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A Non-Static Magnetohydrodynamic Universe in General Relativity

Sudhir Kumar
Research Scholar
Department of Physics
Jai Prakash University, Chapra

ABSTRACT:

The Present paper provides a study on Bianchi type-I non-static magnetohydrodynamic universe with perfect fluid. Taking a suitable metric we have also found and discussed several physical and geometric properties e.g. pressure, density, scalar of expansion and shear tensor. It is found that the electric field, the four current, the fluid density, pressure and scalar of expansion all start with infinite values at the initial singularity (t = 0) and tend to zero when $t \to \infty$.

Key Words : Non-Static, perfect fluid, metric, pressure, density, expansion.

1. Introduction

Various researchers have focussed their mind on the study of the cosmological models in the presence of electromagnetic field in General Relativity. Before starting the work a general idea of magneto-hydrodynamics (MHD) is essential. Infact Magnetohydrodynamics is the study of the motion of an electrically conducting field in the presence of a magnetic field. Electric currents induced in the fluid as a result of its motion modify the fields, at the same time their flow in the magnetic field produces mechanical forces which modify the motion. Magnetohydrodynamics owes its peculiar interest and difficulty to the interaction between the field and the motion. It is well known that galaxies and interatellar space exhibit the presence of strong magnetic field [22] which imparts a sort of viscous effect to the fluid flow. We may assume that in a smaller scale, magnetic field assume an important role for the universe. The behavior of the magnetic field of a star was investigated on the assumption that the field variations caused by the extinction do not create any motion of the stellar matter (Lamb [7]), Wrubel [19]. The important results obtained make the possible interpretation of magneto-hydrodynamical processes in stars only quite roughly. When motions of stellar matter caused by the electromagnetic force are taken into account, new properties may be revealed and the non stability of the magneto-hydrodynamical processes in stars may be studied.

A cosmological model in presence of magnetic field has been studied by Zeldovich [21] and later by Thorne [15]. Shikin [13] also constructed a uniform axially symmetric solution (model) of Einstein's gravitational equation and Maxwell's equations in the case of propagation by a completely ideal substance in the presence of magnetic field directed along the axis of symmetry. Magnetic field in stellar bodies was also discussed by Monoghan [10]. Gravitational collapse of a magnetic star was studied by Ginzburg [2]. Seymour [12] also derived some models of the galactic magnetic field. Jacoba [5, 6] has studied the behavior of the general Bianchi-type I cosmological model in the presence of a spatially homogeneous magnetic field. This problem has been studied again by De [1] with a different approach. This work has been further extended by Tupper [17] to include Einstein-Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI cosmologies with electromagnetic field (Tupper [18]). Roy and Prakash [11], taking the cylindrically symmetric metric of Marder [9], have constructed a spatially homogeneous cosmological model in the presence of an incident magnetic field which is also anisotropic and non-degenerate petrov type-I. Singh and Yadav [14] constructed a spatially homogeneous cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. Some other workers in this field are Hoyos et al. [4a] Roy et. al. [1] (a)] Rahman et. al. [11 (b)], Yadav and Kumar [20] Sharm et. al. [12(a)].

In this paper we have found that a non-static cylindrically symmetric cosmological model with perfect fluid and electromagnetic field. The requirement that the conductivity be positive imposes an additional condition on the metric functions. Taking a suitable metric we have also calculated various physical and geometrical properties e.g. pressure, density, scalar of expansion and components of shear tensor. It is found that the electric field, the four current, the field density, the pressure and scalar of expansion all start with infinite values at the initial singularity (t = 0) and tends to zero when t tends to infinity.

2. The Field Equations and Their Solutions

We consider the most general cylindrically symmetric space-time in the form given by

(2.1) $$ds^2 = A^2 (dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2$$

where the metric potentials A, B, C are functions of time t alone. The distribution consists of a perfect fluid and an electromagnetic field. The energy momentum tensor of the composite field is assumed to be the sum of the corresponding energy momentum tensors. Thus

(2.2) $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \wedge g_{\mu\nu} = -k\left[\left(\rho + P\right)\right] u_\mu u_\nu - P g_{\mu\nu} + E_{\mu\nu}$$

(2.3) $E_{\mu\nu} = -g^{k\ell} F_{\mu\nu} F_{\mu\nu\ell} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$

(2.4) $E_{[\mu\nu.\sigma]} = 0$

(2.5) $F^{\mu\nu}_{;\nu} = J^{\mu}$

where p and ρ pressure and density respectively of the distribution, $E_{\mu\nu}$ is the electromagnetic energy momentum tensor, $F_{\mu\nu}$ is the electromagnetic field tensor, J^{μ} is the current 4-vector, ^ is the cosmological constant and u_{μ} is the flow vector satisfying

(2.6) $g_{\mu\nu} u_{\mu} u_{\nu} = 1$

The co-ordinates are chosen to be commoving so that

(2.7) $u^{\mu} = \left(0, 0, 0 \frac{1}{A}\right)$

and we label the co-ordinates

$$(X, Y, Z, t) \equiv (X^1, X^2, X^3, X^4)$$

We assume the electromagnetic field to be in the direction of X-axis so that F_{14} and F_{23} are the only non-vanishing components of the field tensor $F^{\mu\nu}_{;\nu}$. We write

(2.8) $F^2_{14} A^{-4} + F^2_{23} B^{-2} C^{-2} = M^2$

The equation (2.2) may be written as

(2.9) $\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4 C_4}{AC} - \frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2}\right] - 2\wedge$

$$= -K[M^2 + (\rho + 3P)],$$

(2.10) $-\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} - \frac{A_4^2}{A^2}\right] + 2\wedge = -K[-M^2 + (\rho - P)]$

(2.11) $-\frac{2}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4 C_4}{BC}\right] + 2\wedge = -K[M^2 + (\rho - P)],$

(2.12) $-\frac{2}{A^2}\left[\frac{B_{44}}{C} + \frac{B_4 C_4}{BC}\right] + 2\wedge = -K[M^2 + (\rho - P)],$

where the suffix 4 after the symbols A, B, C denotes ordinary differentiation w.r.t. time t. These equations show that M^2, ρ, P are each functions of t only, and it then follows from equations

(2.4) and (2.8) that F_{23} is a constant and F_{14} is a function of t only i.e.,

(2.13) $$F_{23} = k, F_{14} = \pm A^2(M^2 - k^2B^{-2}C^{-2})^{1/2}$$

where k is a constant.

The case when $F_{14} = 0$, which implies $J^\mu = 0$, we get the model due to Roy and Prakash [11]. We here assume that $F_{14} \neq 0$, and find the only non-vanishing components of J^μ to be

(2.14) $$J^1 = \pm \frac{1}{A^2BC}\frac{\partial}{t}\left[BC(M^2 - k^2B^{-2}C^{-2})^{1/2}\right]$$

Equation (2.14) shows that J^μ is spacelike, unless $M^2 = \ell B^{-2}\, C^{-2}$ where ℓ is a constant in which case $J^\mu = 0$. The 4-current J^μ is in general the sum of the sum of the convection current and conduction current (Licknerowioz [8] and Greenburg [3]).

(2.15) $$J^\mu = \sigma_0 u^\mu + \lambda_{\mu\nu}F^{\mu\nu}$$

where σ_0 is the rest charge density and λ is the conductivity. In the case considered here we have $\sigma_0 = 0$ i.e., magnetohydrodynamics. From equations (2.13), (2.14) and (2.15) we find that the conductivity is given by

(2.16) $$\lambda = -\frac{1}{A}D_4D^{-1}$$

where $D = BC(M^2 - k^2B^{-2}C^{-2})^{1/2}$

The requirement of positive conductivity in (2.16) put further restrictions on A, B, C. Hence in the magnetohydro-dynamics case metric function are restricted not only by the field equations and energy conditions (Hawking and Penrose [14]) they are also restricted by the requirement that the conductivity be positive for a realistic model.

Finally we illustrate the situation described here by an example. Consider the space time with metric.

(2.17) $$ds^2 = t^{4a^2}(dx^2 - dt^2) + t^{2a}(dy^2 + dz^2)$$

Which is obtained from metric (2.1) when

$$A = t^{2a^2},\ \ B = C = t^a,$$

Where a being an arbitrary constant parameter. From field equation we get

(2.18) $$L^2 = a(4a+1)(a-l)t^{-4a^2-2}$$

(2.19) $\rho = \frac{1}{2}a(-4a^2 + 3a - 1)t^{-4a^2-2} + \wedge$

(2.20) $p = \frac{1}{2}a(4a^2 + a - 3)t^{-4a^2-2} - \wedge$

Clearly M^2, ρ, p all are decreasing function of time.

From equation (2.18)

(2.21) $0 > a > -\frac{1}{4}$

Which implies that ρ > 0 and p > 0 (when $\wedge = 0$)

Also we find

(2.22) $\frac{3}{2} < \frac{p}{\rho} < 3$

So that the equation of state for the possible values of a, may range from that ρ = p/3 to ρ =(2/3) p, but can correspond to either of these limits only when the electromagnetic field vanishes. When $a = \pm 1/\sqrt{2}$, then it is the case of disordered radiation i.e., ρ = 3p. Again, when i.e. $a = 1/B \pm 7/8$, then it is the case of stiff matter i.e., g = p.

The electromagnetic field components are

(2.23) $F_{23} = k,\ F_{14} = \pm t^{4a^2-2a} t^{-4a^2+4a-2} a(4a+1)(a-1) - k^2)^{1/2}$

and the magnitude of the magnetic field is restricted by

(2.24) $k^2 < a(a-1)(4a+1)t^{-4a^2+4a-2}$

The non zero component of the current four vactor is

(2.25) $J^1 = +\frac{1}{2}a(a-1)(4a+1)(4a^2 - 2a + 2)t^{-8a^2+2a-3}$

$$[a(a-1)(4a+1)t^{-4a^2+4a-2} - k^2]^{-1/2}$$

Also the conductivity for the model is given by

(2.26) $\lambda = \frac{1}{2}a(a-1)(4a+1)(4a^2 - 4a + 2)t^{-6a^2+4a-3}$

$$[a(a-1)(4a+1)t^{-4a^2+4a-2} - k]^{-1}$$

Scalar of expansion defined by

(2.27) $\Theta = u^{\mu}_{;\mu}$

is given by

(2.28) $\Theta = 2at^{-2a^2-1}[1-a]$

Hence the electric field, the four current, the fluid density, the pressure and scalar of expansion all start with infinite values at the initial singularity (t = 0) and tend to zero.

The components of the shear tensor defined by

(2.29) $\sigma_{\mu\nu} = \frac{1}{2}(u\mu,\nu + \mu_{\nu,\mu}) - \frac{1}{3}\Theta(g_{ij} - u_{\mu}u_{\nu})$

are

(2.30) $\sigma_{11} = 2a(2a-1)t^{2a^2-1}$

$\sigma_{22} = \sigma_{33} = a(2a-3)t^{2a^2+2a-1}$

$\sigma_{44} = 2a(1-a)t^{2a^2-1}$

3. Conclusion

The requirement that the conductivity be positive puts an additional condition on metric potentials. Again we find that the electric field, the four-current, the fluid density, the pressure and scalar of expansion all start with infinite values at the initial singularity (t= 0) and tends towards zero when time $t \to \infty$. Further our investigation is useful in the study of magneto-hydrodynamical processes in stars.

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