg effect, Merrington effect etc. Thus the field of non-Newtonian fluids has a tremendous influence both on technology and science [4-6, 8-10].

In this paper, we have considered the viscous heating effects of a Bingham plastic flow in a circular tube when the wall of the pipe is thermally insulated. We have taken the rheological equation of the Bingham plastic as defined by Oldroyed to be a material obeying Von-Mises yield condition.

2. Hypothesis and Formulation of the Problem

Here we use the following hypothesizes

(a) Rise in temperature is small such that the material properties are assumed to remain constant.

(b) Flow is laminar and fully developed from the instant of entering the tube.

(c) We ignore free convection effects.

(d) We also ignore free convection effects

We take the cylindrical polar coordinates with axis along the axis of the cylinder. The velocity profile for such a material flowing in a circular tube under a constant negative axial pressure gradient (-A) with the axial symmetry is given by

$$u = \frac{A}{4\mu}(a - r_p)^2, O \le r \le r_p$$

$$= \frac{1}{\mu}\left[\frac{A}{4}(a^2 - r^2) - T_y(a - r)\right], r \ge r_p \qquad (2.1)$$

where $r_p\left(= \frac{2\mu T_y}{A}\right)$ is the radius of the plug and is the radius of the cylinder.

By assumptions (b) and (d) the energy equation to the solved may be written as

$$\rho C_v u \frac{\delta T}{\delta Z} = \frac{1}{r}\frac{\delta}{\delta N}(r_{qr}) - \frac{\delta}{\delta Z}(q_z) + p_r Z \frac{du}{dr} \qquad (2.2)$$

where q_r and q_z are the radial and axial components of heat flux vector $\vec{q}$

We assume the fourier law of heat conduction, viz.

$$\vec{q} = -\lambda \vec{\nabla} T \qquad (2.3)$$

where λ is the thermal conductivity of the material. Form (2.2) and (2.3), we get

$$\rho C_v u \frac{\delta r}{\delta Z} = \frac{\lambda}{r}\frac{\delta}{\delta N}\left(r\frac{\delta T}{\delta N}\right) + \lambda\frac{\delta^2 T}{\delta Z^2} + p_r Z \frac{du}{dr} \qquad (2.4)$$

The term on the left hand side represents convection of heat into an element of fluid, the first two terms on the right hand side represent the conduction of heat into the element and the last term represent generation of heat by viscous dissipation of mechanical energy. Since in most cases conduction in the axial direction is negligible compared to the flow of heat in the same direction due to convection, we neglect the term $\lambda \frac{\delta^2 T}{\delta Z^2}$ in equation (2.4).

Thus the energy equation reduces to

$$\rho C_v z \frac{\delta T}{\delta Z} = \frac{\delta}{r} \frac{\delta}{\delta r}\left(r \frac{\delta T}{\delta r} \right) + p_r Z \frac{du}{dr} \tag{2.5}$$

Let to be the uniform temperature at which the fluid enters the pipe. Also the wall is thermally insulated so that the heat flux there is zero. Hence equation (2.5) has to be solved under the boundary condition.

$$T(0, r) = T_0 \tag{2.6}$$

$$\frac{\delta T}{\delta r}(Z, a) = 0 \tag{2.7}$$

$$\frac{\delta T}{\delta r}(r, 0) = 0 \tag{2.8}$$

The boundary condition (2.8) is a statement that the temperature profile is symmetric.

Introduction the dimensionless variables

$$\xi = \frac{Z}{a} \text{and } y = \frac{r}{a} \tag{2.9}$$

equation (2.5) becomes

$$\frac{\rho C_v A a}{4\mu} u_1(y) \frac{\delta T}{\delta \xi} = \frac{\lambda}{a^2} \frac{1}{y} \frac{\delta}{\delta y}\left(y \frac{\delta T}{\delta y} \right) y(y - y_p), y \geq y_p \tag{2.10}$$

$$\frac{\rho C_v A a}{4\mu} u_2(y) \frac{\delta T}{\delta \xi} = \frac{\lambda}{a^2} \frac{1}{y} \frac{\delta}{\delta y}\left(y \frac{\delta T}{\delta y} \right) y \leq y_p \qquad (2.12)$$

where T temperature,

$$u_1(y) = 1 - y^2 + 2y_{yp} - 2y_p \qquad (2.12)$$

and $$u_2(y) = (1 - y_p)^2 \qquad (2.13)$$

We define new dimensionless variables given by

$$x = \frac{3 - 4y_p + y_p^4}{3} \frac{\xi}{p_e}, t = \frac{4\lambda\mu(T - T_0)}{A^2 a^4} \qquad (2.14)$$

where $P_e \left(= \frac{2a < u > \rho C_v}{\lambda} \right)$ is the peclets number (2.15)

and <u> is the mean velocity given by

$$< u > = \frac{ra^2}{24\mu}(y_p^a - 4y_p + b) \qquad (2.16)$$

Equation (2.10) now transforms to

$$u_1(y) \frac{\delta t}{\delta x} = \frac{1}{y} \frac{\delta}{\delta y}\left(y \frac{\delta t}{\delta y} \right) + y(y - y_p), y \geq y_p$$

$$u_2(y) \frac{\delta T}{\delta X} = \frac{1}{y} \frac{\delta}{\delta y}\left(y \frac{\delta t}{\delta y} \right), y \leq y_p \qquad (2.17)$$

The boundary conditions (2.6) to (2.8) reduce to

$$t(0, y) = 0 \qquad (2.18)$$

$$\frac{\delta t}{\delta y}(x, 1) = 0 \qquad (2.19)$$

and $$\frac{\delta t}{\delta y}(x, 0) = 0 \qquad (2.20)$$

Equation (2.17) to (2.20) constitute a boundary value problem.

3. Solution of the Problem

For solution of the above problem, we assume a solution of the form given below

$$t = t_1 + t_2 \tag{3.1}$$

where t_1 is an approximate solution for large value of x.

At larger distances from the entrance of the tube, one expects the initial disturbances in the temperature profile to be damped out and hence the temperature will rise linearly with distance. Hence we take

$$T_1 = H(y) + bx \tag{3.2}$$

Such that $\frac{\delta t_1}{\delta y} = 0$ at y = 0 and y = 1 and t_1 is a solution of (2.17).

We substitute (3.2) into (2.17) and solve the resulting equation for H(y) under the boundary conditions $\frac{\delta H}{\delta y} = 0$ at y = 0 and y = 1 (a direct, consequence of $\frac{\delta t_1}{\delta y} = 0$ at y = 0 and y = 1 the fact that $\frac{\delta H}{\delta y}$ at y = y_p must be unique.

Thus the equation for determing H(y) is

$$\frac{dH}{dy} = \frac{1}{y}\int_0^1 yu_2(y)dy, y \le y_p \tag{3.3}$$

$$= \frac{1}{y}\int_y^1 \left\{y^2(y - y_p) - yu_1(y)\right\} dy, y \ge y_p$$

Integrating (2.3.3), we get

$$H(y) = \frac{(1 - y_p)^2 y^2}{4}; y \le y_p$$

$$= \frac{y_p^4}{2u} + \frac{1 - 2y_p)y^2}{4} + \frac{y_p y^3}{3} - \frac{y^4}{8}; y \ge y_p \tag{3.4}$$

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where H(0) has been set arbitrarily equation zero.

From equations (2.17) to (2.20) and (3.1), we see that t_2 must be the solution of equation (2.17) satisfying the boundary conditions.

$$\frac{\delta t_2}{\delta y} = 0 \text{ at } y = 0 \text{ and } y = 1 \quad (3.5)$$

and $t_2(0, y) = -H(y)$ (3.6)

We take

$T_2 = (x, y) = X(x)\,Y(y)$ (3.7)

This leads to the following pair of ordinary differential equations

$$\frac{dx}{dx} + h\times = 0 \quad (3.8)$$

and

$$\frac{1}{y}\frac{d}{dy}\left(y\frac{dY}{dy}\right) + hu(y)Y = 0 \quad (3.9)$$

where h is a constant and

$u(y) = u_1(y),\ y \geq y_p$

$= u_2(y),\ y \leq y_p$ (3.10)

Equation (3.8) gives

$X(x) = Ae^{-hx}$ (3.11)

Equation (3.9) has to be solved under the boundary conditions

$Y^1(O) = Y^1(1) = 0$

Equations (3.9) and (3.13) constitute a strum Lieuville problem. Hence there is a discrete set of eigen values of h say h_1, h_2……..and a corresponding set of non-zero eigen functions Y_1, Y_2…..satisfying (3.9) and (3.12), the Y_i are orthogonal with respect to the weight function yu(y) on the interval $0 \leq y \leq 1$ i.e.

$$\int_0^1 yu(y)Y_i(y)Y_j(y)dy = 0 \text{ if } i \neq j \quad (3.13)$$

Thus we finally get

$$t_2(x,y) = \sum_{1}^{\infty} Aie^{-hix} Y_i(y) \tag{3.14}$$

Hence

$$t(x,y) = x + H(y) + \sum_{1}^{\infty} A_i e^{-hix} r_i(y) \tag{3.15}$$

Where A_i can be determined from the boundary condition t (0, y) = 0 and are given by

$$A_i = -\frac{\int_0^1 yu(y)H(y)Y_i(y)dy}{\int_0^1 yu(y)Y_i^2(y)dy} \tag{3.16}$$

For large value of x, the exponential terms in (3.15) becomes very small. Also since the first eigen value is zero, t(x, y) for large x is given by

$$t(x, y) = X + H(y) + A_1 \tag{3.17}$$

4. Conclusion and Numerical Results

By using Rayleigh-Ritz method, we have solved the characteristic equation (3.9) numerically and the first four eigen values and eigen functions are found out satisfying the boundary conditions. The coefficients A_i are found out with the help of equation (3.16). The first eigen function corresponding to zero eigen value is taken to be unity. These are tabulated in Table 1

Table 1

Eigen Values and Expansion Coefficients

	i	hi	Ai
$y_p = 0.0$	1	0	0.3125
	2	24.02	-0.00049
	3	69.00	0.05509
	4	250.00	-0.000074
$y_p = 0.10$	1	0	0.13167
	2	27.86	0.00932
	3	80.24	0.07067
	4	286.00	0.00115

$y_p = 0.25$	1	0	0.04869
	2	36.36	0.01454
	3	105.79	-0.07292
	4	366.00	0.00222
$y_p = 0.5$	1	0	0.01939
	2	66.99	-0.00113
	3	212.68	0.01036
	4	659.00	0.00000

(ii) Eiven functions are calculated for different y_p and given in table 2

Table 2

Values of Eigen Function

	Y	Y_2	Y_3	Y_4
$y_p = 0.0$	0.00	132.38232	2.72854890	322.351180
	0.10	121.97977	2.03344200	230.476460
	0.20	93.44222	0.35241200	29.331834
	0.30	53.76113	1.37004600	-119.973650
	0.40	11.68687	2.25797950	-104.811110
	0.50	-25.18337	2.01626700	43.739761
	0.60	-52.43441	1.00439280	175.583530
	0.70	-69.32531	-0.12393720	147.006350
	0.80	-77.87803	-0.39370516	-56.364420
	0.90	-81.23223	-1.22893190	-301.248860
	1.00	-82.01558	-1.30396510	-409.830720
$y_p = 0.1$	0.00	131.89848	1.62045430	318.040050
	0.10	121.65140	1.21081360	227.578570
	0.20	93.48355	0.21907720	29.173794
	0.30	54.16259	-0.80015924	-119.154010
	0.40	12.23738	1.33112800	-106.455620
	0.50	-24.75092	1.19729800	37.846819
	0.60	-52.28519	-0.60614805	167.693070
	0.70	-69.45949	0.06018841	142.544670
	0.80	-78.18661	0.52099365	-52.564340
	0.90	-81.60358	0.726445511	-288.815980
	1.00	-82.39666	0.77414170	-393.733710
	0.00	130.92812	0.25579020	312.727010

$y_p = 0.25$	0.10	120.96223	-0.19084119	324.505560
	0.20	93.46789	-0.33496120	30.253293
	0.30	54.81760	0.12847226	-117.251470
	0.40	13.19989	0.213196.23	-109.760140
	0.50	-23.95327	0.19221800	26.455029
	0.60	-51.95708	0.09781902	152.565290
	0.70	-69.62153	-0.00967500	133.601600
	0.80	-78.66394	-0.8530113	-46.603320
	0.90	-82.20503	-0.1217404	-267.310660
	1.00	-83.02158	-0.12864580	-365.637090
$y_p = 0.5$	0.00	134.36250	4.35747300	270.325640
	0.10	124.38998	3.32910010	183.863960
	0.20	97.01622	-0.81360480	24.161634
	0.30	58.15133	1.84497460	-108.635390
	0.40	15.68202	3.36177260	-110.326960
	0.50	-22.98086	3.23063300	1.090954
	0.60	-52.86582	1.86550130	112.0932160
	0.70	-78.35428	0.15166630	109.309660
	0.80	-82.81326	1.8303600	-24.835835
	0.90	-87.20618	1.89817360	-195.639160
	1.00	-88.30078	2.10379300	-272.507570

(iii) The graphs showing the variation of f(x, y) with y for fixed values of x and y_p have been drawn. They are shown in figures. 2.1-2.3.

(iv) It is observed from the graph that t(x, y) decreases with the increase of y_p. Hence it decreases with the increase of yield stress.

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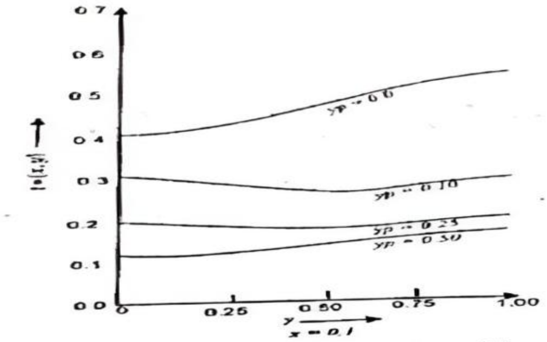

Fig. 1 Temperature Profiles

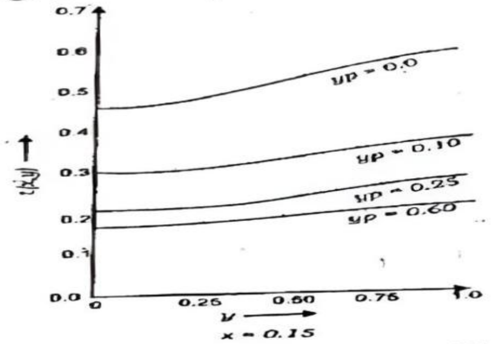

Fig., 2 Temperature Profiles

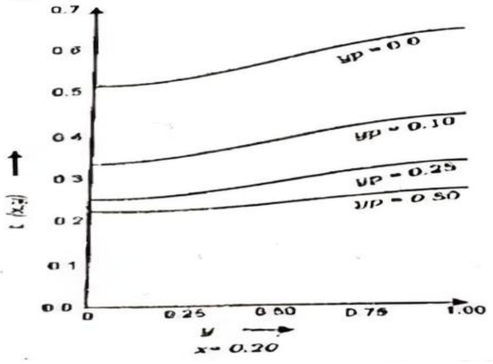

Fig. 3 Temperature Profiles

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