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AN APPLICATION OF FUZZY QUANTIFIER IN SEQUENCING PROBLEM WITH FUZZY RANKING METHOD

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ABSTRACT:
This paper presents a sequencing problem with the aid of triangular or trapezoidal fuzzy numbers. For finding the initial solution of this problem we have preferred the fuzzy quantifier and ranking method, also the optimal solution(order) by using the method of processing n jobs through two machines has been carried out. The solution procedure is illustrated with numerical example.
Key words: ***Fuzzy sequencing problem, Fuzzy quantifier, Trapezoidal fuzzy number, ranking of fuzzy numbers, optimal solution.***

INTRODUCTION

Making decisions in day to day life is vague, ambiguous, incomplete, and imprecise. Crisp logic or conventional logic theory is inadequate for dealing with imprecision, uncertainty and complexity of the real world. This realization that motivated the evolution of fuzzy logic and fuzzy theory. Lofti A. Zadeh(1965) first introduced Fuzzy set as a mathematical way for representing impreciseness or vagueness in every day life.

A series, in which a few jobs or tasks are to be performed following an order, is called sequencing. In such a situation, the effectiveness measure (time, cost, distance etc.,) is a function of the order or sequence of performing a series of jobs. A fuzzy sequencing problem is a problem in which the processing time are fuzzy quantities. The objective of the fuzzy sequencing problem is to determine the orders or sequences of jobs in which they should be performed so as to minimize the total elapsed time, T.

Sakthi et.al [1] adopted Yager's ranking method [2] to transform the fuzzy sequencing problem to a crisp one so that the conventional solution methods may be applied to solve the sequencing problem. In this paper we investigate a sequencing problem with fuzzy processing time t_{ij} represented by fuzzy quantifier which are replaced by triangular or trapezoidal fuzzy numbers.

DEFINITIONS AND FORMULATIONS:

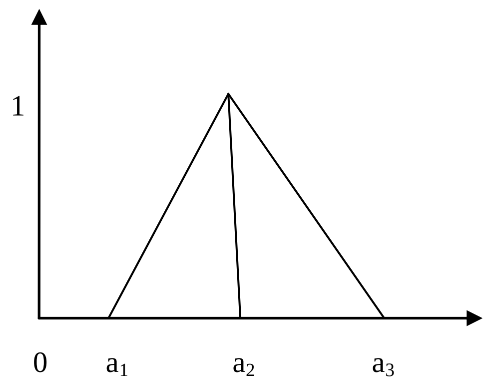
Fuzzy set

A Fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A}=\{(x,\mu(x)/x \in X\}$, where $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \in \tilde{A}$.

Triangular fuzzy number

A triangular fuzzy number is denoted by (a_1,a_2,a_3) where a_1,a_2,a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & x = a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$



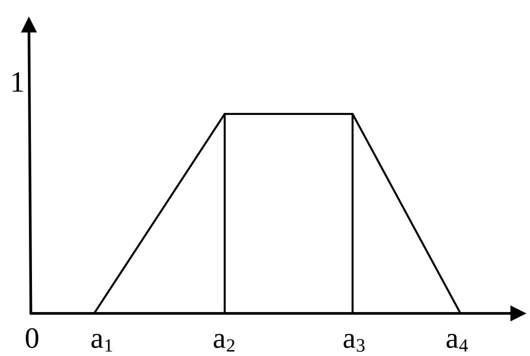
$\mu_{\tilde{a}}(x)$ satisfies the following conditions

1. $\mu_{\tilde{a}}(x)$ is a continuous mapping from R to closed interval [0,1].
2. $\mu_{\tilde{a}}(x) = 0$ for every $x \in (-\infty, a_1]$
3. $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on $[a_1, a_2]$
4. $\mu_{\tilde{a}}(x) = 1$ for every $x = a_2$
5. $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on $[a_3, a_2]$
6. $\mu_{\tilde{a}}(x) = 0$ for every $x \in [a_3, \infty)$.

TRAPEZOIDAL FUZZY NUMBER

A trapezoidal fuzzy number $\tilde{a}$ is denoted by (a_1, a_2, a_3, a_4) where a_1, a_2, a_3, a_4 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \le a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_2 \le x \le a_1 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & x \ge a_4 \end{cases}$$



$\mu_{\tilde{a}}(x)$ satisfies the following conditions

1. $\mu_{\tilde{a}}(x)$ is a continuous mapping from R to closed interval [0,1]
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3. $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on $[a_1, a_2]$
4. $\mu_{\tilde{a}}(x) = 1$ for every $x \in [a_2, a_3]$
5. $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on $[a_3, a_4]$
6. $\mu_{\tilde{a}}(x) = 0$ for every $x \in [a_4, \infty)$.

α-cut

The α -cut set of a fuzzy set Ã is a crisp set defined by $\tilde{A}_{\propto} = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\}$.

ARITHMETIC OPERATIONS

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be triangular fuzzy numbers then

$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1+b_1,\ a_2+b_2,\ a_3+b_3)$$

Similarly, let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers then

$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1+b_1,\ a_2+b_2,\ a_3+b_3, a_4+b_4)$$

FUZZY QUANTIFIER

Fuzzy quantifiers are fuzzy number that take part in fuzzy propositions. Fuzzy quantifiers that characterize linguistic terms such as about 10, much more than 100, atleast about 5 of the first kind and almost all, about half, most and so on of the second kind.

One of them is the form

p: There are Q i's in I such that V(i) is F,

Where V(i) of V is a variable, Q is a fuzzy number expressing linguistic term, F is a fuzzy set of variable V.

Example p: "There are about 10 students in a given class whose fluency in English is high".

Here Given a set of students, I, the value of V(i) of variable V represents the degree of fluency in English of student i, F is a fuzzy set that expresses the linguistic term high, and Q is a fuzzy number expressing the linguistic term about 10.

THE PROPOSED METHOD

Sequencing Problem can be classified into the following groups

1. In the first type of problem, we have n jobs to perform each of which requires processing on some or all m machines. If we analyse the number of sequences, it runs to $(n!)^m$ possible sequences and only a few of them are technologically feasible, i.e., those which satisfy the constraints on the order in which each task has to be processed through m machines.
2. In the second type of problem we have a situation with a number of machine, and a series jobs to perform once a job is finished, we have to take a decision on the next job to be started.

   Now we know solutions only for some special cases of the first type of problems.

PROCESSING N JOBS THROUGH TWO MACHINES

The sequencing problem with n jobs through two machines can be solved easily. S.M.Johnson has developed solution procedure. The problem can be stated as follows;

1. Only two machines are involved, A and B.
2. Each job is processed in the order AB.
3. The exact or expected processing times $A_1, A_2, \ldots, A_n$, $B_1, B_2, \ldots, B_n$ are known.

A decision has to be arrived to find the minimum elapsed time from the start of the first job to the completion of the last job. It has been established that the sequence that minimizes the elapsed time are the same for both machines. The algorithm for solving the problem is as follows:

1. Select the smallest processing time occurring in the list, $A_1, A_2, \ldots, A_n$, $B_1, B_2, \ldots, B_n$ if there is a tie break the tie arbitrarily.
2. If the minimum processing time is Ai, do the ith job first. If it is Bj do the jth job last. This decision is applicable to both machines A and B.
3. Having selected a job to be ordered, there are now n-1, jobs left to be ordered. Apply the steps 1 and 2 to the reduced set of processing times corresponding to the job that is already assigned.
4. Continue in this manner until all jobs have been ordered. The resulting ordering will minimize the elapsed time,T.

NOTATIONS

t_{ij} = processing time(time required) for job i on machinej.

T = Total elapsed time for processing all the jobs including idle time

I_{ij} = Idle time on machine j from the end of job (i-1) to the start of job i.

i.e., t_{ij} = 1, if the ith job is processed on jth machine

= 0, otherwise.

is the decision variable denoting the processing of the machine j to job i, t_{ij} is the time of processing the ith job to the jth machine. Hence, it cannot be determined the required time directly. For solving this problem we first transform the processing time into a crisp one by a Yager's ranking method. It is a robust ranking technique, which satisfies linearity and additive property. Yager's ranking index is defined by

$$\mathrm{Y}(\tilde{t}) = \int_0^1 0.5(c_\propto^L + c_\alpha^U)\, d\alpha$$

where $(c_\propto^L, c_\alpha^U)$ is a ∝-level cut of fuzzy number $\tilde{t}$.

The Yager's ranking index $\mathrm{Y}(\tilde{t})$ gives the representative value of the fuzzy number $\tilde{t}$. Since $Y(\tilde{t}_{ij})$ are crisp values, this problem is obviously the crisp sequencing problem which can be solved by processing n jobs through two machines method.

The steps of the proposed method are

Step 1: Replace the processing time t_{ij} with linguistic variables by triangular or trapezoidal fuzzy numbers.

Step 2: Find Yager's Ranking index.

Step 3: Replace Triangular or Trapezoidal numbers by their respective ranking indices.

Step 4: Solve the resulting SP using processing n jobs through two machines to find optimal sequence and minimum elapsed time,T.

NUMERICAL EXAMPLE

Consider the fuzzy sequencing problem. Here the processing time$(\tilde{t}_{ij})$ of nine jobs is given whose elements are fuzzy quantifiers which characterize the linguistic variables are replaced by fuzzy numbers. The problem is then solved by processing n jobs through two machines.

Table 1

Task	Machine A	Machine B
I	Bad	Medium
II	Low	Average
III	Very low	High
IV	Very high	Very low
V	Medium	Extremely low
VI	Average	Very high
VII	High	Extremely low
VIII	Low	Very high
IX	Very low	Extremely high

SOLUTION

The linguistic variables showing the qualitative data is converted into quantitative data using the following table. As the processing time varies between 0 to 30 minutes the minimum possible value is taken as 0 and the maximum possible value is taken as 30.

Bad	(0,2,3,4)
Extremely low	(6,8,10,12)
Very low	(8,10,12,14)
Low	(10,12,14,16)
Average	(11,12,13,14)
Medium	(16,18,20,23)
High	(14,17,20,23)
Very high	(18,20,22,25)
Extremely high	(22,25,28,30)

The linguistic variables are represented by trapezoidal fuzzy number.

Now from Table 1 we have

Task	Machine A	Machine B
1	(0,2,3,4)	(11,12,13,14)
2	(10,12,14,16)	(16,18,20,23)
3	(8,10,12,14)	(14,17,20,23)
4	(18,20,24,25)	(8,10,12,14)
5	(11,12,13,14)	(6,8,10,12)
6	(16,18,20,23)	(18,20,22,25)
7	(14,17,20,23)	(6,8,10,12)
8	(10,12,14,16)	(18,20,22,25)
9	(8,10,12,14)	(22,25,28,30)

We calculate Y(0,2,3,4) by applying the Yager's ranking method.

The membership function of the trapezoidal fuzzy number (0,2,3,4) is

$$\mu(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x-0}{2-0} & 0 \leq x \leq 2 \\ \frac{4-x}{4-3} & 3 \leq x \leq 4 \\ 0, & x \geq 4 \end{cases}$$

The ∝-cut of the fuzzy number (0,2,3,4) is

$$(c_{\propto}^{L}, c_{\alpha}^{U}) = \big((2-0)\alpha, 4-(4-3)\alpha\big)$$
$$= (2\propto,\ 4-\alpha)$$

Therefore,

$$Y(\tilde{t}_{11}) = Y(0,2,3,4) = \int_0^1 0.5(c_{\propto}^{L} + c_{\alpha}^{U})\ d\propto$$
$$= \int_0^1 0.5(2\propto +4-\alpha)\ d\alpha$$
$$= \int_0^1 0.5(\propto +4)\ d\alpha$$
$$= \int_0^1 (0.5\propto +2)\, d\alpha$$
$$Y(\tilde{t}_{11}) = 2.25$$

Proceeding similarly, the Yager's indices for the processing time $\tilde{t}_{ij}$ are calculated as follows.

$Y(\tilde{t}_{21}) = 13$, $Y(\tilde{t}_{31}) = 11$, $Y(\tilde{t}_{41}) = 20.25$, $Y(\tilde{t}_{51}) = 12.5$, $Y(\tilde{t}_{61}) = 18.25$, $Y(\tilde{t}_{71}) = 18.5$,
$Y(\tilde{t}_{81}) = 13.4$, $Y(\tilde{t}_{9\,1}) = 11$, $Y(\tilde{t}_{12}) = 12.57$, $Y(\tilde{t}_{22}) = 18.25$, $Y(\tilde{t}_{32}) = 18.54$, $Y(\tilde{t}_{42}) = 11$, $Y(\tilde{t}_{52}) = 9$,
$Y(\tilde{t}_{62}) = 20.25$, $Y(\tilde{t}_{72}) = 9$, $Y(\tilde{t}_{82}) = 20.25$, $Y(\tilde{t}_{92}) = 26.25$

Hence, the processing times for the jobs are as follows

Task	I	II	III	IV	V	VI	VII	VIII	IX
Machine A	2.25	13	11	20.25	12.5	18.25	18.5	13	11
Machine B	12.5	18.25	18.5	11	9	20.25	9	20.25	26.25

The smallest processing time between the two machines is 2.25 which corresponds to task I on machine A. Thus task I will be processed first as shown below.

I								

After the task A has been set for processing first, we are left with 8 tasks and their processing times as given below.

Task	II	III	IV	V	VI	VII	VIII	IX
Machine A	13	11	20.25	12.5	18.25	18.5	13	11
Machine B	18.25	18.5	11	9	20.25	9	20.25	26.25

The minimum processing time is 9 which corresponds to task V and VII both on machine B. Therefore task V will be processed in the last and task VII will be penultimate. Thus current partial assignment becomes:

I							VII	V

Further performing the set of processing times now gets reduced to an alternative optimum sequence. The optimum sequences are, therefore, given below:

I	III	IX	II	VIII	VI	IV	VII	V

or

I	III	IX	VIII	II	VI	IV	VII	V

Now the minimum elapsed time can be compute cumulatively as follows:

Tasks	Machine A		Machine B	
	Time in (minutes)	Time out (minutes)	Time in out (minutes)	Time (minutes)
I	0	2.25	2.25	14.75
III	2.25	13.25	14.75	33.25
IX	13.25	24.25	33.25	59.50
II	24.25	37.25	59.50	77.75
VIII	37.25	50.25	77.75	98.00
VI	50.25	68.50	98.00	118.25
IV	68.50	88.75	118.25	129.25
VII	88.75	107.25	129.25	138.25
V	107.25	119.75	138.25	147.25

The minimum elapsed time T = 147.25 minutes

The idle time on machine A is 147.25-119.75=27.50 minutes and the idle time on machine B is 2.25+ 138.25-138.25= 2.25 minutes.

CONCLUSION

In this paper, the processing times are considered as fuzzy quantifiers that characterize linguistic variables represented by trapezoidal fuzzy numbers. The fuzzy sequencing problem has been transformed into crisp sequencing problem using Yager's ranking indices. Hence we have shown that the fuzzy sequencing problems of qualitative nature can be solved in an effective way. Moreover, the solution of fuzzy problems obtained more effectively.

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