
A Problem-Oriented Review of Wave Scattering and Fracture Dynamics in Magnetoelastic Media

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Abstract

Wave scattering and fracture dynamics in magnetoelastic media constitute a class of multi-physics problems whose scientific and technological significance has grown substantially with the proliferation of magnetostrictive sensors, magnetically sensitive structural components, and electromagnetic nondestructive evaluation (NDE) systems. Despite decades of theoretical development, the field retains several unresolved core problems that resist resolution by either analytical or numerical means alone. This problem-oriented review identifies five central scientific challenges: (i) the characterization of crack-tip singularities under combined elastic and magnetic loading; (ii) the formulation and solution of mixed boundary value problems (MBVPs) at crack faces with electromagnetically consistent boundary conditions; (iii) the quantification of magnetoelastic coupling effects on wave speed and stress distribution; (iv) the modeling of dynamic crack propagation under time-varying magnetic fields; and (v) the treatment of material nonlinearity and anisotropy in realistic magnetostrictive media. For each problem, existing analytical methods principally the Wiener–Hopf technique, singular integral equations, and the Stroh formalism are critically assessed alongside finite element (FEM), boundary element (BEM), and extended finite element (XFEM) computational approaches. Key findings reveal that analytical methods provide rigorous asymptotic results for idealized geometries but fail for complex crack networks and nonlinear coupling, while numerical methods offer geometric flexibility at the cost of inadequate treatment of near-tip field singularities in the magnetoelastic setting. Critical gaps include the near-total absence of controlled experimental validation datasets, the underdevelopment of nonlinear magnetoelastic fracture models, and the lack of multi-scale frameworks connecting micromagnetic domain physics to macroscale fracture behavior. A structured research roadmap is proposed, emphasizing hybrid analytical–computational strategies, AI-assisted inverse problems, and standardized experimental protocols.

Keywords: *magnetoelasticity; wave scattering; fracture mechanics; mixed boundary value problems; stress intensity factor; Wiener–Hopf technique; dynamic fracture*

1. Introduction

Magnetoelastic media are materials in which mechanical deformation and magnetic polarization

are energetically coupled deformation alters the magnetic state, and conversely, applied magnetic fields produce mechanical strain (magnetostriction). This coupling, observed in ferromagnetic metals, magnetostrictive alloys such as Terfenol-D and Galfenol, and magnetorheological composites, gives rise to a rich class of multi-physics phenomena with profound implications for structural integrity and wave-based interrogation technologies. When a magnetoelastic body contains defects cracks, inclusions, or interfaces the interplay of elastic wave scattering and fracture mechanics with the magnetic field produces problems of exceptional analytical depth (Eringen & Maugin, 1990; Maugin, 1988).

The scientific importance of these problems spans multiple domains. In structural health monitoring, guided magnetoacoustic waves are used to detect and characterize cracks in ferromagnetic pipelines and pressure vessels (Rose, 2004). In the design of magnetostrictive transducers and actuators, the sensitivity of fracture toughness to bias magnetic field is a primary reliability concern (Gao et al., 2004). In geophysics, the propagation of seismic waves in magnetized crustal rocks involves exactly the class of equations studied here, albeit at very different scales (Knopoff, 1955). Across all these contexts, the core scientific challenge is the same: to predict, from first principles, how a coupled elastic–electromagnetic field interacts with geometric or material discontinuities in the medium.

A problem-oriented review differs from a conventional narrative review in that it organizes the literature around unresolved scientific problems rather than chronological or topical categories (Grant & Booth, 2009). This approach forces critical evaluation of what is and is not known, surfaces genuine inconsistencies between competing analyses, and produces actionable research priorities. The present review adopts this methodology, organizing its analysis around five core problems that remain only partially resolved despite substantial accumulated effort. For each problem, we assess: what analytical and numerical tools have been applied; what results have been obtained; where those results conflict or remain incomplete; and what methodological advances would most efficiently advance the field.

The review is organized as follows. Section 2 establishes the mathematical framework. Sections 3–7 address the five core problems in turn. Section 8 provides a critical comparative analysis of solution methods. Section 9 synthesizes the literature thematically. Section 10 identifies critical gaps. Section 11 proposes a research roadmap. Section 12 concludes.

2. Governing Physics and Mathematical Framework

2.1 The Magnetoelastic Coupled System

The field equations of linear magnetoelasticity constitute a coupled system in which the elastic and electromagnetic subsystems interact through constitutive relations encoding the magnetomechanical

coupling. We adopt the quasi-static electromagnetic approximation (displacement current neglected), appropriate for the frequency ranges of practical interest in elastodynamics (10^2 – 10^6 Hz) with materials of moderate conductivity. The governing equations are:

Elastic equation of motion (Achenbach, 1973):

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i \quad (2.1)$$

Quasi-static Maxwell equations (Eringen & Maugin, 1990):

$$\nabla \times H = J, \quad \nabla \cdot B = 0, \quad \nabla \times E = -\partial B / \partial t \quad (2.2)$$

Magnetoelastic constitutive relations (Tiersten, 1964):

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} + \beta_{ijk} H_k \quad (2.3a)$$

$$B_i = \beta_{kij} \varepsilon_{jk} + \mu_{ij} H_j \quad (2.3b)$$

In Equation (2.1), σ_{ij} is the Cauchy stress tensor (Pa), $F_i = \mu_0(M \cdot \nabla)H_i + (J \times B)_i$ is the body force per unit volume from electromagnetic interactions (N/m^3), ρ is mass density (kg/m^3), and \ddot{u}_i is the material acceleration. In (2.3a), C_{ijkl} is the elastic stiffness tensor (Pa), $\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2$ is the infinitesimal strain tensor, and β_{ijk} is the third-order piezomagnetic coupling tensor (T or $N/A \cdot m$). In (2.3b), μ_{ij} is the magnetic permeability tensor (H/m). The coupling tensor β_{ijk} is the source of all magnetomechanical interaction: it vanishes for non-magnetic materials, and its symmetry class is determined by the crystal group of the medium.

Two fundamental assumptions underpin Equations (2.1)–(2.3): (i) linearity in both the elastic and magnetic responses valid for small strains and fields below magnetic saturation; and (ii) the quasi-static electromagnetic approximation valid when the electromagnetic wavelength is much larger than the elastic wavelength, a condition satisfied for frequencies below approximately 10^8 Hz in typical metals. The ramifications of relaxing these assumptions are addressed in Section 7.

A critical issue, inadequately resolved in the existing literature, concerns the correct form of the electromagnetic body force F_i for a deformable magnetic solid. At least three distinct formulations are in use the Kelvin, Helmholtz, and Chu formulations which differ in how they partition the total electromagnetic momentum between field and matter (Pao & Yeh, 1973). These formulations are energetically equivalent for the total force but differ in the body force/body couple decomposition, leading to different stress tensors that can produce different stress intensity factors when applied to fracture problems (Pak & Herrmann, 1986). This ambiguity represents a genuine unresolved problem in the foundations of the subject.

2.2 Wave Propagation and Dispersion Analysis

For an unbounded magnetoelastic medium with uniform bias field H_0 , plane harmonic wave solutions of the form:

$$u_i = U_i \exp[i(k_j x_j - \omega t)] \quad (2.4)$$

are substituted into Equations (2.1)–(2.3), yielding the eigenvalue problem:

$$[\rho\omega^2\delta_{ij} - Q_{ij}(k, H_0)] U_j = 0 \quad (2.5)$$

where $Q_{ij}(k, H_0) = C_{ijkl} k_j k_k + \mu_0 H_0^2 \Lambda_{ij}(n)$ is the acoustic tensor modified by the magnetic field, with $\Lambda_{ij}(n)$ a tensorial function of the propagation direction $n = k/|k|$. Setting $\det[Q_{ij} - \rho\omega^2\delta_{ij}] = 0$ yields the dispersion relation, which for an isotropic magnetoelastic body with bias field transverse to the propagation direction gives effective wave speeds (Dunkin & Eringen, 1963):

$$c_{-T}^2 = (G + \mu_0 H_0^2)/\rho, \quad c_{-L}^2 = (\lambda + 2G + \mu_0 H_0^2)/\rho \quad (2.6)$$

demonstrating that the magnetic field augments both transverse and longitudinal wave speeds by the Alfvén pressure $\mu_0 H_0^2$. The ratio $\alpha = \mu_0 H_0^2/G$ the Alfvén-to-shear modulus ratio serves as the primary dimensionless coupling parameter. For typical structural steels at $B = 1$ T, $\alpha \approx 3 \times 10^{-7}$, indicating that the wave speed modification is small but measurable and potentially significant for resonance-based NDE. For magnetostrictive alloys such as Terfenol-D, the coupling is orders of magnitude stronger, and the linear approximation itself becomes questionable.

3. Core Scientific Problem 1: Wave Scattering by Cracks and the Crack-Tip Singularity

3.1 Problem Statement and Scientific Challenge

The scattering of elastic waves by cracks in magnetoelastic media is the archetypal problem of the field. Its difficulty resides in the incompatibility between the singular near-tip stress field, which requires special treatment, and the magnetoelastic coupling, which modifies the constitutive relations and hence the structure of the singularity. In classical linear elastic fracture mechanics (LEFM), the crack-tip stress field is characterized by the stress intensity factor (SIF) (Irwin, 1957):

$$K_I = \lim_{r \rightarrow 0} \sqrt{(2\pi r)} \sigma_{22}(r, 0), \quad K_{III} = \lim_{r \rightarrow 0} \sqrt{(2\pi r)} \sigma_{12}(r, 0) \quad (3.1)$$

where r is the distance ahead of the crack tip and the stresses are evaluated in the crack plane ($\theta = 0$). The central scientific challenge is: how is this $r^{-1/2}$ singularity modified in amplitude and structure by the presence of the magnetic field and the magnetoelastic coupling?

3.2 Existing Solutions and Their Limitations

Shindo (1977) provided the first rigorous answer for Mode III (antiplane) loading: the dynamic SIF for SH wave scattering by a finite crack in an infinite magnetoelastic body is $K_{III} = \tau_0 \sqrt{(\pi a)} F(\beta, \theta)$, where

$F(\beta, \theta)$ is a dimensionless frequency-dependent scattering function computed via the Fourier transform and the residue theorem. The magnetic field enters F exclusively through the augmented shear wave speed c_T (Equation 2.6), leading to the conclusion that K_{III} increases monotonically with H_0 for fixed displacement loading. This result has been confirmed independently by Wang and Shen (2002) and Ing and Wang (2004) for related geometries.

However, Sih and Song (2003) disputed this monotonic relationship for mixed-mode loading in magnetoelastic composites, arguing that the magnetic field introduces a biaxial pre-stress state that can reduce K_I even while increasing K_{III} . This contradiction has not been resolved in the literature and arises from differing assumptions about the electromagnetic boundary conditions on the crack faces a point we address as a separate core problem in Section 4. A further limitation of all analytical SIF analyses to date is their restriction to geometrically simple configurations: a single planar crack in an infinite or semi-infinite body. Multiple interacting cracks, kinked cracks, and surface-breaking cracks of realistic aspect ratios remain analytically intractable.

From the numerical side, Bhargava and Sharma (2013) applied XFEM to magnetoelastic crack problems, using the enrichment:

$$u^h(x) = \sum_I N_I u_I + \sum_{J \in Sc} N_J H(x) a_J + \sum_{K \in St} N_K [\sum_{\alpha} F_{\alpha}(x) b_{K\alpha}] \quad (3.2)$$

where N_I are standard shape functions, $H(x)$ is the Heaviside enrichment, and $F_{\alpha}(x) \in \{\sqrt{r} \sin(\theta/2), \sqrt{r} \cos(\theta/2), \sqrt{r} \sin(\theta/2)\sin\theta, \sqrt{r} \cos(\theta/2)\sin\theta\}$ are the crack-tip enrichment functions. Their XFEM results agreed with analytical benchmarks to within 2–5% for the simple geometries studied but required careful treatment of the interaction integral to extract magnetoelastic SIFs. The key unresolved issue is whether the standard crack-tip enrichment functions F_{α} remain optimal for the magnetoelastic case they were derived from purely elastic asymptotic fields or whether enrichment functions incorporating the electromagnetic coupling are needed for better accuracy near strongly coupled crack tips.

3.3 Critical Evaluation

The existing literature on crack-tip singularities in magnetoelastic media has established the qualitative picture but left critical quantitative questions open. The $r^{-1/2}$ singularity structure survives magnetoelastic coupling in the linear regime (Shindo, 1977; Pan, 2002), but the angular distribution of the singular field and hence the mixed-mode character of the crack tip is modified by the coupling tensor β_{ijk} . For strongly coupled materials (large α), purely elastic enrichment functions in XFEM may introduce systematic SIF errors that have not been quantified. Moreover, the role of the electromagnetic singularity at the crack tip the magnetic field H also develops a weak singularity at the crack tip in conducting media

has been analyzed by Pak and Herrmann (1986) but has not been incorporated into numerical enrichment strategies. This represents a tractable but unaddressed research gap.

4. Core Scientific Problem 2: Mixed Boundary Value Problems and Electromagnetic Crack Face Conditions

4.1 Problem Statement

Mixed boundary value problems (MBVPs) in magnetoelastic fracture mechanics are doubly mixed: mechanically, because traction-free conditions on the crack face differ from the applied loading conditions on the remote boundary; and electromagnetically, because the crack cavity (air or vacuum) has different electromagnetic properties from the surrounding solid. The latter creates an additional degree of freedom in problem formulation that does not exist in purely elastic fracture mechanics: the electromagnetic boundary conditions on the crack faces must be specified, and the choice has significant quantitative consequences for the computed SIF.

4.2 The Electromagnetic Crack Face Condition Problem

Three electromagnetic crack face models have been proposed and analyzed in the literature: (i) the ‘magnetically impermeable’ model ($B_n = 0$, H_t discontinuous) corresponding to a crack cavity with zero permeability; (ii) the ‘magnetically permeable’ model (B_n and H_t continuous) corresponding to a crack cavity with the same permeability as the surrounding medium; and (iii) the ‘realistic’ or Hao–Shen model in which the cavity permeability μ_c is treated as a free parameter (Zhong & Li, 2006; Gao et al., 2003).

Zhong and Li (2006) showed analytically that the impermeable and permeable models yield qualitatively different K_I – H_0 relationships: the impermeable model predicts K_I decreasing with H_0 , while the permeable model predicts K_I increasing with H_0 a 180° reversal in the predicted magnetic effect on fracture severity. This is not a minor quantitative discrepancy; it represents a fundamental ambiguity about whether magnetic fields promote or retard fracture in a given configuration. The realistic model of Gao et al. (2003) interpolates between these extremes as μ_c/μ varies from 0 to 1, providing a physically consistent framework. However, experimental measurements of the effective permeability of crack cavities in magnetostrictive materials are essentially absent from the literature, making it impossible to validate any of these models against physical reality.

4.3 Mathematical Framework for MBVPs

The solution of MBVPs in magnetoelastic media typically proceeds through integral transform methods that reduce the problem to a functional equation in the transform domain. For a semi-infinite crack in an infinite magnetoelastic medium, the Fourier transform of the boundary conditions on the crack face and the requirement of continuity across the crack line leads to a Wiener–Hopf equation (Noble, 1958):

$$\Phi_{+}(\xi) K(\xi) + \Phi_{-}(-\xi) = F(\xi), \quad \text{Im}(\xi) \in (-\beta, \alpha) \quad (4.1)$$

or equivalently in the convolution integral form:

$$\varphi(x) + \int_0^\infty K(x-t)\varphi(t) dt = f(x), \quad x > 0 \quad (4.2)$$

where $\Phi_{+}(\xi)$ and $\Phi_{-}(\xi)$ are the one-sided Fourier transforms of the unknown boundary data, $K(\xi)$ is the kernel function encoding the magnetoelastic physics, and $F(\xi)$ represents the loading. The crux of the Wiener–Hopf method is the multiplicative factorization $K(\xi) = K_{+}(\xi) \cdot K_{-}(\xi)$, where $K_{+}(\xi)$ and $K_{-}(\xi)$ are analytic and non-zero in the upper and lower halves of a strip of analyticity in the complex ξ -plane respectively. For scalar (Mode III) magnetoelastic problems, $K(\xi) = c_T \sqrt{(\xi^2 - k_T^2)/(\omega + \mu_0 \sigma_c T^2 \xi^2/\omega)}$, and the factorization can be carried out analytically (Shindo, 1977). For in-plane (P-SV) problems, $K(\xi)$ is a 2×2 matrix kernel that involves the coupling between all magnetoacoustic modes; matrix Wiener–Hopf factorization in closed form is not generally possible and requires approximate or numerical procedures (Abrahams & Wickham, 1990).

This limitation the inaccessibility of exact matrix Wiener–Hopf factorization for in-plane magnetoelastic crack problems is a major unresolved mathematical challenge. Approximate factorization methods (Padé approximants, iterative schemes) have been developed for analogous problems in fluid mechanics and anisotropic elasticity (Norris & Abrahams, 2008) but have not been systematically applied to magnetoelastic MBVPs.

4.4 Singular Integral Equation Approach for Finite Cracks

For finite cracks, the dislocation density method (Erdogan, 1978) transforms the MBVP into a Cauchy-type singular integral equation:

$$\int_{-a}^a [\mu\omega(t-x) + L(x,t)] \varphi(t) dt = f(x), \quad |x| < a \quad (4.3)$$

where $\varphi(t)$ is the dislocation density (proportional to the displacement jump gradient across the crack face), $L(x,t)$ is a bounded kernel incorporating the magnetoelastic coupling, and $f(x)$ is the crack face traction. The Chebyshev polynomial expansion $\varphi(t) = \sum_n a_n T_n(t/a)/\sqrt{1-(t/a)^2}$, combined with Gauss–

Chebyshev quadrature, provides a numerically stable and well-validated solution strategy (Erdogan, 1978).

The SIFs at the crack tips are recovered from:

$$K_{I(\pm a)} = \pm \sqrt{\pi a} \mu_0 \lim_{t \rightarrow \pm a} \sqrt{a \mp t} \varphi(t) \quad (4.4)$$

This approach has been successfully applied to magnetoelastic crack problems by Shindo et al. (1997) and Wang and Shen (2002). However, it shares the limitation of all singular integral equation methods: it requires the Green's function of the magnetoelastic problem to be explicitly known, restricting applicability to geometrically simple (infinite or semi-infinite) domains.

5. Core Scientific Problem 3: Magnetoelastic Coupling and Its Effect on Wave Speed and Stress

5.1 The Quantification Problem

A fundamental scientific problem deceptively simple to state is to determine, quantitatively and as a function of field magnitude and orientation, how the applied magnetic field modifies the propagation and scattering of elastic waves in a cracked body. The challenge has two components: (i) accurately computing the wave speed modification, which requires precise knowledge of the piezomagnetic coupling tensor β_{ijk} ; and (ii) translating this wave speed modification into a change in the scattered field amplitude and the dynamic SIF, which requires solving the scattering problem with the modified dispersion relation.

5.2 State of Knowledge and Inconsistencies

The augmented wave speed result (Equation 2.6) is well-established for isotropic media and has been extended to transversely isotropic magnetoelastic media by Dhua and Chattopadhyay (2016) and to hexagonal magnetostrictive crystals by Bera and Lahiri (1984). In all cases, the bias magnetic field introduces an effective increase in the elastic modulus of order $\mu_0 H_0^2$. The key discrepancy in the literature concerns the sign and magnitude of the coupling effect on the SIF: whether the magnetic field increases or decreases fracture severity depends critically on the loading mode (mechanical vs. magnetic) and the electromagnetic crack face conditions (as discussed in Section 4). Studies by Shindo (1977), Ing and Wang (2004), and Zhou et al. (2004) using the impermeable crack model find K increasing with H_0 for displacement-controlled loading; studies by Song and Sih (2003) and Zhong and Li (2006) using the permeable or realistic model find the opposite for load-controlled configurations.

This inconsistency reflects a genuine ambiguity in the physical problem, not merely a computational error. It underscores the urgency of the electromagnetic boundary condition problem (Section 4) and the need for experimental validation (Section 10). The practical implication whether applying a magnetic field makes a cracked structure safer or more dangerous cannot be answered definitively from existing theory alone.

5.3 Dimensionless Analysis and Scaling

A useful framework for comparing results across different studies is dimensionless analysis. The key dimensionless parameters governing the magnetoelastic scattering problem are: the Alfvén-to-shear modulus ratio $\alpha = \mu_0 H_0^2 / G$ (coupling strength), the magnetic Reynolds number $R_m = \mu_0 \sigma_e c_T a$ (induction significance), the dimensionless frequency $\beta = \omega a / c_T$ (dynamic parameter), and the crack density $\varepsilon = Na^2$ (for crack arrays). Most existing analytical solutions explore one-dimensional parameter spaces (varying β at fixed α), while the experimentally relevant regime involves simultaneous variation of α and R_m a two-dimensional parameter space that is largely unexplored. This represents a systematic gap in the existing literature that could be addressed by combined analytical and numerical means without requiring new experimental data.

6. Core Scientific Problem 4: Dynamic Fracture Under Time-Varying Magnetic Fields

6.1 Problem Statement and Physical Complexity

Dynamic fracture the propagation of a crack at non-negligible velocity under transient loading in magnetoelastic media involves a hierarchy of coupled dynamic effects: elastic wave radiation, electromagnetic induction (Faraday's law applied to the moving crack), and the modification of the near-tip energy balance by both mechanical and electromagnetic energy fluxes. The energy release rate, generalized to the magnetoelastic setting (Pak & Herrmann, 1986), takes the form:

$$G_{\{total\}} = G_{\{mech\}} + G_{\{em\}} = -d(U_{\{elastic\}} + U_{\{magnetic\}})/da \quad (6.1)$$

where $G_{\{em\}} = -dU_{\{magnetic\}}/da$ represents the rate of change of stored magnetic energy with crack advance. The sign of $G_{\{em\}}$ depends on whether the crack opening expels or concentrates magnetic flux, which in turn depends on the crack orientation relative to the applied field a configuration-dependent result with no universal sign. The crack propagation criterion $G_{\{total\}} = G_c$ (critical energy release rate) thus involves a competition between mechanical driving force and electromagnetic retardation or amplification.

6.2 Existing Analyses and Their Scope

The most complete dynamic analyses in the magnetoelastic fracture literature are those of Li and Mataga (1996) and Ing and Wang (2004), both employing the Wiener–Hopf technique combined with the Cagniard–de Hoop inversion method for the transient Laplace transform. Ing and Wang (2004) analyzed a crack in a magnetoelastic strip subjected to a suddenly applied crack face traction, obtaining exact closed-form expressions for the dynamic SIF as a function of time t and magnetic field H_0 . Their results show that the SIF first overshoots its static value (dynamic amplification) and then settles, with the magnetic field affecting the overshoot amplitude and the settling time a result relevant to impact loading of

magnetostrictive components.

However, both analyses assume steady-state crack propagation (crack advancing at constant velocity v_c) or a stationary crack under transient loading. The coupled problem of a crack accelerating from rest relevant to fracture initiation under a rising magnetic field has not been solved analytically and presents formidable mathematical challenges. Furthermore, the electromagnetic radiation associated with a moving crack in a conducting magnetoelastic medium (analogous to Cherenkov radiation in electrodynamics) has been discussed qualitatively (Maugin, 1988) but not quantitatively analyzed within a rigorous boundary value problem framework.

6.3 Crack Propagation Instability Under Magnetic Loading

A physically important and analytically underexplored phenomenon is the possible instability of crack propagation in magnetoelastic media. In purely elastic media, crack branching and oscillatory propagation occur when the crack speed exceeds approximately 60–70% of the Rayleigh wave speed (Freund, 1990). In magnetoelastic media, the effective Rayleigh wave speed is modified by the bias field through Equation (2.6), so the instability threshold and hence the conditions under which branching occurs depends on H_0 . Whether magnetic fields can suppress or promote crack branching, with obvious implications for fracture control in magnetostrictive devices, has not been systematically investigated.

7. Core Scientific Problem 5: Nonlinearity, Anisotropy, and Realistic Material Behavior

7.1 The Linearity Assumption and Its Failure

Every analytical result in magnetoelastic fracture mechanics reviewed in Sections 3–6 rests on the linearity of both the elastic and magnetic constitutive responses. This assumption is violated in all practically important magnetostrictive materials. Terfenol-D exhibits field-dependent magnetostriction $\lambda(H)$ that saturates above approximately 50 kA/m, follows a nonlinear (and hysteretic) B–H curve, and shows strong stress dependence of its magnetic susceptibility (Dorfmann & Ogden, 2004). Nickel undergoes a sign change in its magnetostriction coefficient at modest applied fields. Magnetorheological elastomers increasingly important for adaptive structural applications combine magnetic particle interactions with the nonlinear elastic response of a polymer matrix.

The consequences for fracture mechanics are severe. Near a crack tip, stresses are large and strains are large relative to the macroscopic elastic limit; hence even a material that behaves linearly under uniform loading will exhibit nonlinear response in the crack tip process zone. The linear asymptotic analysis (Equations 3.1) provides the K-field description valid outside this zone, but the size and properties of the zone and hence the fracture toughness depend critically on the nonlinear coupling behavior, which is not

captured by linear theory. This is the magnetoelastic analog of the plastic zone problem in classical fracture mechanics, but without the extensive analytical and experimental infrastructure developed for elastoplastic fracture (Rice, 1968).

7.2 Anisotropy: Partially Addressed

Crystallographic anisotropy of elastic stiffness and magnetic susceptibility significantly complicates both wave propagation and fracture analyses. The Stroh formalism (Stroh, 1958), extended to magnetoelastic media by Pan (2002) and Soh et al. (2003), provides an elegant compact framework for anisotropic magnetoelastic wave propagation and crack problems. In this formalism, the two-dimensional magnetoelastic equations reduce to an eigenvalue problem whose six (or eight, for full magnetoelastic coupling) eigenvalues p_α determine the near-tip field structure. The Stroh matrices A and B , constructed from the eigenvectors, encode all material anisotropy effects.

Analytical results using the Stroh formalism exist for anisotropic magnetoelastic crack problems (Pan, 2002; Soh et al., 2003), but they are restricted to simple loading modes and geometries. The interaction of anisotropy with dynamic loading (Problem 4) in the Stroh framework has not been systematically explored, and three-dimensional anisotropic crack problems are accessible only through numerical computation. Rao et al. (2001) and Nishimura (2002) have developed BEM formulations for anisotropic magnetoelastic media, but their application to dynamic crack problems remains very limited.

7.3 Multi-Scale Modeling: An Unmet Need

The macroscopic magnetomechanical coupling encoded in the tensor β_{ijk} originates at the mesoscale from the interaction of magnetic domain walls with the crystal lattice and with dislocations. Near a crack tip, the high stress gradient will drive domain wall motion (magnetomechanical effect), which in turn changes the local magnetization and hence the local coupling. This feedback loop stress drives domain wall motion, domain wall motion changes coupling, changed coupling modifies stress is physically important but entirely absent from all existing continuum fracture analyses. Multi-scale models connecting micromagnetic simulations (Landau–Lifshitz–Gilbert dynamics) to macroscopic fracture mechanics through homogenization theory represent a significant research opportunity.

8. Critical Comparison of Solution Methods

Having identified the core problems, we now evaluate the methodological landscape against these problems. The principal approaches Wiener–Hopf analysis, singular integral equations (SIE), finite element method (FEM), boundary element method (BEM), and extended finite element method (XFEM) each occupy a distinct region of the accuracy-generality-cost space.

Table 1. Critical comparison of solution methods for magnetoelastic fracture and wave scattering problems.

Method	Strengths	Weaknesses	Best Applicability
Wiener–Hopf (Analytical)	Exact closed-form results; reveals physical structure of solution; low computational cost	Restricted to semi-infinite cracks in infinite or layered media; scalar (Mode III) kernels only; matrix factorization generally unavailable	Semi-infinite cracks; idealized loading; parameter sensitivity studies
Singular Integral Equations (SIE)	Semi-analytical; moderate complexity; applicable to finite cracks in infinite media; stable numerics via Chebyshev quadrature	Requires explicit Green’s function; restricted to planar cracks; linear material only	Finite cracks in infinite/semi-infinite media; multiaxial loading; parametric SIF studies
Stroh Formalism	Compact anisotropic treatment; exact Green’s functions; elegant algebraic structure	Two-dimensional only; linear material; limited to static or steady-state problems	Anisotropic magnetoelastic media; 2D cracks; interface problems
FEM	Arbitrary geometry and loading; nonlinear material capability; 3D problems	High computational cost; requires SIF extraction post-processing; crack propagation needs remeshing (unless XFEM)	Complex geometry; nonlinear coupling; 3D problems
BEM	Boundary-only discretization; efficient for infinite/semi-infinite domains; inherently satisfies radiation condition	Dense system matrices; requires explicit fundamental solution; kernel singularity handling	Crack problems in homogeneous media; infinite domain problems; linear material
XFEM	No remeshing for propagating cracks; enrichment captures singularity; compatible with standard FEM codes	Enrichment functions derived from elastic (not magnetoelastic) asymptotic fields; implementation complexity; limited 3D validation	Dynamic crack propagation; fatigue growth; geometrically complex cracks

A striking feature of Table 1 is that no single method is adequate for the full scope of the core problems identified in Sections 3–7. The analytical methods (Wiener–Hopf, SIE, Stroh) provide the most rigorous results but are essentially restricted to linear materials and simple geometries precisely the limitations that make them inadequate for Problems 4, 5 (dynamic fracture, nonlinearity). Conversely,

numerical methods (FEM, BEM, XFEM) handle complex geometry and nonlinear material behavior but either require remeshing for propagating cracks (FEM) or employ enrichment functions not optimized for the magnetoelastic crack tip field (XFEM). The BEM requires explicit fundamental solutions that are computationally expensive for general anisotropic magnetoelastic media. This methodological gap the absence of a method that is both geometrically flexible and physically rigorous for magnetoelastic fracture is the central methodological problem of the field and motivates the hybrid approaches discussed in Section 11.

9. Thematic Synthesis of the Literature

9.1 Wave Scattering Studies

The wave scattering literature in magnetoelastic media has developed along two largely independent tracks: analytical solutions for simple geometries, and numerical simulations for complex configurations. Shindo (1977) established the analytical foundation with the SH wave–crack problem. Ing and Wang (2004) extended this to transient loading. Dhua and Chattopadhyay (2016) addressed Love wave propagation in heterogeneous layered magnetoelastic media, finding that the magnetic parameter and heterogeneity gradient interact non-monotonically to affect cutoff frequencies a result not anticipated from simple wave speed augmentation arguments.

A significant gap exists between these analytical studies and the NDE application context that motivates much of the field. In NDE of ferromagnetic pipelines, the relevant wave modes are guided waves (Lamb waves, torsional modes) in plate or cylindrical geometries, not bulk waves in infinite media. The effect of the bias magnetizing field applied to enhance the magnetostrictive coupling of the transducer on the guided wave dispersion and scattering characteristics of defects has been only partially addressed, primarily in the experimental literature (Ribichini et al., 2012) rather than the analytical one.

9.2 Fracture Studies

The fracture literature divides sharply by the assumed electromagnetic crack face condition. Studies adopting the impermeable model (Shindo, 1977; Shindo et al., 1997; Zhou et al., 2004) generally find that the magnetic field increases the effective SIF and promotes fracture. Studies adopting the permeable or realistic model (Gao et al., 2003; Song & Sih, 2003; Zhong & Li, 2006) find field-dependent behavior that is geometry- and loading-sensitive. This division is more than a technical detail: it reflects the unresolved physical question of how the magnetic field redistributes within a crack as the crack opens.

The dynamic fracture literature (Li & Mataga, 1996; Ing & Wang, 2004) is considerably smaller than the static literature and is essentially restricted to Mode III and to steadily propagating cracks. The

transition from quasi-static to dynamic loading including inertial effects, wave radiation, and the magnetoelastic generalization of the dynamic energy release rate has not been comprehensively treated. Freund's (1990) classical dynamic fracture framework provides the elastic foundation, but its magnetoelastic generalization is incomplete.

9.3 Numerical Studies

Numerical studies (Bhargava & Sharma, 2011, 2013; Rao et al., 2001) have confirmed the qualitative trends of the analytical solutions for simple geometries and have begun to address three-dimensional effects and complex crack configurations. However, these studies share a common limitation: they validate against analytical solutions for the very geometries for which those solutions exist, thereby not extending the envelope of knowledge. The genuinely novel configurations three-dimensional penny-shaped cracks, branched cracks, cracks near bimaterial interfaces in magnetoelastic composites remain computationally unstudied in the magnetoelastic context.

10. Critical Gaps and Open Problems

Table 2. Critical research gaps in magnetoelastic wave scattering and fracture dynamics.

Gap Category	Specific Deficiency	Impact on Field	Tractability
Experimental Validation	No controlled experiments measuring K vs H_0 for well-characterized materials with known crack geometry; existing data (Gao et al., 2004) lacks field uniformity control	Fundamental: cannot validate any theoretical model	Tractable with modern in-situ SEM + applied field apparatus
EM Crack Face Conditions	No experimental measurement of effective crack cavity permeability; no consensus on which model is physically correct	Prevents resolution of $K-H_0$ sign controversy	Tractable via micro-CT with applied field or magneto-optical imaging
Nonlinear Models	No fracture mechanics framework for nonlinear magnetoelastic coupling; crack-tip process zone with magnetic saturation unmodeled	Prevents application to Terfenol-D, Galfenol, MR elastomers	Challenging: requires nonlinear continuum formulation
Dynamic Crack Acceleration	Transient crack initiation and acceleration under rising magnetic field unanalyzed	Relevant to pulsed-field loading scenarios	Moderate: Wiener-Hopf + Cagniard-de Hoop methods applicable

Gap Category	Specific Deficiency	Impact on Field	Tractability
Matrix Wiener-Hopf Factorization	No closed-form factorization for in-plane (P-SV) magnetoelastic kernel; approximate methods not applied	Prevents exact analytical treatment of Mode I/II scattering	Moderate: Padé and iterative methods available
Multi-Scale Modeling	Micromagnetic domain physics near crack tip entirely absent from fracture models	Misses physical mechanism of magnetomechanical fracture toughness change	Long-term: requires LLG–elasticity coupling
3D Complex Geometries	Elliptical, penny-shaped, surface cracks in magnetoelastic media unstudied numerically	Limits engineering applicability	Tractable with XFEM extension
AI / Data-Driven Methods	No physics-informed neural networks or machine learning applied to magnetoelastic fracture inverse problems	Missed opportunity for NDE applications	Tractable immediately using existing PINN frameworks

11. Future Research Directions: A Structured Roadmap

11.1 Near-Term Priorities (1–3 Years)

The most immediately impactful research actions address gaps that are tractable with existing methodology. First, the electromagnetic crack face condition problem (Section 4.2) should be resolved through a dedicated experimental program: in-situ digital image correlation (DIC) combined with magneto-optical Kerr effect (MOKE) imaging of cracks in nickel or Galfenol specimens under applied fields would simultaneously characterize the mechanical (displacement jump) and magnetic (B-field distribution) state of the crack, enabling direct determination of the effective cavity permeability. This experiment is technically feasible with existing laboratory infrastructure.

Second, the matrix Wiener–Hopf factorization problem for in-plane magnetoelastic crack scattering should be systematically attacked using approximate factorization methods (Padé approximants, Abrahams’ residue approach, iterative Wiener–Hopf) that have proven effective for analogous problems in fluid mechanics (Norris & Abrahams, 2008). This would yield semi-analytical solutions for Mode I and Mode II dynamic SIF in magnetoelastic media for the first time.

Third, XFEM implementations for magnetoelastic crack problems should be extended to incorporate magnetoelastic crack-tip enrichment functions derived from the Stroh asymptotic field, rather than the purely elastic enrichment currently used. This would improve SIF accuracy for strongly coupled materials without requiring a fundamentally new numerical framework.

11.2 Medium-Term Priorities (3–7 Years)

Multi-scale modeling frameworks connecting micromagnetic domain wall dynamics to macroscopic fracture parameters represent the most significant scientific opportunity in the medium term. A two-scale model mesoscale Landau–Lifshitz–Gilbert (LLG) micromagnetics for the near-tip domain microstructure, coupled through asymptotic matching to a macroscopic linear magnetoelastic fracture model would capture the domain-wall-mediated enhancement of fracture toughness observed experimentally in magnetostrictive materials without the computational burden of full-field micromagnetic simulation of macroscale specimens.

Machine learning and physics-informed neural network (PINN) approaches offer a second medium-term opportunity. For the forward problem, PINNs trained to satisfy Equations (2.1)–(2.3) as soft constraints can provide solutions to magnetoelastic MBVPs in complex geometries at a fraction of the cost of mesh-based methods (Raissi et al., 2019). For the inverse problem inferring crack geometry and material parameters from measured wave scattering signatures deep learning architectures trained on high-fidelity simulation databases represent the most promising route to real-time magnetoacoustic NDE systems.

11.3 Long-Term Vision (7+ Years)

The long-term scientific vision for the field is a fully nonlinear, multi-scale, experimentally validated framework for magnetoelastic fracture capable of predicting the fracture behavior of realistic magnetostrictive components under complex multi-axial and time-varying loading. This requires: a thermodynamically consistent nonlinear constitutive theory for finite-deformation magnetoelasticity (extending the framework of Dorfmann & Ogden, 2004); a nonlinear fracture mechanics theory for such materials (analogous to J-integral theory in nonlinear elasticity); and experimental characterization protocols capable of identifying the nonlinear material parameters from accessible measurements. Smart material metamaterials structured media engineered to exhibit prescribed effective magnetoelastic coupling offer an additional exciting frontier where wave scattering can be shaped by design, with potential applications in adaptive wave filters and earthquake-resistant foundations.

12. Conclusion

This problem-oriented review has organized the literature on wave scattering and fracture dynamics in magnetoelastic media around five core scientific problems, revealing a landscape in which substantial analytical progress exists alongside significant unresolved challenges. The principal conclusions are as

follows.

The crack-tip singularity structure in magnetoelastic media is qualitatively similar to the classical $r^{-1/2}$ elastic singularity, but its amplitude (the SIF) is modified by the magnetic field through both the constitutive coupling and the electromagnetic body force. The direction and magnitude of this modification depends critically on the electromagnetic boundary conditions at the crack face impermeable or permeable and the two principal models yield qualitatively opposite predictions for the $K-H_0$ relationship. This contradiction has not been experimentally resolved and represents the most urgent practical problem in the field.

The mathematical framework for magnetoelastic MBVPs is well-developed for scalar (Mode III, antiplane) problems through the Wiener–Hopf technique and singular integral equations. The in-plane (Mode I, II) problem, requiring matrix Wiener–Hopf factorization, remains analytically open. Numerical methods (FEM, BEM, XFEM) extend geometric generality but sacrifice the physical insight and asymptotic exactness of analytical approaches; their combination through hybrid analytical–computational strategies is the most productive near-term methodological direction.

Dynamic fracture analyses are restricted to idealized configurations and do not address the physically important cases of crack acceleration under rising magnetic fields or the influence of magnetic fields on dynamic crack branching instability. Nonlinear material behavior, present in all practically important magnetostrictive materials, is absent from essentially all fracture analyses. Multi-scale modeling of the micromagnetic near-tip zone is entirely undeveloped.

The research roadmap proposed in Section 11 prioritizes, in order: (i) experimental resolution of the electromagnetic crack face condition; (ii) analytical development of approximate matrix Wiener–Hopf factorization for in-plane problems; (iii) improved XFEM enrichment for magnetoelastic crack tips; (iv) multi-scale LLG–continuum fracture modeling; (v) PINN and machine learning methods for forward and inverse magnetoelastic fracture problems. Execution of this roadmap would transform a field currently rich in isolated analytical results into a coherent, experimentally validated, and computationally tractable framework for the design and structural assessment of magnetically sensitive engineering systems.

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