
Some Non-Static Conformally Flat Solutions of Einstein-Maxwell Fields with Spherical Symmetry

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Abstract :

The present paper deals with conformally flat non-static solutions for spherically symmetric charged perfect fluid distribution under different conditions. In one case we have discussed the solution when field tensor component F_{14} vanishes while in other case we have taken $F_{14} \neq 0$ and using commoving coordinates i.e. $u_1 = 0$. We have found and discussed various physical and geometrical properties.

Key Words : Conformally flat, non-static, perfect fluid, field tensor, spherical symmetry.

1. INTRODUCTION

Many relativists have shown their keen interest in study of conformally flat metrics. In 1971 Buchdahl [1] and later on in 1977 Gurese [7] have shown that the only static distribution of the fluid with positive density and pressure which would generate a conformally flat metric through the Einstein's equations without cosmological term is that described by the Schwarzschild interior solution. Burman [2] discussed the motion of the particles in conformally flat space time. Singh and Abdussattar [17] have obtained non-static generalization of the Schwarzschild interior solution which is conformal to flat space time. Shikin [16] and Zalcev and Shikin [23] have obtained conformally flat non-static solution in general relativity and scalar tensor theories of gravitation.

Roy and Raj Bali [14] have obtained the solutions of Einstein's field equations representing non-static spherically symmetric perfect fluid distribution which is conformally flat. Chang [4] has found some conformally flat interior solutions of the Einstein Maxwell equations for charged stable static sphere. These solutions satisfy physical conditions inside the spheres.

Kallyanshetti and Waghmode [10] have considered the static conformally flat spherically symmetric perfect fluid distribution in the framework of Einstein-Cartan theory and obtained the field equations. They have solved these field equations and discussed the reality conditions for

their solutions. They have observed that density ρ will not be constant as observed by Narlikar [12] for conformally flat spherically symmetric perfect fluid distribution. But they have shown that $\bar{\rho} - (\rho - 2\sigma k^2)$ will be constant. Yadav and Prasad [20] have obtained general solution representing conformally flat, non-static spherically symmetric perfect fluid distribution with spin. Yadav and Sinha [21], [22] have also found some solutions for static charged dust sphere with conformal flatness. Some other workers in this field are Coley and Tupper [3], Hansraj [8], Moopnar and Maharaj [11], Shee D. et al. [15], Tupper et al. [19], Pandya et al. [24], Rao and Reddy [19], Gupta [6] and Ishikawa [9].

This paper provides some non-static conformally flat spherically symmetric solutions with perfect charged fluid distribution under different conditions. In case I, the solution has been found when field tensor component F^{14} vanishes and followed by two subcases (i) when $u_1 \neq 0$ and (ii) when $u_1 \neq 0, A_{14} = 0$, In case II solution is found using commoving coordinates i.e. $u_1 = 0$ but $F_{14} \neq 0$. Different physical and geometrical features have been also found and discussed.

2. THE FIELD EQUATIONS

Here take the metric in the form given by

$$(2.1) \quad ds^2 = e^A (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi - dt^2)$$

Where $A = A(r, t)$

For charged fluid distribution the Einstein Maxwell equations are

$$(2.3) \quad G_{\mu\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}, G_{\alpha\beta} + \Lambda G_{\alpha\beta} = -8\pi \bar{T}_{\alpha\beta}$$

$$(2.4) \quad [(-g)^{1/2} F^{\alpha\beta}]_{;\beta} = 4\pi J^\alpha (-g)^{1/2}, F_{[\alpha\beta,\gamma]} = 0$$

where Λ is cosmological constant, \bar{T}_{ij} is the energy momentum tensor, J^i is the current four vector and $R_{\alpha\beta}$ is the Ricci tensor and R the scalar of curvature tensor and $G_{\alpha\beta}$ is Einstein tensor.

For the system under study the energy momentum tensor \bar{T}_β^α has two parts viz. T_β^α and E_β^α for matter and charges respectively i.e. $\bar{T}_\beta^\alpha = T_\beta^\alpha + E_\beta^\alpha$

$$(2.5) \quad T_{\beta}^{\alpha} = [(e + p)u^{\alpha}u_{\beta} - \delta_{\beta}^{\alpha}p]$$

$$(2.6) \quad g_{\alpha\beta}u^{\alpha}u^{\beta} = -1$$

The electromagnetic energy momentum tensor E_{β}^{α} in terms of field tensor $F_{\alpha\beta}$ is given by

$$(2.7) \quad E_{\beta}^{\alpha} = -F_{\beta\mu}F^{\alpha\mu} + \frac{1}{4}\delta_{\beta}^{\alpha}F_{\mu\nu}F^{\mu\nu}$$

Due to spherical symmetry the only vanishing components of field tensor $F^{\alpha\beta}$ are F^{14}

Also we have

$$(2.8) \quad R = e^{-A} \left[3A_{11} + \frac{3A_1^2}{2} + \frac{6A_1}{r} - 3A_{44} - \frac{3A_4^2}{2} \right]$$

The field equation are given by

$$(2.9) \quad \frac{3A_1^2}{4} + \frac{2A_1}{r} - A_{44} - \frac{A_4^2}{4} - Ae^A = 8\pi \{ (\rho + p)u_1^2 + pe^A - e^{-A}(F_{14})^2 \}$$

$$(2.10) \quad A_{11} \frac{A_1^2}{4} + \frac{A_1}{r} - A_{44} - \frac{A_4^2}{4} - Ae^A = 8\pi \{ (\rho + p)u_1^2 + pe^A - e^{-A}(F_{14})^2 \}$$

$$(2.11) \quad -A_{11} \frac{A_1^2}{4} + \frac{2A_1}{r} + \frac{3A_4^2}{4} + Ae^A = 8\pi \{ (\rho + p)u_4^2 - pe^A - e^{-A}(F_{14})^2 \}$$

$$(2.12) \quad -A_{14} + \frac{A_1A_4}{2} = 8\pi(\rho + p)u_1u_4$$

Equation (2.6) is turned into

$$(3.2.13) \quad u_4^2 - u_1^2 = e^A$$

In the above suffices 1 and 4 after the symbols denote differentiation with respect to r and t respectively.

3. SOLUTIONS OF THE FIELD EQUATION

From equations (2.9) and (2.10), we have

$$(3.1) \quad 8\pi \{ (\rho + p)u_1^2 - 2e^{-A}(F_{14})^2 \} = \frac{A_1^2}{2} + \frac{A_1}{r} - A_{11}$$

From equation (2.10) and (2.11), we have

$$(3.2) \quad 8\pi \left\{ (\rho + p)u_1^2 + 2e^{-A} (F_{14}^2)^2 \right\} = \frac{A_4^2}{2} - \frac{A_1}{r} - A_{44}$$

Equations (2.13), (3.1) and (3.2) lead no

$$(3.3) \quad 8\pi \left\{ (\rho + p)e^A 4E^{-A} (F_{14})^2 \right\} = \frac{A_4^2}{2} - \lambda_{44} - \frac{A_1^2}{2} + \lambda_{11} - \frac{2A_1}{r}$$

From equations (2.10) and (3.3) we have

$$(3.4) \quad 8\pi \left\{ \rho + 3F_{14}^2 \right\} = e^{-A} \left[\frac{3A_4^2}{4} - \frac{3A_1^2}{4} - \frac{3A_1}{r} \right] + \wedge$$

Now we attempt the solution in different flowing cases

Case I : When field tensor compound F^{14} vanishes and

- (i) when $u_1 \neq 0$
- (ii) when $\mu_1 \neq 0$ and $F_{14} = 0$

Case II : Solution using co-moving coordinates

i.e. when $u_1 = 0$ and $F_{14} \neq 0$

Case I : The field tensor component F^{14} vanishes and (i) when $u_1 \neq 0$ then using equation (3.1),

(3.2) and (2.12) we get

$$(3.5) \quad \frac{A_1}{2r} \left(\frac{2A_1}{r} + A_1^2 - A_4^2 \right) = A_{11} \left(\frac{A_1}{r} - \frac{A_4^2}{2} \right) + A_{14}A_1A_4 + A_{44} \left(\frac{-A_1^2}{2} - \frac{A_1}{r} \right)$$

Using Monge's method, solution of (3.5) is found to be

$$(3.6) \quad \exp(A) = 16\mu^2 \left[\phi(\mu(r^2 - t^2) - t) - vt \right]^{-2}$$

Where μ and v are constants.

Hence the metric (2.1) takes the form

$$(3.7) \quad ds^2 = 16\mu^2 \left[\phi(\mu(r^2 - t^2) - t) - vt \right]^{-2} \left(dr^2 + r^2 d\theta^2 + r^2 + r^2 \sin^2 \theta d\phi^2 - dt^2 \right)$$

The solution obtained by Singh and Abdussattar [28] is a particular case of (3.7) with $v = 0$. Also when $v = 0$, the metric (3.7) transforms to Robertson Walker metric of constant negative curvature.

The pressure and density for the mode (3.7) are given by

$$(3.8) \quad 8\pi p = \left[2\phi''(\phi - vt) - 3\phi'^2 \right] \left[\frac{1 - 4\mu(\mu(r^2 - t^2) - t) - \Lambda}{16\mu^2} \right]$$

$$- \left[3v^2 + 12\mu\phi'(\phi - vt) + 6v(2\mu t + 1) \right] / 16\mu^2$$

$$(3.9) \quad 8\pi\rho = \frac{1}{16\mu^2} \left[\frac{3\phi'(\phi - vt)}{4\mu} + 6v(2\mu t + 1)\phi' + 3(1 - 4\mu(\mu(r^2 - t^2) - t)) \right] + \Lambda$$

Where a prime denotes differentiation with respect to its argument.

The non-zero components of flow vector are u_1 and u_4 given by

$$(3.10) \quad u_1 = \frac{8\pi\mu^2 r}{(\phi - vt)G}$$

$$(3.11) \quad u_4 = \frac{4\pi(2\mu t + 1)}{(\phi - vt)G}$$

The first reality condition $e + p > 0$ gives

$$(3.12) \quad G^2(\phi - vt)\phi'' > 0$$

where $G = [1 - 4\mu(\mu(r^2 - t^2) - t)]^{1/2}$

since $G^2 > 0$, we have from equation (3.12) that $(\phi - vt)$ and ϕ'' should have the same sign.

Also the second reality conditions $\rho + 3p > 0$ yields

$$(3.13) \quad > 3 \left[\left(\frac{G}{4\mu} \right)^2 \left\{ \phi''(\phi - vt) - \phi'^2 \right\} + \left(\frac{v}{4\mu} \right)^2 \right] - \frac{3}{8\mu^2} [2\mu\phi'(\phi - vt) + v(2\mu t + 1)\phi']$$

The expression for expansion θ , rotation w_{ij} and shear σ_{ij} are given by

$$(3.14) \quad \theta = \frac{3}{4\mu G} \left[2\mu(\phi - vt) + \phi'(G^2) - 2v(2\mu t + 1) \right]$$

$$(3.15) \quad w_{14} = \frac{2\mu^2 rv}{(\phi - vt)^2 G}$$

$$(3.16) \quad \sigma_{11} = \frac{8\mu v(2\mu t + 1)(G^2 + 4\mu^2 r^2)}{(\phi - vt)^2 G^3}$$

$$\sigma_{22} = \sin^2 \theta \sigma_{33} = \frac{8\mu v r^2 (2\mu t + 1)}{(\phi - vt)^2 G^3}$$

$$\sigma_{44} = 4\mu v(2\mu t + 1) \frac{\{2(2\mu t + 1)^2 - G^2\}}{(\phi - vt)^2 G^3}$$

$$\sigma_{14} = \frac{4\mu^2 vt \{4(2\mu t + 1)^2 + G^2\}}{(\phi - vt)^2 G^3}$$

It can also be seen that the flow vector u_i does not satisfy $u_{ij} u^i = 0$ in general which means that flow is non-geodetic in general. Hence the model is expanding, rotating, shearing but not-geodetic in general.

Case I

(ii) when $u_1 \neq 0$ and $A_{14} = 0$

In the case equation (3.5) goes to the form

$$(3.17) \quad A_{11} \left(\frac{A_1}{r} - \frac{A_4^2}{2} \right) + A_{44} \left[\frac{A_1^2}{2} - \frac{A_1}{r} \right] + A_{11} A_{44} = \frac{A_1}{2r} \left(A_1^2 - A_4^2 + \frac{2A_1}{r} \right)$$

Integration of $A_{14} = 0$ gives

$$(3.18) \quad A = \alpha(r) + \beta(t)$$

Equation (3.17) and (3.18) gives

$$(3.19) \quad -\frac{\alpha'}{r} \left(\frac{\alpha'}{r} - \alpha'' \right) \ddot{\beta} \left(\frac{\alpha'^2}{2} + \frac{\alpha'}{r} - \alpha'' \right) + \dot{\beta}^2 \left(\frac{\alpha''}{2} - \frac{\alpha'}{2r} \right)$$

Here dash and dot denote ordinary partial differentiation with respect to r and t respectively.

For the solution of (3.19). We consider two possible subcases

Sub case (a) : choose $\beta = gt^2 + ct + k$

where g, k, c are constants

sub case (b) : $\alpha'' = \frac{\lambda r^2}{2}, \ddot{\beta} = -\lambda, \lambda \neq 0$

where λ is a constant.

Sub case (a) : Here to avoid complexity, we take $g = 0$

So that $\beta = ct + k$ which on differentiating gives

$$(3.20) \quad \dot{\beta} = c$$

where c is a non-zero constant. Equation (3.19) and (3.20) provides

$$(3.21) \quad \left(\frac{\alpha'}{r}\right)' \left[\alpha' - \frac{c^2 r}{2}\right] = \frac{\alpha'}{r} \cdot \frac{\alpha'^2}{2}$$

Taking $\frac{\alpha'}{r} = y(r)$ in equation (3.21), we get

$$(3.22) \quad 2yy' - ry^3 = c^2 y'$$

Integrating (3.22), we get

$$(3.23) \quad y = \frac{\sqrt{4 + cz}}{z} - \frac{2}{z}$$

where $z = r^2 + D$, D being a constant of integration. Putting $y = \frac{\alpha'}{r}$ in equation (3.23). We have

$$(3.24) \quad d\alpha = \frac{-2rdr}{z} + \frac{r\sqrt{4 + cz}}{z}$$

Integrating (3.24), we have

$$(3.25) \quad \alpha - \sqrt{4 + cz} + \log \left[\frac{E\sqrt{4 + cz} - 2}{z\{\sqrt{ez + 4} + 2\}} \right]$$

E being a constant of integration. From (3.20) we get

$$(3.26) \quad \beta = ct + k$$

where k is a constant of integration. From (3.18), (3.25) and (3.26) we have

$$(3.27) \quad \exp(A) = \frac{E\{\sqrt{(4+cZ)} - 2\}}{Z\{\sqrt{4+cZ} + 2\}} \times \exp\left[ct + k + \sqrt{(4+cZ)}\right]$$

Therefore the metric (2.1) goes to the form.

$$(3.28) \quad ds^2 = \frac{E\{\sqrt{(4+cZ)} - 2\}}{Z\{\sqrt{(4+cZ)} + 2\}} \times \exp\left[ct + k + \sqrt{(4+cZ)}\right] \\ \times \left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - dt^2\right\}$$

The distribution in the model (3.28) is given by

$$(3.29) \quad 8\pi\rho = \frac{Z(\sqrt{4+cZ} + 2)}{E(\sqrt{(4+cZ)} + 2)} e^{-[ct+k+\sqrt{(4+cZ)}]} \\ \times \left[\frac{2r^2}{Z^2} - \frac{4D}{Z^2} + \frac{cr^2}{4Z} + \frac{D\sqrt{(4+cZ)}}{Z^2} + \frac{4(D-r^2) + CDZ}{\sqrt{(4+cZ)}.Z} \right]$$

$$(3.30) \quad 8\pi\rho = \frac{Z(\sqrt{(4+cZ)} + Z)}{(\sqrt{(4+cZ)} + 2)} e^{-[ct+k+\sqrt{(4+cZ)}]} \\ \times \left[\frac{3c^2}{4} + \frac{6D}{2Z} - \frac{3cr^2}{4Z} - \frac{3D\sqrt{4+cZ}}{Z^2} \right]$$

The reality condition $\rho + p > 0$ provides

$$(3.31) \quad \frac{CDZ + 4(D-r^2)}{2Z^2\sqrt{4+cZ}} - \frac{D\sqrt{4+cZ}}{Z^2} + \frac{c(c-1)r^2 + 4 + DC^2}{4Z} > 0$$

Condition $\rho + 3p > 0$ gives

$$(3.32) \quad \frac{3CDZ + 12(D-r^2)}{Z^2\sqrt{4+cZ}} + \frac{6(D-r^2)}{Z^2} + 2 \frac{E(\sqrt{4+cZ} - 2)}{Z(\sqrt{4+cZ} + 2)} e^{[ctg+k+\sqrt{4+cZ}]} > 0$$

The non-vanishing components of the flow vector are given by

$$(3.33) \quad u_1^2 = \frac{E(\sqrt{4+cz} - 2)}{Z(\sqrt{4+cz} + 2)} \exp(ct + k + \sqrt{4+cz}) \times \left[\left\{ \frac{c^2}{4} + \frac{1}{z} - \frac{\sqrt{4+cz}}{2z} \right\} \times \left\{ \frac{CDz + 4(D-r^2)}{2Z^2\sqrt{4+cz}} + \frac{C(c-1)r^2 + 4 + Dc^2}{4Z} - \frac{D\sqrt{4+cz}}{z^2} \right\}^{-1} - 1 \right]$$

and

$$(3.34) \quad u_4^2 = \frac{E(\sqrt{4+cz} - 2)}{Z(\sqrt{4+cz} + 2)} e^{[ct+kt+\sqrt{(4+cz)}]} \times \left[\left\{ \frac{c^2}{4} + \frac{1}{z} - \frac{\sqrt{4+cz}}{2z} \right\} \times \left\{ \frac{CDz + 4(D-r^2)}{2Z^2\sqrt{4+cz}} + \frac{c(c-1)r^2 + 4 + DC^2}{4Z} - \frac{D\sqrt{(4+cz)}}{z^2} \right\}^{-1} \right]$$

Sub case (b) :

$$\text{Here } \alpha'' = \frac{\lambda r^2}{2}, \ddot{B} = -\lambda, \lambda \neq 0$$

where λ is a constant

In this case we get

$$(3.35) \quad A = \bar{\lambda}(r^2 - t^2) + \lambda_1 t + \lambda_2$$

where $\bar{\lambda} = \frac{\lambda}{2}$ and λ_1 and λ_2 are another constants. In this case metric (2.1) may be cast into the

form

$$(3.36) \quad ds^2 = \exp\left[-\left\{\bar{\lambda}(r^2 - t^2) + \lambda_1 t + \lambda_2\right\}\right] \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - dt^2 \right]$$

Physical and Geometrical features can be studied as in sub case (a). With suitable adjustment of constants, this model can be considered as that given by Singh and Abdussattar [17] for non-static spherically symmetric perfect fluid distribution which is conformally flat. Further if we take $t = 0$ in (3.35), it given $A = \bar{\lambda}r^2 + \lambda_2$ which shows that A is a function of r only and the

model reduces to static one. This shown that solution given by kalyanshetti and Waghmode [15] without spin is a particular case of the solution given by us.

Case II : Solution in commoving Co-ordinates

i.e. $u_1 = 0$ but $F_{14} \neq 0$

use of $u_1 = 0$ in (3.3.1) provides

$$(3.37) \quad e^{-A} 8\pi(F_{14})^2 = \frac{A_1^2}{4} + \frac{A_1}{2r} - \frac{A_{11}}{2}$$

From equation (2.12) we have

$$(3.38) \quad 2A_{14} - A_1 - A_4 = 0$$

On integrating we can find

$$(3.39) \quad e^A = [f(r) + \psi(t)]^{-2}$$

Where f and ψ are functions of r and t respectively.

Here the line element is given by

$$(3.40) \quad ds^2 = (f + \psi)^{-2} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - dt^2 \right)$$

Pressure and density for the metric (3.40) are given by

$$(3.41) \quad 8\pi p = (f + \psi) \left(2\psi_{44} - f_{11} - \frac{3f_1}{r} \right) + 3(f_1^2 + \psi_4^2)$$

$$(3.42) \quad 8\pi \rho = 3 \left[(f + \psi) \left(f_{11} + \frac{f_1}{r} \right) + (\psi_4^2 - f_1^2) \right]$$

The non-vanishing component of the flow vector u_4 is given by

$$(3.43) \quad u_4 = (f + \psi)^{-1}$$

The reality conditions (Ellis) [5] given by $\rho + p > 0$ and $\rho + 3p > 0$ provide.

$$(3.44) \quad 3\psi_4^2 + (f + \psi)(\psi_{44} + f_{11}) > 0$$

$$(3.45) \quad (f + \psi) \left(\psi_{44} - \frac{f_1}{r} \right) + (f_1^2 + 2\psi_4^2) > 0$$

F_{14} is given by

$$(3.46) \quad F_{14} = (f + \psi)^{-3/4} \left(\frac{f_1}{r} - f_{11} \right)^{1/2}$$

The expansion Θ , rotation w_{ij} and shear σ_{ij} given by

$$(3.47) \quad \Theta = u_{;i}^i$$

$$(3.48) \quad w_{ij} = u_{i,j} - u_{j,i}$$

$$(3.49) \quad \sigma_{ij} = (u_{j,j} + u_{j,i}) - \frac{1}{3} \Theta h_{ij}$$

where $h_{ij} = (g_{ij} - u_i u_j)$

are obtained as

$$(3.50) \quad \Theta = 3\psi_4$$

$$(3.51) \quad w_{ij} = 0$$

$$(3.52) \quad \sigma_{ij} = 0$$

4. Discussion

Here we see that model (3.40) representing the distribution of charged perfect fluid is expanding with time but non-rotating and non-shearing. Our study made in this paper is very useful and significant for deeper knowledge and investigation in astrophysics.

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