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## **An Investigation on Electromagnetic Scattering by an Inhomogeneous Plasma Anisotropic Sphere**

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### **Abstract:**

The present paper provides an investigation on electromagnetic scattering by an inhomogeneous plasma anisotropic sphere of multilayers. Some general numerical results have been presented in the case of lossy plasma sphere and resonance region.

**Key words:** Scattering, plasma, anisotropic, inhomogeneous, electromagnetic field.

### **Introduction:**

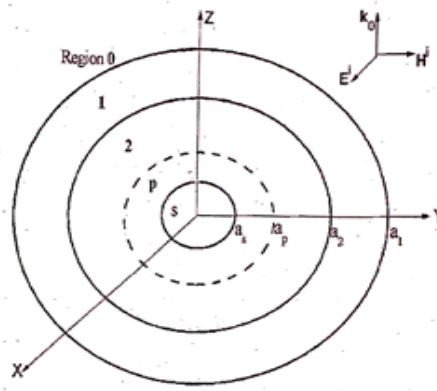
In recent years, there have been growing interests in characterizing interactions between electromagnetic fields and composite materials and especially anisotropic media, primarily because of its wide applications in the design and analysis of various novel antenna and microwave devices of high performance. Specifically, numerical approaches based on integral equations [1], [2] as well as differential equations [3] were developed and the analytical methods based on eigenvector wave functions [4] have been also adopted to analyze this kind of problems. Although the efforts were primarily spent in the past on two-dimensional (2-D) geometries, some progresses have been made in the characteristic analysis of three-dimensional (3-D) anisotropic scatterers using the method of (MoM) [5], [6] combined with the field (surface) integral equation (CFIE) formulation [7], the couple dipole approximation method [8], the integral equation [9], the field expansion method [10], [11], and the spectrum domain Fourier transform approach [12] - [16]. In some earlier works of the authors, electromagnetic fields scattered by a plasma anisotropic sphere [15], an uniaxial anisotropic sphere [16], and a plasma anisotropic spherical shell [17], were obtained based on the eigenfunction expansions in terms of spherical vector wave functions in [15], [16]. This work will extend those earlier research to a multilayered plasma anisotropic sphere. Although the analytical solution procedure is the same as before, the formulation to be made here is more generalized and more complicated. Hence, a detailed formulation is still necessary for the three-dimensional plane wave scattering by the multilayered plasma anisotropic sphere. After the analytical solution is

obtained, we also present numerical results for physical insight into the problem. To validate our formulation and code, we consider a comparison of the results obtained using the MOM with CG-FFT algorithm and our results newly obtained using the analytical approach and a good agreement between the two sets of results is demonstrated.

## 2. Formulation

Consider a geometry depicted in Fig. 1 which shows a cross-sectional view of an inhomogeneous plasma anisotropic sphere. The sphere can be divided into  $(s-1)$  homogeneous spherical layers and a core sphere whose radii from outside to inside are  $a_1, a_2, \dots, a_s$  and where  $(s+1)$  regions are denoted as region 0, region 1, ..., and regions. In our formulation and later numerical calculations, region 0 ( $r \leq a_1$ ) and regions ( $r \leq a_s$ ) are assumed to be free spaces. In the subsequent analysis, a time dependence of the form  $\exp(-i\omega t)$  is assumed for the electromagnetic field quantities but is suppressed throughout the treatment.

This composite structure is illuminated by a plane wave which is assumed to have an electric-field amplitude equal to unity, to be polarized in parallel to  $\hat{X}$ , and to propagate in the  $+\hat{Z}$  direction. The electrical vector wave equations in such a



**Fig. 1 Geometry for plane wave scattering by a plasma anisotropic sphere of  $(s-1)$ -layers.**

source-free inhomogeneous plasma anisotropic medium can be written in the following form

$$(1) \quad \nabla \times \nabla \times E_p - \omega^2 \bar{\epsilon}_0 \mu_0 E_p = 0, P = 1, 2, \dots, s-1$$

where  $E$  denotes the electric field, and  $\mu_0$  represents the free-space permeability while, (where  $p = 1, 2, \dots, s-1$ ) identifies the permittivity tensor of the  $p$ -th plasma anisotropic medium. If an external constant magnetic field is applied in the  $+z$ -direction, the permittivity tensor  $\bar{\epsilon}_p$  takes the following form -[15] :

$$(2) \quad \bar{\epsilon}_p = \begin{bmatrix} \epsilon_{1p} & -\epsilon_{2p} & 0 \\ \epsilon_{2p} & \epsilon_{1p} & 0 \\ 0 & 0 & \epsilon_{3p} \end{bmatrix} \quad p = 1, 2, \dots, s-1$$

By using Fourier transformation], [4], [9], [21]-[16], the expansion of plane wave factors in spherical vector wave functions in isotropic medium [18], and the characteristics of spherical Bessel functions, the electromagnetic fields (designated by the subscript  $p$ ,  $p = 1, 2, \dots, s-1$ ) in homogeneous plasma anisotropic spherically layered regions can be obtained [15]

According to the radiation condition of an outgoing wave (attenuated to zero at infinity) and the asymptotic behavior of spherical Bessel functions, only  $h$  should be retained in the radial functions, therefore the expansion of scattered fields (designated by the superscript SC) are

$$(3) \quad E^{sca} = \sum_{mn} \left[ A_{mn}^{SCA} M_{mn}^{(3)}(r, K_0) + B_{mn}^{SCA} N_{mn}^{(3)}(r, k_0) \right]$$

$$(4) \quad H^{sca} = \frac{k_0}{i\omega\mu_0} \sum_{mn} \left[ A_{mn}^{SCA} M_{mn}^{(3)}(r, K_0) + B_{mn}^{SCA} N_{mn}^{(3)}(r, k_0) \right]$$

Where the coefficients,  $A_{mn}^{SCO}$  and  $B_{mn}^{SCO}$  (whereas  $n$  varies from 0 to  $+\infty$  while  $m$  changes from  $-n$  to  $n$ ), are unknowns, again  $M_{mn}^{(1)}(r, k_0)$  and  $N_{mn}^{(1)}(r, k_0)$  denote the spherical vector, wave functions and  $k_0 = \omega(\epsilon_0\mu_0)^{1/2}$  identifies the wave number in free space respectively.

Because the region  $s$  represents free space and electromagnetic fields are finite at the coordinate origin, the fields (designated by the subscript  $s$ ) can be therefore expanded and expressed in terms of spherical vector functions as follows:

$$(5) \quad E_s = \sum_{mn} \left[ A_{mn}^S M_{mn}^{(1)}(r, K_0) + B_{mn}^S N_{mn}^{(1)}(r, k_0) \right]$$

$$(6) \quad H, \frac{k_0}{i\omega\mu_0} \sum_{mn} \left[ A_{mn}^S N_{mn}^{(1)}(r, K_0) + B_{mn}^S N_{mn}^{(1)}(r, k_0) \right]$$

The coefficients of A and B are unknowns to be determined. The expressions of electromagnetic fields which are expanded in spherical vector wave functions in plasma anisotropic medium and isotropic medium in region p (where  $p = 0, 1, 2, \dots, s$ ) have been derived. The unknown coefficients of electromagnetic fields in region p can be determined by the tangential-component continuity of electromagnetic fields on the spherical surfaces of (s - 1)-layered plasma anisotropic sphere. The boundary conditions (where  $p = 1, 2, \dots, s$ ) at  $r = a_p$  are written as

$$(7) \quad E_{p-1,t} = E_{pt}$$

$$(8) \quad H_{p-1,t} = H_{pt}$$

From the boundary conditions, we obtain the following :

$$(9) \quad \bar{Z}_p, X_p = \begin{cases} [J, I], & r = a_1 (p = 1); \\ 0, & r = a_s (p = s - 1) \end{cases}$$

$$(10) \quad \bar{Z}_p = \begin{bmatrix} Z_{11}^p & Z_{12}^p & Z_{21}^p & Z_{22}^p \\ Y_{11}^p & Y_{12}^p & Y_{21}^p & Y_{22}^p \end{bmatrix}$$

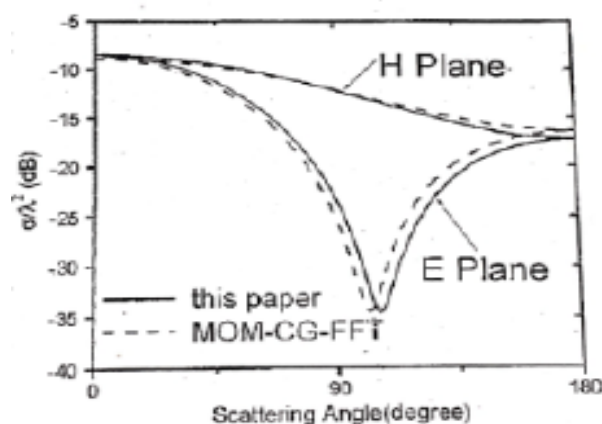


Fig. 2 RCSs versus scattering angle  $\theta$  (in degrees) : results of this paper (solid curve) and of MoM with CG-FFT fast algorithm (dashed curve)

### 3. Numerical Results and Discussion

In the previous section, we have presented the necessary theoretical formulation of the electromagnetic fields of a plane wave scattered by an inhomogeneous plasma anisotropic sphere of multilayers. To gain more physical insight into the problem, we will provide in this section some numerical solutions to the problem of electromagnetic scattering by an (s-1) layered homogenous plasma sphere of multilayers whose radii are assumed to be  $a_1, a_2, \dots, a_p, \dots, a_s$  from outside to inside.

Numerical computations are performed by applying the theoretical formulae developed earlier in the previous sections. In order to check the accuracy of the newly obtained numerical results, we perform two trials. Firstly, we calculate the radar cross sections using the present method of a single-layered homogeneous plasma anisotropic shell and the MoM speeded up with the CG-FFT technique [6]. The results are shown in Fig.2 where the electric dimensions are chosen as  $k_0 a_1 = 0.3\pi$  and  $k_0 a_2 = 0.2\pi$ ,  $k_0 a_2 = 0.2\pi$ , while the permittivity tensor elements are assumed to be  $\epsilon_1 = 7\epsilon_0$ ,  $\epsilon_2 = i\epsilon_0$ , and  $\epsilon_3 = 5\epsilon_0$  (where and subsequently,  $\epsilon_0$  stands for the free space permittivity).

From Fig. 2, it is seen apparently that the radar cross sections calculated by using the two methods (i.e., the present method in this paper and the MoM with the CG-FFT fast algorithm) are in a very good agreement in both the E- and H- planes. Second, in Fig. 3, we calculate the two-layered plasma spherical shell where the parameters of the two layered plasma media are the same and then we use one-layered theory of this method for computing the radar cross sections where the electric dimensions are chosen as  $k_0 a_1 = \pi$ ,  $k_0 a_2 = 0.9\pi$ ,  $k_0 a_3 = 0.8\pi$  while the permittivity tensor elements are assumed to be  $\epsilon_{11} \epsilon_{12} = 3, \epsilon_0$ ,  $\epsilon_{21} = \epsilon_{22} = 1.5 i\epsilon_0$ , and  $\epsilon_{31} = \epsilon_{32} = 2\epsilon_0$ . The results (the single-layered and the double-layered plasma spherical shells) are in a very good agreement in both E-and H-planes. From these two figures (4, 5), it partially verifies the correctness and applicability of our theory as well as the program codes.

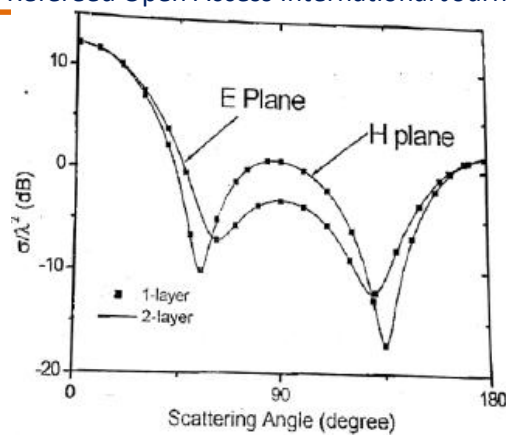


Fig. 3 RCSs versus scattering angle  $\theta$  (in degrees) in the E and H planes. Comparison of RCSs between a single layered plasma shell and a 2-layered plasma shell where the medium parameters are similar.

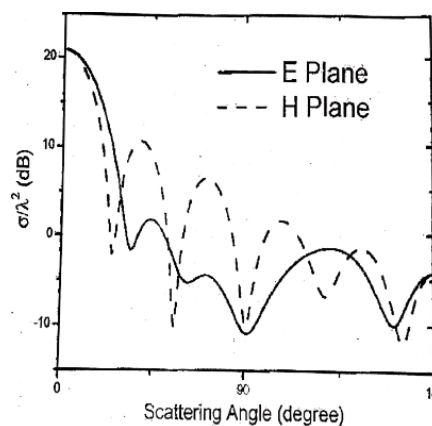


Fig. 4 RCSs of a 5-layered plasma spherical shell versus scattering angle  $\theta$  (in degrees) in the E-plane (solid curve) and in the H-plane (dashed curve)

After this, we obtain some new results unavailable elsewhere in literature. Two examples are considered herein, and their radar cross sections are plotted in Figs. 4 and 5.

Fig. 4 presents radar cross sections of a 5-layered homogeneous plasma sphere of more general plasma material, where the permittivity tensor elements are characterized by various values. The maximum number  $n'$  in [14]-[18] to achieve good convergent results is found to be 10. To illustrate further applicability of the scattering solution to an electrically large sized 8-layered plasma anisotropic spheres (for example, in its resonance region), the radar cross

sections of a relatively large plasma anisotropic sphere under the illumination by an incident plane wave, are obtained and depicted in both the E-plane and

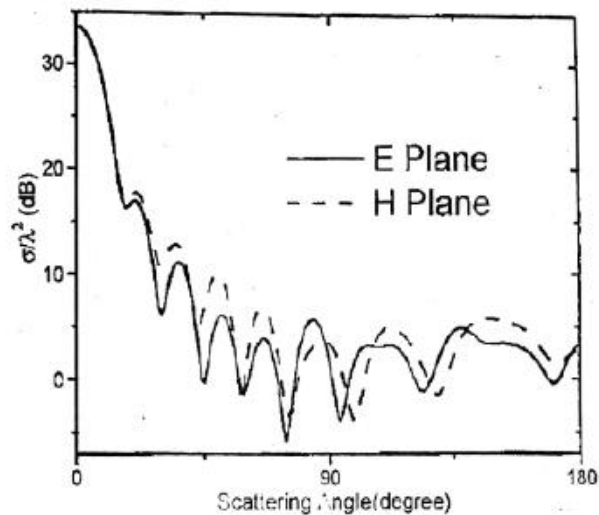


Fig. 5 RCSs of an 8-layered plasma spherical shell versus scattering angle  $\theta$  (in degrees) in the E-plane (solid curve) and in the H-plane (dashed curve). the H-plane in Fig. 8.5. As the electric dimension of the sphere is increased.

It is seen from the above figures and discussions that the radar section crosses (RCSs) ( $\sigma/\lambda^2$ ) in both the E- and H-planes vary with the angles of incidence. For a small-sized plasma anisotropic spherical shell, the RCS in the H-plane decays slightly with the scattering angle from  $0^\circ$  to  $180^\circ$  while it changes drastically with the scattering angle from  $0^\circ$  to  $180^\circ$  in the E-plane (decaying rapidly until  $90^\circ$  and then increasing quickly back to the same power level of its H-plane counterpart at  $180^\circ$ ). For a medium-sized plasma anisotropic sphere, the RCSS in both the E- and H-planes vary oscillatingly with amplitude decreasing with the scattering angle. For a large-sized plasma anisotropic sphere, the RCSS in the E- and H-planes almost have the same main beam width.

#### 4. Discussion

Solutions to scattering of plane waves by an inhomogeneous plasma sphere are derived by employing the eigenfunction expansions of the fields in terms of spherical vector wave functions. The inhomogeneous plasma sphere is divided into  $(s-1)$  layered homogeneous plasma anisotropic spherical regions in the formulation. It is found that the solutions can be obtained and expressed using only one- dimensional integrals which can be evaluated easily. Numerical results are then yielded from the new formulas and found to agree very well with



these obtained using the MoM accelerated with CG-FFT technique, as expected. Some general numerical results are demonstrated, for instance, in the cases of lossy plasma sphere and resonance region.

## 5. References

1. Al-Kanhal, M.A. and E. Arvas (1996), IEEE Trans. Antenna's Propagat., vol. 44, p. 1041-1048.
2. Zhang, M, and W.X. Zhang (1995), "Microwave eu Opt. Technol. Lett., Vol. 10, No. 1 22-25.
3. Manzon, J. C. (1995) "IEEE Trans. Antennas Propagat., Vol. 43, 1288-1296.
4. Olyslager, F. (1997) "Time-harmonic two and three dimensional closed-form Green's dyadic for gyrotropic, bianisotropic and anisotropoc media, 'Electromagn., Vol, 17, No. 4, 369-386.
5. Zhang, M. and W. Hong. 1997, Proc. IEEE Antennas Propagat. Sco. Int. Synip., 910-913, Montreal Canada.
6. Cheng, D. J. (1997). Phys. Rev. E., Vol. 56, No. 2, 2321-2324.
7. Yin, W.Y. and L.W., Li, (1999). Phys. Rev. E., Vol. 60, No. 1, 918-925.
8. Shanker, B., S.K. Han, and E. Michielssen, (1998) Radio Sci., vol. 33, No. 1, 17-31, 1998.
9. Hafner, C. (1991) The Generalized Multiploe Technique for Computational Electromagnetics., Artech House, Inc.
10. Leviatan, Y. et. al. IEEE Trans. Microwave Theory Tech., Vol. 31, 806-811.
11. Leviatan., l and A. Boag (1987) IEEE Trans. Antennas Propagat., Vol. 35, 1119-1127.
12. Leviatan, Y., Am. Boag and Al. Boag, (1988) IEEE Trans. Antennas Propagat., Vol. 36, 1722-1734.
13. Cheng, C. H. (1993) "GMT/SDT for underground EM scattering" Ph.D. dissertation, Southeast University, China.



14. Na, H.G. and H.T. Kim (1995) IEEE Trans. Antennas Propagat., Vol. 43, 426-430.
15. Zhang, M. and I. Shu (1995) Microwave & Opt. Technol. Lett., Vol. 10, No. 6, 363-365.
16. Na, H.O. and H.T. Kim (1996) IEE. Proc. H, Vol. 143 No. 2, 163-168.
17. Kang, T.W. and H.T. Kim, (1999) IEEE Trans. Antennas Propagat., Vol. 47, No. 6, 1118-1120.
18. Beker, B., K.R. et. al. (1989) IEEE, Trans. Antennas Propagat., Vol. 37, No. 12, p. 1573-1581.

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