

TSALLIS HOLOGRAPHIC DARK ENERGY'S COSMOLOGICAL CONSEQUENCES IN LRS BIANCHI TYPE-I SPACETIME IN $f(T)$ THEORY OF GRAVITY

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Abstract

In this paper, we investigated a locally rotationally symmetric Bianchi Type-I cosmological model incorporating dark matter and Tsallis holographic dark energy (THDE) within the framework of $f(T)$ modified gravity theory. Using the analytic relation $A = B^n$ and hybrid expansion form to obtain exact solutions of the field equations. The model's physical characteristics are contrasted with the available observational data. It has been observed that the universe is in an accelerating stage for the model, and it reaches isotropy at $n = 1$, and the universe achieves residual anisotropy at a late time.

Keywords: LRS Bianchi type-I, Tsallis holographic dark energy, Cosmic Time, $f(T)$ gravity.

I. Introduction

$f(T)$ theory of gravity is an adjustment theory of gravity in which tele parallelism is used to explain the phenomenon of gravitation in terms of torsion instead of curvature. In contrast to general relativity, where gravity is represented using the Ricci scalar (R), $f(T)$ gravity replaces torsion scalar (T) with an arbitrary function $f(T)$. This is a change that gives us a natural model of explaining the late-time accelerated expansion of the Universe without the introduction of a cosmological constant. The gravity of $f(T)$ is mathematically simple as compared to other modified theories like $f(R)$ gravity due to its second-order field equations. due to this, it has found extensive use in cosmology to examine dark energy, dark matter interactions, anisotropic universe models, and early and late time cosmic evolution.

$f(T)$ gravity model in a Locally Rotationally Symmetric Bianchi type-I model to get a better idea of cosmic evolution. It compares interacting and non-interacting dark energy and dark matter cases, in terms of the energy density that they contain at different redshifts. The model seeks to solve the weaknesses of the Λ CDM model, especially the coincidence problem. Among the main findings, there is an early singularity, a decelerating early expansion and a transition to late-time accelerated expansion and the model is applicable in the explanation of the behavior of early universe as well as current cosmic acceleration [1]. The author has been explored Tsallis holographic dark energy in a Bianchi-I anisotropic universe using a hybrid expansion law. It shows that the equation of state lies in the k-essence region and applies state finder diagnostics to analyze dark energy behavior. The study also reconstructs the scalar field potential, offering insights into late-time cosmic acceleration [2]. The author examined an LRS Bianchi type-I model of cosmology whereby matter is minimally interacting with the dark energy that is holographic and the gravity is of a minimally

interacting linear form of $f(T)$. It makes a comparison between power-law model and exponential model, which reveals that power-law model is unstable with time, exponential model is stable and nonsingular. The two models have faster expansion and project a smooth evolution towards an isotropic Universe toward the end times [3]. Shekh, S. H examined an LRS Bianchi type-I cosmological model with holographic dark energy that is used to explain the expansion of the universe. It compares the time- and redshift-dependent EoS parameters with the observations and diagnostic tools in order to differentiate the model and Λ CDM. It also and discussed thawing/freezing behavior, and reconstructions quintessence and tachyon scalar field dynamics [4]. Pradhan, A & at.el. analyses a $f(T)$ gravity model that includes dark matter and modified holographic Ricci dark energy on an LRS Bianchi type-I Universe to described the cosmic acceleration. It examined the feasibility and permanency of solutions in various expansion laws. The phases of evolution of the Universe are characterized with the help of state finder diagnostics and jerk parameter [5]. The author investigated an LRS Bianchi type-I dark energy model in $f(T)$ gravity, which concentrates on the role played by the torsion scalar. It demonstrates that higher-order torsion terms are excluded in which case there is stronger early-time anisotropy, which is anisotropic in a preferred direction of non-linear $f(T)$ models. It suggested a reasonable structure that is in line with the dark energy activity and cosmic development [6,7].

Srivastava, V., & Sharma, U. K. explained the accelerated expansion of the universe, the Granda-Liveros horizon is used to study the Tsallis holographic dark energy (THDE) model. It highlights the need for more investigation and validation while examining the relationship between Tsallis statistics and dark energy and adding to the current research on alternative dark energy models [8]. The author investigated the interactions between dark matter and dark energy in a homogeneous and anisotropic LRS Bianchi type-I universe. It examines the deceleration parameter and displays a change from previous deceleration to present acceleration. The study contributes to the investigation of alternative gravity theories in cosmology by deriving exact solutions and examining the physical and geometric properties of the universe using the Rényi holographic dark energy model within $f(R)$ gravity [9]. The author examined an anisotropic accelerating Bianchi type-I universe in $f(T)$ gravity, taking into account two non-interacting elements: dark energy and cosmic strings. It concludes that the universe transitions from a matter-dominated phase to a quintessence phase using a matter-dominated power-law expansion. According to CMBR observations, cosmic strings are present in the early universe but disappear as the universe expands [10]. The Hubble horizon is used as the infrared cutoff in the study of a Bianchi type-I anisotropic universe using Barrow holographic dark energy. It operates in $f(Q)$ gravity and concentrates on the $f(Q) = \lambda Q^2$ model. The research looks at important cosmological parameters, uses Hubble parameter observations to constrain the model, and analyses cosmic expansion using the deceleration parameter in [11]. The author examined the Tsallis holographic dark energy on an anisotropic Bianchi type-I universe with an interacting dark matter based on the Hubble law by Berman. It concludes that the ensuing dark energy EoS may act as quintessence or phantom, be brought to the Λ CDM paradigm, and be in agreement with observational data, which underlines the importance of Tsallis statistics and holography in the cosmic evolution [12,13]. Bhattacharjee, S analyzed Tsallis and Renyi holographic dark energy models in non-linear interaction in FRW spacetime based on a hybrid expansion law. It obtains a deceleration acceleration changeover that is similar to the observations, opposite of EoS behaviors near -1 at $z=0$, and indicates that THDE is stable and RHDE is unstable in perturbations [14]. Fayaz, V & at.el. discussed the $f(T)$ changed teleparallel gravity instead of the dark energy in a Bianchi type I universe. It also builds $f(T)$ -model which is consistent

with holographic dark energy, study of the EoS, and discovery of de Sitter and power-law solutions of the phantom phase, provides insight into late-time cosmic acceleration [15]. The author reviewed Tsallis holographic dark energy in general relativity of Marder space-time based on the Hubble horizon cutoff. Three models of THDE are built using the various deceleration parameter strategies, exhibit phantom and quintessence behavior, and model I and II are both very similar to the Λ CDM model [16]. Bharali, J., & Das, K., Modified Tsallis Holographic Dark energy in general relativity is introduced in the Bianchi type-III space-time. Applying the Hubble law of Berman, two models with constant deceleration parameters, one quintessence-like and the other Λ -like, are obtained and determine a correspondence with quintessence scalar fields and are consistent with observations of the current cosmos [17]. The author examined the evolution of the dark energy in a Bianchi type-I universe when using $f(T)$ gravity. It derives time-varying EoS with WMAP, supernovae, galaxy clustering, and CMB anisotropy congruous and physical behavior through important quantities such as the torsion scalar using a Hubble parameter law that gives a constant deceleration parameter [18]. Charjan, S. S explored the cosmological models in Bianchi type- I with a cosmological constant, dark energy magnetized. It discussed the geometrical and physical properties of the universe in two scenarios of scalar expansion using the field equations and displays how the magnetic fields, together with the dark energy, affect the dynamics of the universe expansion [19]. In [20] a $f(T)$ gravity model with a Hubble cutoff is developed based on Tsallis holographic dark energy. It highlights implications for dark energy and cosmic acceleration, demonstrates accelerated expansion, verifies model stability via sound speed, and finds violations of the strong and weak energy conditions in the phantom regime. Using the Hubble horizon as the infrared cutoff, Aktaş, C explored Tsallis holographic dark energy in a homogeneous, anisotropic Marder universe. It uses state finder diagnostics to differentiate dark energy models and examines deceleration, anisotropy, and higher-order cosmological parameters, demonstrating a tendency toward isotropy [21]. With an emphasis on the function of the interaction constant between dark matter and dark energy, Jawad, A.& at el. investigated Tsallis holographic dark energy in a non-flat FRW universe. It examines the deceleration parameter and how it relates to the apparent horizon, demonstrating how well the model captures the transition phase of the universe in [22]. [23] had focused on sign-changeable interactions in a Bianchi type I universe with pressure less dark matter and Tsallis holographic dark energy. It employs state finder and $\omega_D - \omega'_D$ diagnostics to demonstrate how model parameters impact evolution trajectories, analyses different infrared cutoffs, and identifies future classical instability and possible phantom crossing. The Tsallis–Cirto holographic dark energy model in inhomogeneous Lemaitre-Tolman-Bondi (LTB) universes is studied in [24]. Its implications for cosmological evolution are examined, and knowledge gaps regarding dark energy behavior in inhomogeneous settings are filled.

The author examined the Tsallis holographic dark energy (THDE) in the Nojiri-Odintsov gravity, with the infrared cutoff being the event horizon. It brings to fore oscillations in the Hubble parameter, the possibility of singularities as well as the comparison of the model to Λ CDM in terms of fitting the Ia supernova data. The effect of THDE on cosmological variance, matter fluctuations and growth of cosmic structure at low redshifts. The models are widely consistent with observations, such as the cosmological constant at 1σ , and can relieve 8 tensions, but are not very popular in terms of model selection [25,26]. Dhore, A. O., & Ugale, M. R. investigated modified holographic Ricci dark energy in $f(T)$ gravity, the study examined cosmic acceleration, energy conditions and thermodynamic consistency in a FRW universe. The physical and geometrical aspects of the model are analyzed to determine the viability of the model in cosmology [27]. The author discussed the Granda

Olivero and Hubble cutoffs of the Tsallis Holographic Dark Energy (THDE) model, in which the dark energy is a dynamical vacuum. It explains how to move between decelerated expansion to accelerated expansion, is quintessence-like and coincides with Λ CDM in non-interacting cases. The consistency, stability and correspondence of the model to quintessence is verified using observations and dynamical analysis [28,29]. The author examined the process of the matter to dark energy through a Tsallis holographic dark energy model with conformal time and a range of IR cutoffs. It explores the dynamics of the dark energy, its equation of state and its deceleration, and stability in both interacting and non-interacting cases are studied. In some instances, the model corresponds to observational data and determines the generalized second law of thermodynamics [30,31]. The author examined Tsallis holographic dark energy in cosmologies of Kantowski-Sachs and higher dimensions Kaluza-Klein models including interacting, non-interacting models, with Hubble radius as an infrared cutoff. It discusses the deceleration parameter, equation of state and energy densities, demonstrating a change of deceleration to acceleration and a potential change of a phantom phase. Stability and coupling parameters are checked to stabilize it in accordance with the observations [32,33]. The Ghaffari, S.& at.el. explored Tsallis holographic hazy universe, which exhibits a change of deceleration to acceleration, and is in agreement with observations [34]. The author discussed a model of the fractal universe in Tsallis holographic dark energy and pressure less dark matter in their interaction. This interaction term is recreated, with the Hubble length being used as the infrared cut-off. The paper looks at how the universe would have had to shift to accelerated expansion and evaluates several cosmological parameters between the time when the universe was dominated by matter and the late acceleration period. The stability of the model is checked with small perturbations with the help of the squared sound speed which measures its robustness [35,36].

This paper is divided into several section: Section II deals with equation of motion in the framework of $f(T)$ gravity. In section III considering LRS Bianchi type-I metric, we have obtained the corresponding field equations in the framework of $f(T)$ gravity. In section IV, we obtained the exact solutions to the field equations along with various physical and kinematical quantities. In section V we have discussed the physical and kinematical properties through graphical representations. Lastly, in section VI, we have concluded the investigations.

II. Gravitational field equation for $f(T)$ Theory

In $F(T)$ theory [37], the field equations are obtained from an action which is expressed as

$$S = \int [T + f(T) + L_m] e^{\theta} dx^4 \tag{1}$$

Where T is the torsion scalar, $f(T)$ is a differentiable function of T and L_m is Lagrangian density for ordinary matter.

The variation of equation (1) with respect to tetrad h_{μ}^i to the following field equation.

$$S_{\mu}^{\theta\rho} \partial_{\rho} T f_{TT} + [e^{-1} e_{\mu}^i \partial_{\rho} (e e_i^{\alpha} S_{\alpha}^{\theta\rho}) + T_{\lambda\mu}^{\alpha} S_{\alpha}^{\theta\lambda}] (1 + f_T) + \frac{1}{4} \delta_{\mu}^{\theta} (T + f) = 4\pi T_{\mu}^{\theta} \tag{2}$$

The torsion scalar is defined by

$$T = S_{\rho}^{\mu\theta} T_{\mu\theta}^{\rho} \tag{3}$$

Where $T_{\mu\theta}^\rho$ is the torsion tensor and $S_\rho^{\mu\theta}$ is the antisymmetric tensor as.

$$T_{\mu\theta}^\rho = h_i^\rho (\partial_\mu h_\theta^i - \partial_\theta h_\mu^i)$$

$$S_\rho^{\mu\theta} = \frac{1}{2} [K_\rho^{\mu\theta} + \delta_\rho^\mu T_\theta^{\theta\theta} - \delta_\rho^\theta T_\theta^{\theta\mu}]$$

III. Metric and Field equation

LRS-Bianchi type-I metric is expressed as

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2)$$

(4)

where A and B are Scale factor which is a function of cosmic time (t).

The torsion scalar for above metric is obtained as

$$T = -2 \left[2 \left(\frac{\dot{A}\dot{B}}{AB} \right) + \left(\frac{\dot{B}}{B} \right)^2 \right]$$

(5)

For the matter energy density (ρ_m). The tensor for matter energy density momentum (T_{ij}) is

$$T_{ij} = \text{diag}[\rho_m, 0, 0, 0]$$

(6)

In addition, tensor for THDE energy momentum is supposed as

$$\begin{aligned} \bar{T}_{ij} &= \text{diag}[\rho_T, -p_T, -p_T, -p_T] \\ &= \text{diag}[1, -w_T, -w_T, -w_T](\rho_T) \end{aligned}$$

(7)

and

$$T = T_{ij} + \bar{T}_{ij}$$

(8)

The field equations for discussed metric can be written as

$$(f + T) + 4(1 + f_T) \left[\left(\frac{\dot{B}}{B} \right)^2 + 2 \left(\frac{\dot{A}\dot{B}}{AB} \right) \right] = - \left(\frac{16\pi + 3\lambda_2}{2\lambda_1} \right) (\rho_m + \rho_T) + \frac{\lambda_2}{\lambda_1} \rho_T$$

(9)

$$(f + T) + 4(1 + f_T) \left[\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B} \right)^2 + \frac{\dot{A}\dot{B}}{AB} \right] + 4 \frac{\dot{B}}{B} \dot{T} f_{TT} = \left(\frac{16\pi + 3\lambda_2}{2\lambda_1} \right) p_T - \frac{\lambda_2}{\lambda_1} (\rho_m + \rho_T)$$

$$(10) \quad (f + T) + 2(1 + f_T) \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B} \right)^2 + 3 \left(\frac{\dot{A}\dot{B}}{AB} \right) \right] + 2 \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{T} f_{TT} = \left(\frac{16\pi + 3\lambda_2}{2\lambda_1} \right) p_T - \frac{\lambda_2}{\lambda_1} (\rho_m + \rho_T)$$

(11)

Also, the conservation law of energy $T_{,j}^{ij} = 0$ gives

$$\rho_m + \rho_T + 3H(\rho_m + \rho_T + p_T) = 0$$

(12)

IV. Solution of the field equation

For the metric (1), scale factor, spatial volume, are expressed as

$$V = a^3 = AB^2$$

(13)

We have considered the hybrid exponential form of an average scale factor as

$$a = te^t$$

(14)

from equation (13) and (14)

$$AB^2 = t^3 e^{3t}$$

(15)

We now have five unknowns $A, B, \rho_m, \rho_T, p_T$.

Now for solving, the system of equation completely. We assume that the expansion scalar is proportional to the shear, this gives the relation between the metric potential as [38],

$$A = B^n \quad (16)$$

Where, n is an arbitrary constant.

From equation (15) and (16)

$$A = (te^t)^{\frac{3n}{n+2}}$$

(17)

$$B = (te^t)^{\frac{3}{n+2}} \quad (18)$$

Hubble Parameter:

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right)$$

$$H = \left(1 + \frac{1}{t} \right) \quad (19)$$

Scalar expansion

$$\theta = \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) = 3$$

$$\theta = 3 \left(1 + \frac{1}{t} \right) \quad (20)$$

Deceleration Parameter

$$q = -1 - \frac{d}{dt} \left(\frac{1}{H} \right)$$

$$q = -1 + \frac{1}{(t+1)^2} \quad (21)$$

Anisotropy Parameter

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \quad (22)$$

Shear Scalar

$$\sigma(t)^2 = \frac{3}{2} H^2 A_m$$

From (17) and (20)

$$\sigma(t) = \sqrt{3} \frac{(n-1)(t+1)}{(n+2)t} \quad (23)$$

We have considered that DE and Dark matter from (12)

$$\dot{\rho}_m + 3H\rho_m = 0$$

$$\rho_m = ce^{-3(t+\log t)} \quad (24)$$

Where c is constant

THDE density as

$$\rho_T = DH^{4-2\beta}$$

From (17)

$$\rho_T = D \left(1 + \frac{1}{t}\right)^{4-2\beta} \quad (25)$$

Where D and β are constant.

Also, from (17) $\dot{\rho}_T + 3H(\rho_T + p_T) = 0$

$$p_T = -D \left(1 + \frac{1}{t}\right)^{4-2\beta} \quad (26)$$

From (23) and (24)

The barotropic equation of state (EOS): $w_T = -1$ (27)

Torsion scalar

$$T = -2(2n + 1) \left(\frac{3}{n+2}\right)^2 \left(1 + \frac{1}{t}\right)^2 \quad (28)$$

The metric filled with DM and THDE within the framework of $f(T)$ gravity becomes

$$ds^2 = dt^2 - (te^t)^{\frac{6n}{n+2}} dx^2 - (te^t)^{\frac{6}{n+2}} (dy^2 + dz^2) \quad (29)$$

The metric potentials of this model remain positive and finite. They keep spatial extensions non-zero (in any of the directions) and constant and non-vanishing at $t = 0$ and never vanish at any later t . This implies that the model is not directionally singular.

V. Graphical discussion

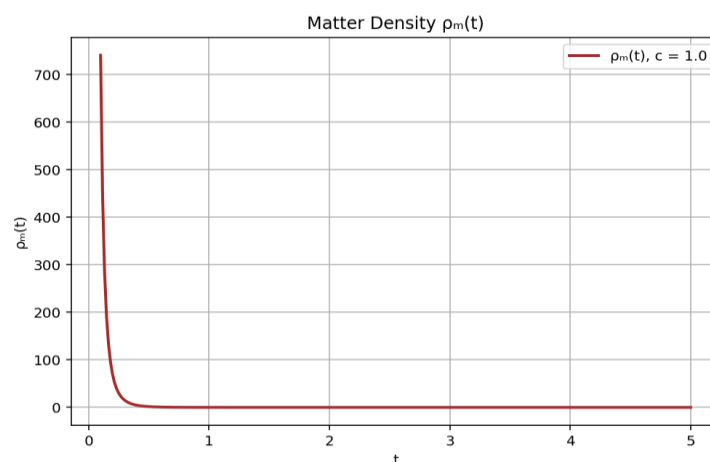


Figure 1. Variation of the Matter's Energy Density with time.

Equation (24) gives the matter’s energy density for the model and its variation with time (t) is represented by Figure 1. The graphical behavior of ρ_m shows that it rapidly decreases with time and approaches zero, indicating that matter becomes diluted and negligible at late times, likely due to the accelerated expansion of the metric potentials.

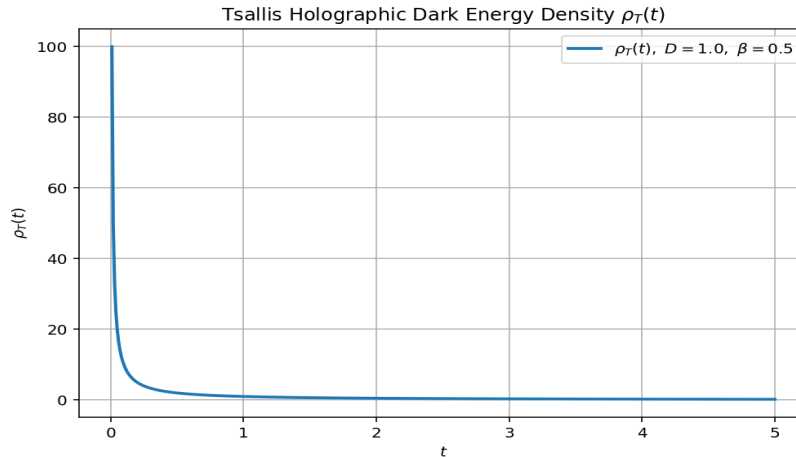


Figure 2. Variation of the THDE Density with time.

Equation (25) gives the THDE density for the model and its variation with time (t) is represented by Figure 2. The graphical behavior of ρ_T shows that it reduces rapidly and become constant for $\beta = 2$.

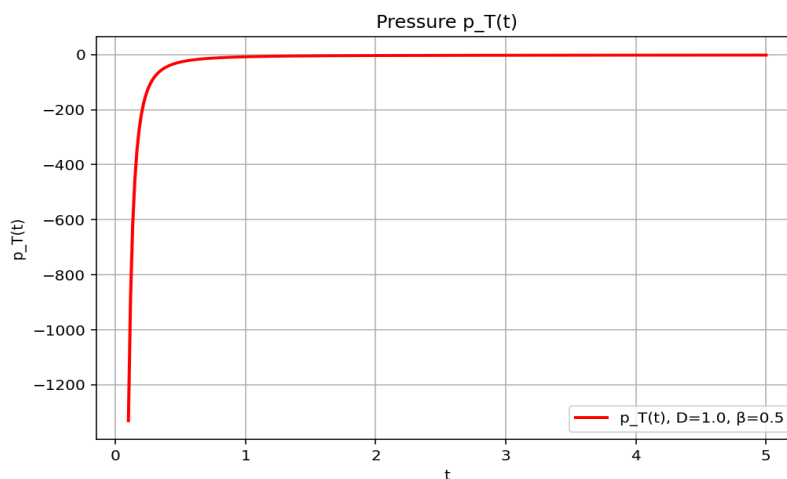


Figure 3. Variation of THDE Pressure with time.

Equation (26) gives the THDE pressure for the model and is represented by Figure 3. The graphical behavior of p_T shows that it is negative, monotonic pressure approaching $-D$ at late times, with early-time behavior governed by β .

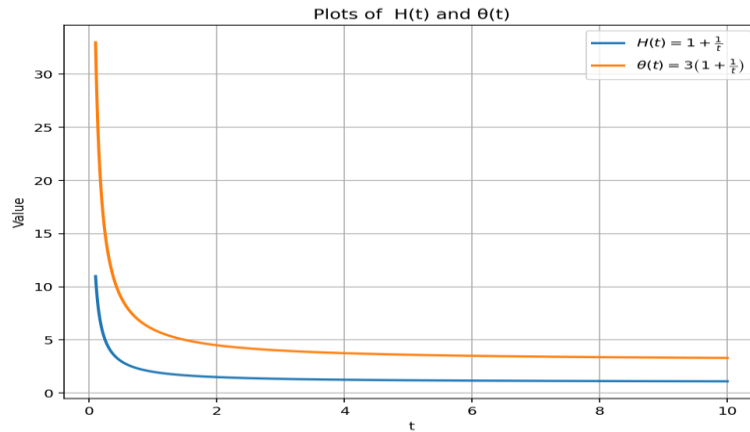


Figure 4. Variation of Hubble Parameter and Scalar Expansion with time.

Equation (19) and (20) gives the Hubble Parameter and Scalar expansion for the model and its variation with time (t) is represented by Figure 4. The graphical behavior of H and θ shows rapid early expansion slowing to a steady late-time expansion, isotropic in all directions.

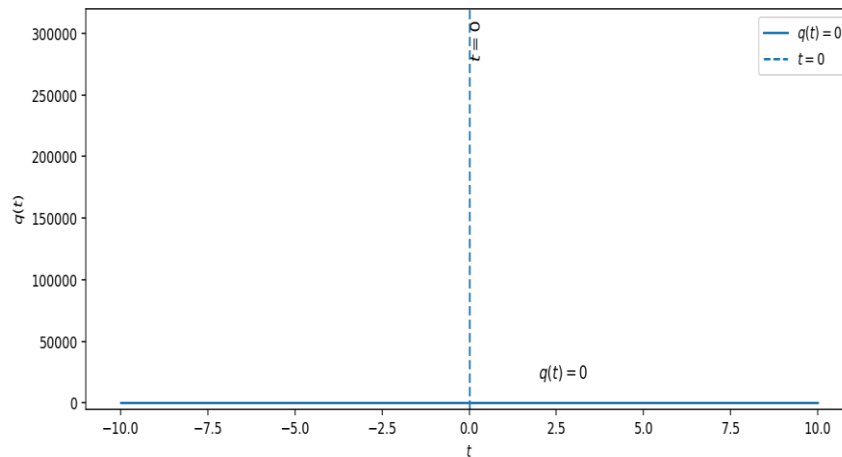


Figure 5. Variation of Deceleration Parameter with time.

Equation (21) gives the Deceleration Parameter for the model and its variation with time (t) is represented by Figure 5. The graphical behavior of q shows early universe is nearly decelerating ($q \approx 0$), and at late times the universe accelerates toward $q = -1$.

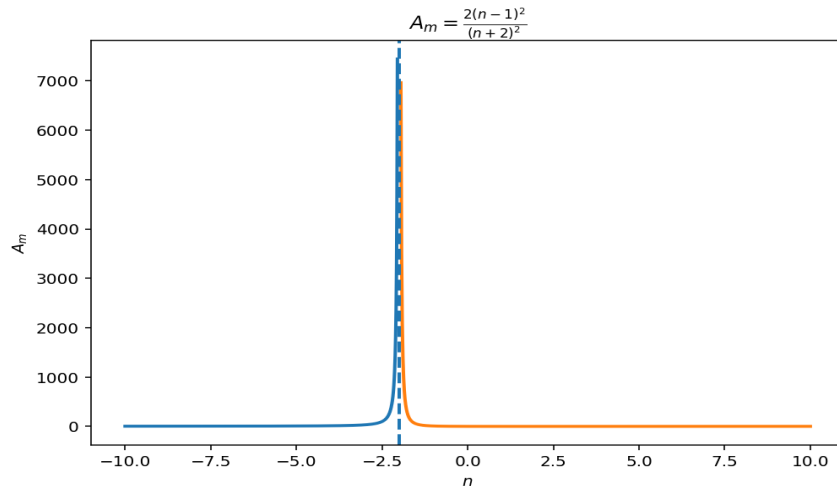


Figure 6. Variation of Anisotropy Parameter with time.

Equation (22) gives the Anisotropy Parameter for the model and its variation with parameter n is represented by Figure 6. The graphical behavior of A_m shows that

$A_m = 0$ for $n = 1$ (isotropic universe), and $A_m > 0$ for $n \neq 1$ (anisotropic universe).

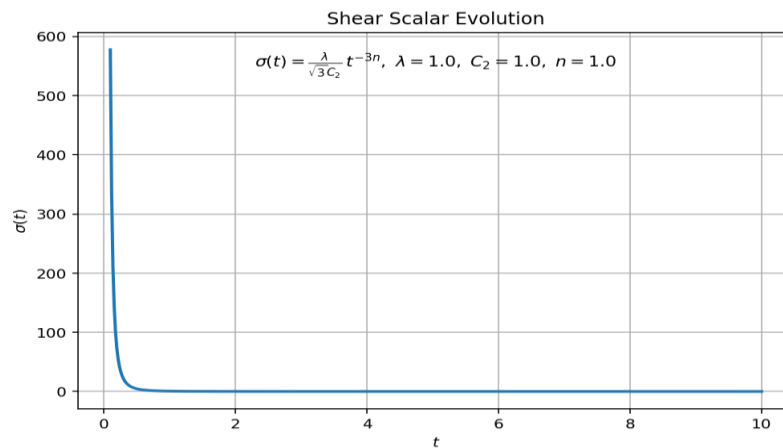


Figure7. Variation of Shear Scalar with time.

Equation (23) gives the Shear Scalar for the model and its variation with time (t) is represented by Figure7. The graphical behavior of $\sigma(t)$ shows that for ($n=1$) isotropic universe($\sigma=0$), $t \rightarrow 0$: highly anisotropic early universe ($\sigma \rightarrow \infty$), $t \rightarrow \infty$: residual anisotropy.

VI. Conclusions

In this paper, we expounded the Bianchi type-I model. We used THDE density and the analytic relation $A = B^n$ and hybrid expansion form to give the specific solution to the field equations. Each physical parameter, including Hubble's parameter, the deceleration parameter, the anisotropy parameter, scalar expansion, and Share Scalar, has been derived. It is observed that in the model universe achieves isotropic at $n=1$. And universe achieves residual anisotropy at a late time. In this expanding universe Hubble's parameter and scalar expansion are reducing function of time, and the deceleration parameter indicates accelerating expansion of the universe at a late time. Furthermore, the matter's density rapidly decreases with time and approaches zero, indicating that matter becomes diluted and negligible at late times. For the case $\beta=2$, the density of the THDE approaches a constant value. This indicates that dark energy dominates the late-time dynamics of the universe. Since the THDE pressure is negative, it drives the accelerating expansion of the universe. Recent observations support the result presented here.

Although $f(T)=T$ is aTEGR, the cosmic dynamics is not that of standard GR due to the fact that the model incorporates a Tsallis holographic dark energy (THDE). Recent studies indicate that THDE within theTEGR model is possible to accelerate the cosmic universe, and our findings further support its practicality.

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