A Theoretical Model to Predict Effective Heat Storage Coefficient of Two Phase Systems by Introducing Interfacial Layer

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ABSTRACT

Effective Heat Storage Coefficient (EHSC) for granular substances is a function of Heat Storage Coefficient (HSC) of the constituent phases and their volume fractions. The system is considered as a two-phase system: solid phase, fluid phase and interfacial layer between solid and fluid phases. The resistor model is used to find effective heat storage coefficient from the values of HSC of the constituent phases. We have developed a semi empirical expression for EHSC based on resistor model for two-phase systems which is being comprised of contributions from both the phase with interfacial layer. In the proposed model, a correction term for interfacial layer is developed. A new correlation term F is introduced in place of fractional volume of fluid/gas phase. Best fitted relation for F is presented here. The calculations of EHSC for granular substances carried out by the proposed model gives an average deviation of 5.0% from experimental values reported in the literature. The values predicted by the model are in good agreement with experimental values. The theoretical HSC values are determined from present model shows least deviation from the experimental values. Comparison of the proposed relation with different models has also been made.

KEYWORDS – Effective heat storage coefficient (EHSC); series and parallel registors; correlation factor; interfacial layer; two-phase system.

1. INTRODUCTION

Granular substances have emerged as promising materials for use in heat sink and heat exchanger application. Although highly desirable, accurate prediction of the effective heat storage coefficient for granular substances has remained to be a challenging problem. An ever increasing interest has been focused on heat and mass transfer processes in porous media due to their growing importance in functional material design, thermal managements of micro systems, and even in bio-medical engineering. Among them some materials are novel types of industrial materials with low density and unique transport properties, different from those of conventional porous media, which bring them to special and important applications. For examples, the metal and ceramic foams have been used in design of aircraft wing structure in the aerospace industry, in catalytic surfaces for chemical reactions, as the core structure for high strength panels, and the containment matrix and burn rate enhancer for solid propellants.

Theoretical modeling for granular substances in two phase system is very important in determining their ability to storage of heat. For engineers and physicists, the thermal characteristics for these substances of industrial importance are a very challenging. For explosive material industry, nuclear reactors and in oil exploration, the study of thermal parameters of two-phase systems is also valuable. The thermal characteristics of metal foams a very important in determining their ability to store heat. To determine the solution of this problem, we have to need a set of thermal parameters. These are

(a) The thermal conductivity λ,

(b) The volumetric specific heat C which is the product of the specific heat c and the density ρ,

(c) The thermal diffusivity α ($\alpha=\lambda/C$) and

(d) The heat storage coefficient (HSC) β. So we get the following relation, which is defined as-

$$\beta = \sqrt{\lambda \rho c} = \sqrt{\lambda C} = \frac{\lambda}{\sqrt{\alpha}} \qquad (1.1)$$

Lichtenecker[1] also investigated a very simple and effective working empirical relation for porous mixture. In the literature one finds that the effective HSC of composites is an additive property and considering various components as resistors one can take a combination of these to predict effective HSC. This is a very common practice adopted to predict effective heat storage coefficient from the thermal conductivity of different phases for materials. Accepting the similarity, a geometry dependent resistor model has been proposed for heat storage coefficient of granular substances. The study of the effective HSC is necessary in calculating the heat accumulating capacity of a medium when in a transient state. A detailed study of β is given in Verma et. al.[2] In the literature[3-6], one finds very little importance attached knowledge of the effective HSC of various materials. The heat storage coefficient characterizes a medium from the viewpoint of its heat storage ability. If we take a section of the medium, then some of the heat entering is retained by it and the rest is transferred to subsequent layers. But when the steady state is reached no heat is retained and all is transferred to the subsequent layers. So here we notice that during the transient state the heat retained by a particular layer is a function of its heat storage coefficient.

In literature, we have lot of theoretical models for the determination of effective HSC of materials. Shrotriya et al.[7] proposed cubic particles in a cubic unit cell. They considered a theoretical model for the prediction of effective HSC of loose granular substances and compared theoretical values of HSC obtained from the model with values obtained by experiments performed with plane heat source. Misra et al.[8] produce a resistor model to determine effective HSC of two phase systems. They presented the grains of the medium as spherical in shape and by replacing porosity (ϕ) by porosity correction factor (F_P). In similar manner, Zhang et al.[9] have defined a model for HSC of soil. For this they used randomly mixed model to stimulate the spatial structure of the multi-phase media and observed, the significant effect of the degree of saturation on heat storage coefficient. Recently, Usha Singh et al.[10] investigated a theoretical model to predict the effective HSC of fruits. They considered cubic array has been divided into unit cells and resistor model is applied to determine effective HSC of unit cell. All the models involved a geometric parameter that was evaluated using the experimental data.

All these applications require accurate evaluations of the effective heat storage coefficient of such materials. To predict the EHSC of metal foams using previous work done is very difficult because of the complexity of geometry encountered in materials, along with the large difference in heat storage coefficient of the constituents. As we know that the EHSC of two phase systems also depends upon various factors such as HSC of constituent phases, porosity, shape factor, size of particles their distribution etc. and, incorporating all these factors in the prediction of EHSC of two phase system is a complex affair. As it is not often possible to conduct experiments on EHSC, a theoretical expression is needed to predict its value.

In the present paper, we have developed an empirical relation for quick estimation of EHSC of granular substances. In order to incorporate varying individual geometries and non-linear flow of heat flux lines generated due to the different in thermal conductivity of the constituent phases, a correlation term F has been introduced. Parameter estimation technique has been used to optimize the value of F. Expressions for F has been obtained by simulating experimental data reported in the literature. Our approach is simpler and provides wider applicability of the proposed relation and enhances its ability to predict correctly the HSC of granular substances.

2. FORMULATION OF THE PROPOSED MODEL

In many excellent articleswe find that the EHSC of a composite is an additive property. Considering various components as resistors one can take a combination of such resistors to predict EHSC. This is a very common practice adopted to predict HSC from the thermal conductivity of the constituent phases. Accepting this similar method, a relation is proposed here in the following manner.

Consider a two-phase medium made up of solid material (subscript s); a fluid (subscript f) and interfacial layer between solid and fluid (subscript sf) filling the pore space having volume fractions $\phi_s, \phi_f and \phi_{sf}$ respectively. Here we suppose that the matrix is to be made up of layers oriented parallel and perpendicular to the direction of heat flow, alternately as depicted in Fig. 1.

The heat storage coefficient of parallel layers $\beta_{\parallel}$ is given by the weighted arithmetic mean and perpendicular layers $\beta_{\perp}$ by weighted harmonic mean. The corresponding expressions are

$$\beta_{\parallel} = \phi_f\beta_f + \phi_s\beta_s + \phi_{sf}\beta_{sf} \qquad (2.1)$$

$$\beta_{\perp} = \frac{\beta_f\beta_s\beta_{sf}}{\phi_f\beta_s\beta_{sf} + \phi_s\beta_f\beta_{sf} + \phi_{sf}\beta_s\beta_f} \qquad (2.2)$$

Where β_s = heat storage coefficient of solid phase, β_f = heat storage coefficient of fluid phase, β_{sf} = heat storage coefficient of interfacial layer, ϕ_s = volume fraction of solid phase, ϕ_f = volume fraction of fluid phase, ϕ_{sf} = volume fraction of interfacial layer.

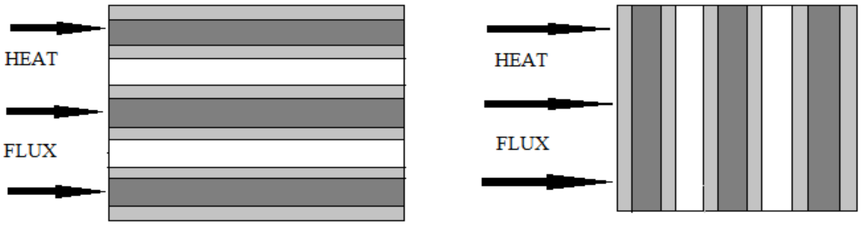

Fig. 1- Configuration of the resistors in a two-phase system.

The $\beta_{\parallel}$ and$\beta_{\perp}$ are the upper (parallel) and lower (perpendicular) bounds on the HSC of a two phase system, respectively, therefore, EHSC will obviously lie between these two bounds. Mean to say that EHSC is also depends on interfacial layer between these two bounds. As these relations do not predict the EHSC of a real two-phase system correctly, so here we introduce a different kind of weighted geometric mean as shown in given expression

$$\beta_e = \left(\beta_{\parallel}^{F}\beta_{\perp}^{(1-F)}\right) ; 0 \le \mathrm{F} \le 1 \qquad (2.3)$$

Where F$^{\text{th}}$ fraction of the material is oriented in the direction of heat flow means in parallel and remaining (1 – F)$^{\text{th}}$ fraction is oriented in the perpendicular direction. The EHSC of a two-phase system is found to depend on $\frac{\beta_s}{}\beta_f$ of the constituent phases. A higher ratio favours a larger fraction of the heat storage coefficient in a direction perpendicular to heat flow.

Eq. (2.3) is solved for F in terms of $\beta_{\parallel}$, $\beta_{\perp} and \beta_e$. The solution is

$$F = \frac{ln\,[\,\phi_f\frac{\beta_e}{\beta_f} + \phi_s\frac{\beta_e}{\beta_s} + \phi_{sf}\frac{\beta_e}{\beta_{sf}}\,]}{ln\,[\,\phi_f^2 + \phi_s^2 + \phi_{sf}^2 + \phi_f\phi_s\left(\frac{\beta_f}{\beta_s} + \frac{\beta_s}{\beta_f}\right) + \phi_f\phi_{sf}\left(\frac{\beta_f}{\beta_{sf}} + \frac{\beta_{sf}}{\beta_f}\right) + \phi_s\phi_{sf}\left(\frac{\beta_s}{\beta_{sf}} + \frac{\beta_{sf}}{\beta_s}\right)\,]} \qquad (2.4)$$

The correlation term F is a function of ratio of heat storage coefficient of the constituent phases and porosity of the system. Remember that, we have tried many combinations. One such plot of F versus R = ln ($\phi_f \frac{\beta_s}{\beta_f}$) is shown in Fig. 2, which is the best suited one for our proposed model. We have used a curve fitting technique and found that the expression

$$F = 0.073\,R^2 - 0.457\,R + 1.012 \qquad (2.5)$$

$$F = 0.073(ln\phi_f \frac{\beta_s}{\beta_f})^2 - 0.457\left(ln\phi_f \frac{\beta_s}{\beta_f}\right) + 1.012 \qquad (2.6)$$

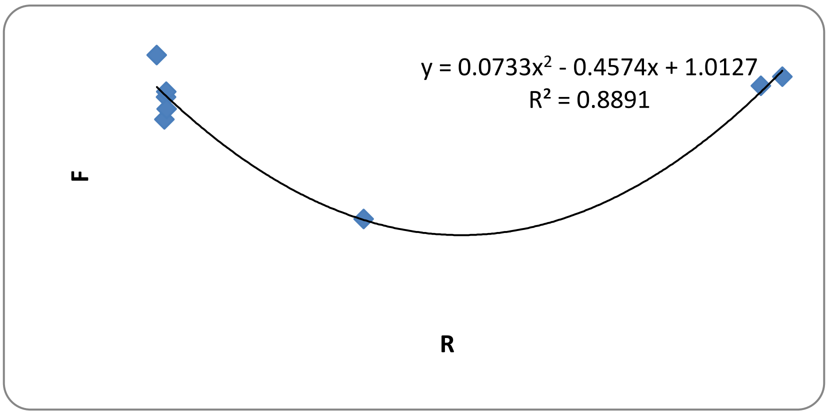

Fig. 2 - Variation of F with R

Best fitted the curve in Fig. 2. It is also observed from experimental results that the expression (2.5) or (2.6) represent the true state of affairs of a real system. F is calculated using Eq. (2.6).

As we know, $\lambda_{sf}= 2\lambda_f$ (i.e. $K_{layer} = 2K_f$) the same value as that given by Leong et al.[11] is used in the calculation of the thermal conductivity. So in the present paper, we use the same phenomena for heat storage coefficient β_{sf} means $\beta_{sf} = 2\beta_f$ (i.e. $\beta_{sf} = \lambda_{sf} /\sqrt{\alpha} = 2\lambda_f/\sqrt{\alpha}$). As we know that the total volume of any system is unity. So we get the volume fraction of solid, fluid and interfacial layer is $\phi_s + \phi_f + \phi_{sf} = 1$. According to this phenomenon we can easily calculate the volume fraction of interfacial layer (ϕ_{sf}), and we get a very small range for interfacial layer. Here, the average value of ϕ_{sf} is just 0.001 only for all samples which we have in Table-1.

3. RESULTS and DISCUSSION

We have tested the validity of our empirical model as discussed above on materials for two-phase systems, for which the characteristics of the constituent phases, including heat storage coefficient of solid phase, fluid phase and interfacial layer between solid and fluid phases, porosity and the experimental results for the EHSC have been cited in the literature[9,12] . So now putting equation (2.6) in equation (2.3), EHSC for a large number of samples given in the literature has been calculated. On applying above equation in Eq. (2.3) we have calculated the values of heat storage coefficient for a number of samples in Table-1. Figure 2 shows a comparison of the experimental results of heat storage coefficient and calculated values from Eq. (2.3). It is seen from this plot that experimental values and proposed model values show an average deviation of just 5.0%. So this proposed model can be used successfully to predict the effective heat storage coefficients of similar systems when heat storage coefficients of their constituent phases and the porosity values are known.

In Table-2, the samples under study are compared with other models for effective heat storage coefficients for materials.Thus, EHSC using K. J. Singh et al.[12] ,Usha Singh et al.[9] has been determined. Figure 2 also shows comparison of experimental values of given samples with these models. The average deviation in EHSC for given samples is 9.1% and 47.5%, for K. J. Singh et al.[12] , Usha Singh et al.[9] models, respectively. However, the proposed model shows only 5.0% deviation. Thus, our model provides better results for granular substances than the other models. So we get, the results using our model show least deviation from the experimental values.

Table-1. Comparison of HSC values for two-phase systems using Eq. (2.3). The Effective Heat Storage Coefficient β is in $Wm^{-2}C^{-1}sec^{1/2}$.

Sample No.	Sample	ϕ_s	β_s	ϕ_f	β_f	ϕ_{sf}	β_{sf}	β(expt)	β(expt)	% err
1	Glass/ICB	0.431	1335	0.568	462.5	0.001	925	820.2	799.4	2.8
2	Silica/water	0.431	4900.1	0.568	1566.3	0.001	3132.6	2802.6	2804	0
3	Silica/water	0.439	4900.1	0.56	1566.3	0.001	3132.6	2761.8	2834.8	2.6
4	Silica/water	0.43	4900.1	0.569	1566.3	0.001	3132.6	2815.2	2800.2	0.4
5	Silica/water	0.428	4900.1	0.571	1566.3	0.001	3132.6	2759.4	2792.6	1.2
6	Riverbase sand/air	0.4	3108	0.599	6.2	0.001	12.4	499.5	542.8	12.7
7	Dry dune	0.35	3495	0.6	6.2	0.0	12.4	576.	573.	6.2

	sand/air	8		41		01		8	9	
8	Dune sand/air	0.42	3495	0.579	6.2	0.001	12.4	561	584.3	4.1
9	Dry cement/air	0.56	3041	0.441	6.2	0.001	12.4	285.6	341.6	19.6
10	S.Steel/IC8	0.476	8604.7	0.525	462.5	0.001	925	1502.4	1484.3	0.7

Average error 5.0%

Table-2. Comparison of EHSC values for two-phase systems with different models[12,9] .

No.	Samples	βs	β (exp)	β(Our model)	error	β(KJS)	erro	β(US)	error
1	Glass/IC8	1335	820.2	799.4	2.8	890.7	8.7	637.1	22.3
2	Silica/water	4900.1	2802.6	2804.1	0	2987.7	6.6	1713.9	38.8
3	Silica/water	4900.1	2761.8	2834.8	2.6	2868.3	3.8	1733.4	37.2
4	Silica/water	4900.1	2815.2	2800.2	0.4	2954.1	4.9	1708.1	39.3
5	Silica/water	4900.1	2759.4	2792.6	1.2	2945	6.7	1734.5	37.1
6	Riverbase sand/air	3108	499.5	542.8	12.7	551.8	10.4	207.8	58.3
7	Dry dune sand/air	3495	576.8	573.9	6.2	534	7.4	207.6	64
8	Dune sand/air	3495	561	584.3	4.1	620.9	10.	207.6	62.9

							6		
9	Dry cement/air	3041	285.6	341.6	19.6	325.4	13.9	208.2	27.1
10	S.Steel/IC8	8604.7	1502.4	1484.3	0.7	1777.6	18.3	169.6	88.7
	Average Error				5.03		9.13		47.57

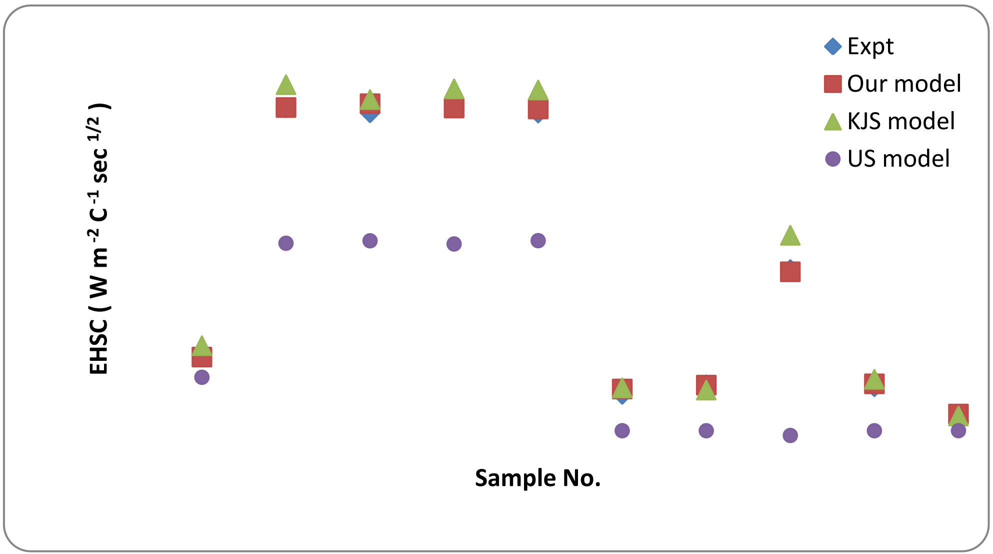

Fig. 2: Comparison of experimental and theoretical values of effective HSC.

From above discussion and presentation we easily predict that the EHSC strongly depends on porosity and the ratio of thermal conductivity of the constituents. Other factors have small role on the EHSC. The parameters of fluids, such as the size, volume fraction, the thickness of the interfacial layer, are shown to play important roles in the enhancement of effective heat storage coefficient. The model predictions have been shown to be reasonable and are in good agreement with the available experimental data. It has one new term, correlation factor and which is valid for granular substances. In present model proposed here is capable of predicting EHSC values closer to the experimental results for all samples which are given in Table-1. Compared with the existing models, our method is highly promising as a realistic, reliable and robust tool predicting properties for various materials without restoring any empirical parameters which have to be determined case by case. This work helped to clarify the impact of this key interfacial layer on the thermal analysis of different type of materials. Here we predict that experimental data for the EHSC of granular substances with well characterized micro-structures having wide range of porosity are still in short supply. From above whole discussion, it is expected that the experimentally validated model will be helpful in the evaluation of the EHSC for foam like materials in the whole range of porosity.

4. CONCLUSIONS

The effective heat storage coefficient of granular substances for two phase systems may be determined with empirical correction to porosity in the theoretical model. The correlation factor (F) in the proposed model for prediction of heat storage coefficient is found to be dependent on the ratio of the EHSC of the constituent phases of the system. And, using the resistor model and the EHSC of constituent phases, the solid phase EHSC may be known. Our proposed model with correlation factor (F) shows an average deviation of 5.0% from the experimental values. Thus, the values of EHSC predicted by the present model are very close to experimental results than obtained from other models cited in the literature. Thus, using this theoretical model one can find out the EHSC of materials.

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