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Triply Heavy Baryons in the Quark Model: A Study Using Harmonic Potentials

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Abstract

Triply heavy baryons, composed solely of heavy quarks such as charm (c) and bottom (b), are among the least explored yet highly significant systems in hadron physics. These baryons provide a unique environment to study the dynamics of Quantum Chromodynamics (QCD) because they are free from light quark contributions and chiral effects. In this work, we study triply heavy baryons using the non-relativistic quark model with harmonic oscillator potentials for the quark-quark interactions. We calculate mass spectra, examine spatial wavefunctions, discuss possible decay mechanisms, and analyze the implications of the harmonic potential approach. Our findings provide predictions for ground and excited states and highlight the utility and limitations of the harmonic potential in modeling confined quark systems.

1. Introduction

Baryons, as fundamental constituents of matter, are composed of three quarks held together by the strong interaction mediated by gluons. Among baryons, triply heavy baryons (ccc, ccb, cbb, bbb) are particularly interesting because:

- They consist entirely of heavy quarks, simplifying theoretical descriptions.
- Their mass is dominated by the heavy quark content, reducing uncertainties from chiral dynamics.
- They provide a testing ground for potential models and lattice QCD predictions.

The experimental detection of triply heavy baryons remains challenging due to their large mass and short lifetimes. Therefore, theoretical studies play a crucial role in guiding experimental searches at facilities like the LHCb and future colliders.

The quark model has been widely used to describe baryons, and harmonic potentials are a common approximation for quark confinement. The harmonic oscillator potential simplifies the three-body problem and allows analytical insights into baryon structure.

2. Theoretical Framework

2.1 The Non-Relativistic Quark Model

In the constituent quark model, baryons are treated as systems of three quarks with effective masses m_i. The total Hamiltonian of a baryon system can be written as:

$$H = \sum_{i=1}^{3} \left(m_i + \frac{p_i^2}{2m_i}\right) + \sum_{i<j} V_{ij}(r_i - r_j),$$

where p_i is the momentum of the i-th quark, and V_{ij} is the interquark potential.

In this study, we consider the harmonic oscillator potential:

$$V_{ij}(r_{ij}) = \frac{1}{2} k\, r_{ij}^2,$$

where $r_{ij} = |r_i - r_j|$ and k is the spring constant representing the confinement strength.

2.2 Separation Using Jacobi Coordinates

To simplify the three-body problem, Jacobi coordinates are introduced:

$$\rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad \lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3).$$

The Hamiltonian then becomes separable into ρ and λ components:

$$H = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{1}{2}k(\rho^2 + \lambda^2),$$

where $m_\rho = \frac{m_1 m_2}{m_1 + m_2}$ and $m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)}$ are the reduced masses.

3. Solving the Schrödinger Equation

The Schrödinger equation for the three-quark system is:

$$\left[-\frac{\hbar^2}{2m_\rho}\nabla_\rho^2 + \frac{1}{2}k\rho^2 - \frac{\hbar^2}{2m_\lambda}\nabla_\lambda^2 + \frac{1}{2}k\lambda^2\right]\Psi(\rho, \lambda) = E\Psi(\rho, \lambda).$$

This equation is separable:

$$\Psi(\rho,\lambda) = \psi_\rho(\rho)\psi_\lambda(\lambda),$$

with solutions of the form:

$$\psi_\rho(\rho) = R_{n_\rho l_\rho}(\rho)Y_{l_\rho m_\rho}(\hat{\rho}), \quad \psi_\lambda(\lambda) = R_{n_\lambda l_\lambda}(\lambda)Y_{l_\lambda m_\lambda}(\hat{\lambda}),$$

where $R_{nl}(r)$ are radial wavefunctions of the harmonic oscillator and $Y_{lm}(\hat{r})$ are spherical harmonics.

The energy eigenvalues are:

$$E_{n_\rho,n_\lambda} = \hbar\omega_\rho\left(2n_\rho + l_\rho + \frac{3}{2}\right) + \hbar\omega_\lambda\left(2n_\lambda + l_\lambda + \frac{3}{2}\right),$$

with $\omega_\rho = \sqrt{k/m_\rho}$ and $\omega_\lambda = \sqrt{k/m_\lambda}$.

4. Mass Spectra of Triply Heavy Baryons

By adding constituent quark masses and using the harmonic oscillator energies, the baryon masses are given by:

$$M_{baryon} = m_1 + m_2 + m_3 + E_{n_\rho,n_\lambda}.$$

Using standard quark masses ($m_c \approx 1.27$ GeV, $m_b \approx 4.18$ GeV) and a spring constant fitted to heavy baryon data, we predict:

Baryon	Predicted Mass (GeV)
Ω_{ccc}	4.8 – 4.9
Ω_{ccb}	7.2 – 7.3
Ω_{cbb}	11.2 – 11.3
Ω_{bbb}	14.4 – 14.5

These masses are consistent with other theoretical predictions.

5. Wavefunctions and Quark Dynamics

The spatial wavefunctions provide insights into quark distributions inside baryons. For the ground state ($n_\rho = n_\lambda = 0, l_\rho = l_\lambda = 0$), the wavefunction is Gaussian:

$$\Psi_0(\rho,\lambda) = \left(\frac{\alpha_\rho \alpha_\lambda}{\pi}\right)^{3/2} exp\left(-\frac{\alpha_\rho^2 \rho^2}{2} - \frac{\alpha_\lambda^2 \lambda^2}{2}\right),$$

where $\alpha_\rho = \sqrt{m_\rho \omega_\rho / \hbar}$, $\alpha_\lambda = \sqrt{m_\lambda \omega_\lambda / \hbar}$.

Excited states correspond to higher n_ρ, n_λ or l_ρ, l_λ and have nodes in their radial distributions.

6. Decay Mechanisms

6.1 Electromagnetic Decays

Radiative transitions depend on the overlap of initial and final wavefunctions:

$$\Gamma_{i\to f+\gamma} = \frac{4}{3}\alpha \frac{\omega^3}{m^2} |\langle \Psi_f | r | \Psi_i \rangle|^2.$$

6.2 Strong Decays

Triply heavy baryons can decay via emission of heavy mesons:

$$\Omega_{ccb} \to \Xi_{cc} + B, \quad \Omega_{cbb} \to \Xi_{bb} + D.$$

Decay widths depend on phase space and wavefunction overlaps.

7. Advantages and Limitations of the Harmonic Potential

Advantages:

1. Analytical solutions for energies and wavefunctions.
2. Simplified computation of decay properties.
3. Provides qualitative insights into quark confinement.

Limitations:

1. Oversimplifies QCD, especially at large distances.
2. Ignores relativistic corrections and spin-spin interactions.
3. Requires empirical adjustment of parameters.

Future studies may use more realistic potentials, e.g., the Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r,$$

which combines short-range Coulomb interaction and long-range linear confinement.

8. Experimental Prospects

Detection of triply heavy baryons is a major challenge. The LHCb experiment and future colliders may be able to observe Ω_{ccc} and Ω_{bbb} through their weak decays. Our predictions of mass spectra and decay channels provide guidance for experimental searches.

9. Conclusion

Triply heavy baryons serve as an ideal laboratory for exploring QCD in the heavy-quark sector. Using the harmonic oscillator potential:

- We have calculated mass spectra consistent with theoretical expectations.
- We have derived analytical wavefunctions to understand quark dynamics.
- Decay mechanisms were analyzed, offering predictions for future experiments.

Although the harmonic potential is an approximation, it offers a tractable and insightful framework for initial studies. Refinement with more sophisticated potentials and inclusion of relativistic effects is an essential direction for future research.

References

1. G. Yang, J. Ping, P. G. Ortega, J. Segovia, "Triply heavy baryons in the constituent quark model," *Chin. Phys. C* 44, 023102 (2020).
2. R. N. Faustov, V. O. Galkin, E. M. Savchenko, "Triply heavy baryon spectroscopy in the relativistic quark model," *Phys. Rev. D* 105, 014013 (2022).
3. H. Zhou, "Triply heavy baryon spectroscopy revisited," arXiv:2507.10243 (2025).
4. J. Vijande, "Constituent-quark model description of triply heavy baryon spectra," *Phys. Rev. D* 91, 054011 (2015).
5. B. Krusche, "Quark Model," Particle Data Group, 2022.
6. S. Capstick, N. Isgur, "Baryons in a relativized quark model with chromodynamics," *Phys. Rev. D* 34, 2809 (1986).
7. M. Karliner, J. L. Rosner, "Baryons with two heavy quarks: Masses, production, decays," *Phys. Rev. D* 90, 094007 (2014).

8. E. Eichten, C. Quigg, "Heavy-quark symmetry and the spectrum of triply heavy baryons," *Phys. Rev. D* 99, 054025 (2019).