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BI-OBJECTIVE CRITERIA IN TWO MACHINES SCHEDULING PROBLEM WITH FUZZY DUE DATE

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ABSTRACT:
The present study investigates a different approach to nx2 flow shop scheduling problem under fuzzy environment with bi-objective criteria. On one side to minimize the tardiness of jobs and on the other side to find the satisfaction level of demand maker with the help of fuzzy due date to each job. The membership function of fuzzy due time assigned to each job shows the satisfaction level of demand maker. The analysis has been made through heuristic algorithm as well as graphically. The study may be helpful for a company of huge demand of market since a check can be applied on early expiry of jobs, storage problem etc.
Key Words: ***Fuzzy processing time, trapezoidal, tardiness, <AHR>.***

INTRODUCTION:

Scheduling has been examined in Operation Research literature since early 50's through the remarkable work done by Johnson (1954). In general way scheduling can be defined as the problem of sequencing a set of operations and allocating them to temporal slots without violating technical and capacitive constraints .These operations may be operations on machines in a factory, train, car or aeroplane movements in transportation, lesson plans at college, computer processes perform on the CPU etc. A good scheduling strategy may help companies to respond the market demand quickly and to run the plant efficiently. This helps the plant to be more competitive in the market.
The multi-criteria project scheduling problem is characterized by four components, a set of resources R, a set of activities A, a set of procedure constraints C and a set of project performance measures Q i.e. to find an allocation (considered in time) of resources from set R to activities (jobs or tasks) from set A such that all activities in A are completed, the constraints are satisfied and best compromise between criteria from set C to Q is reached. A fuzzy scheduling algorithm builds into the real time system flexibility adaptation to the uncertainty inherent in real time environments and offers a means to improve several important characteristics of real time systems. The fuzzy membership function is a continuous function possessing fractional degree of membership.

LITERATURE SURVEY:

The concept of fuzzy processing time, fuzzy due dates, fuzzy precedence relation etc. is being introduced by various researchers in the field of scheduling. Mohd. Ikram (1986), Conway(1965) discussed about earliness & lateness of jobs in flow shop scheduling. Mc Cahon & Lee (1992) studied the job sequencing problem when job processing times are represented with fuzzy numbers. Chong et al (1955), Shibuchi et al (1996) and Ishii(1995) have studied fuzzified scheduling problems by using fuzzy due date.T.P. Singh, Sunita (2008,2010) made an attempt to find the demand maker satisfaction level under triangular fuzzy rule in 2 and 3 stage scheduling problems.
This paper is an extension work done by Sunita et al (2010) in the sense that the fuzzy environment has been considered trapezoidal in nature and graded mean integration representation has been applied. The transportation time from first machine to second is also being included. In the concept of fuzzy due date, the membership function of fuzzy due date assigned to each job represents the human user's satisfaction grade for the completion time of the job. Thus the fuzzy due date is directly related to the earliness and tardiness penalty in conventional scheduling problems and an algorithm has been constructed. In support of our algorithm, a numerical illustration is also

presented and satisfaction level has been shown graphically. The paper investigates a different approach to nx2 flow shop scheduling with bi-objective criteria. On one side to minimize the tardiness of jobs and on the other side to find the satisfaction level of demand maker with the help of fuzzy due date to each job. The membership function of fuzzy due time assign to each job shows the satisfaction level of demand maker. A comparative analysis between completion time and due time of each job in order to find out the satisfaction level of demand maker has been made. The processing time are taken in trapezoidal fuzzy no. The fuzzy decision rules are used to express a performance evaluation for the decision which can be made to solve a conflict. Thus the paper is wider and more applicable in production concern.

When we can not define a fixed due date for a job, we use the concept of fuzzy due date. A fuzzy due date is associated with each job and membership of the fuzzy due date of job i is denoted by μ_i which represents the grade of satisfaction level of demand maker when the completion time of job C_i is being already calculated.

GRADED MEAN ITEGRATION REPRESENTATION METHOD:

Chen S. H & Hsiel C.H.(1999) introduced graded mean integration representation method based on the integral value of graded mean h level of generalized fuzzy number for defuzzifying generalized fuzzy member. The generalized fuzzy number is defined as follows:

Suppose $\tilde{A}$ is a generalized fuzzy number. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

- $\mu_{\tilde{A}}$ (x) is a continuous mapping from R to [0,1],
- $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq\cong a_{1,}$
- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
- $\mu_{\tilde{A}}(x) = w_{A,} a_2 \leq x \leq a_3$,
- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
- $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

Where $0< w_{A,} \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers.

Generalized fuzzy numbers are denoted as $\tilde{A}= (a_1, a_2, a_3, a_4 ; w_A)_{LR}$. The graded mean h-level value of generalized fuzzy number $\tilde{A}= (a_1, a_2, a_3, a_4 ; w_A)_{LR}$ is given by $\frac{h}{2}\{L^{-1}(h) + R^{-1}(h)\}$. Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{w_A} \frac{h}{2}\{L^{-1}(h)+R^{-1}(h)\}\, dh}{\int_0^{w_A} h\, dh},$$

Where $0 < h \leq w_A$ and $0< w_A \leq 1$.

Let $\tilde{A}$ be a trapezoidal fuzzy number and denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$. Then we can get the Graded mean Integration Representation of $\tilde{A}$ by the formula as:

$$P(\tilde{A}) = \frac{\int_0^1 \frac{h}{2}\{(a_1+a_4)+h(a_2-a_1-a_4+a_3)\}dh}{\int_0^1 h\, dh} = \frac{a_1+2a_2+2a_3+a_4}{6}$$

SATISFACTION LEVEL:

As demand maker wants that the job should be completed at a particular time but in real situation, it has been observed that the output generally falls before or after the fixed time, which creates dissatisfaction to the demand maker. His satisfaction level depends on the time that how much early or later it has been reached. The satisfaction level is defined as follows

$$\mu_i = \begin{cases} 0 & ;\text{ if } x<d_{i1} \\ (x-d_{i1})/(d_{i2}-d_{i1}) & ;\text{ if } d_{i1}<x<d_{i2} \\ 1 & ;\text{ if } d_{i2}<x<d_{i3} \\ (d_{i4}-x)/(d_{i4}-d_{i3}) & ;\text{ if } d_{i3}<x<d_{i4} \end{cases}$$

0 ; if $x>d_{i4}$

ASSUPTIONS AND NOTATIONS:

Assumptions:

- No job pre- emption is allowed.
- The machine can process one job at a time.
- All jobs are available at time zero.
- The machine set up time is negligible.
- Due date are fuzzy.
- Fixed transportation time is taken.

Notations:

A-<AHR> Average high ranking of fuzzy numbers (a, b, c, d) =(a +2b+2c+d)/6

A_{i1}-Average high ranking of the processing time of i^{th} job on 1^{st} machine.

A_{i2}-Average high ranking of the processing time of i^{th} job on 2^{nd} machine.

P_i – Processing time of i^{th} job.

L – Total tardiness of all the jobs.

K- Constant factor for which total tardiness is optimized.

d_{ij} - Due time for i^{th} job and j^{th} fuzzy time

C_i – Completion time of i^{th} job.

t_i – Transportation time from first machine to second machine.

μ_i – Degree of satisfaction level of i^{th} job for the demand maker.

THEOREM ON TARDINESS OF THE JOBS:

Let Pi be the process time of i^{th} job on both machines and KA_{i1} is the due time of the job. Then tardiness of the job is $L^2 = \sum(P_i - KA_{i1})^2$ where K is the factor depending on P_i & A_i is minimum for K $=\sum \frac{P_i A_{i1}}{A_{i1}^2}$.

THEOREM:

The theorem is proved on the basis of work done by Singh T.P., Sunita (2010). To find the value of the constant K, for which the tardiness of jobs L is minimum. This result can be proved by simple calculus of maxima and minima. Total tardiness of the jobs is the time in which the jobs are idle i.e. $L^2 = \sum(P_i - KA_{i1})^2$ where i =1 to n.

By using simple differentiation and the fact the first differential equating to zero we have K $=\sum \frac{P_i A_{i1}}{A_{i1}^2}$. Now for the minimum value, second differentiation should be positive.

$\frac{d^2L}{dK^2} = 2\sum A_{i1}^2 > 0$ being square .

L^2 is minimum for K $=\sum \frac{P_i A_{i1}}{A_{i1}^2}$ .Since due time depends upon the processing time of jobs on the first machine, so we desire to get the job early whose due time is early . As smaller the processing time early is the due time. Hence the sequence of jobs can be made in non decreasing order.

ALGORITHM:

Step 1:Find <AHR> of the processing time of each job in fuzzy environment (a, b, c, d) =(a +2b+2c+d)/6 on the basis of grade mean integration representation.

Step 2: Add transportation time as follows: $A_{i1}^{'} = A_{i1} + t_i$ & $A_{i2}^{'} = A_{i2} + t_i$.

Step 3: Arrange the sequence in non decreasing order of $A_{i1}^{'}$.

Step 4: Find the processing time P_i of each job i.

Step 5: Find the value of K by using the formula K $=\sum \frac{P_i A_{i1}}{A_{i1}^2}$

Step 6: Using the value of K, find due times of each job in fuzzy environment d_{ij} =K*processing time of i^{th} job in j^{th} fuzzy time.

Step 7: Find the completion time C_i of each job in fuzzy environment. We find the completion time C_i by using the formula $C_i = (C_{i1}+2C_{i2} + 2C_{i3} +C_{i4})/6$ on the basis of grade mean integration representation.

Step 8: Comparing completion time and due time of the jobs, we can find the satisfaction of demand maker.

NUMERICAL EXAMPLE:

Let there are 5 jobs & 2 machines whose processing time is given in trapezoidal fuzzy number and fixed transportation time is given from first machine to second machine.

JOB	M1	ti	M2
1	(8,9,11,13)	5	(8,10,11,12)
2	(8,9,10,11)	3	(10,11,12,13)
3	(6,8,9,10)	4	(15,18,20,22)
4	(9,10,12,14)	2	(6,8,9,12)
5	(10,12,13,14)	6	(6,7,9,11)

Our objective is to find the completion time and due time of each job also find the degree of satisfaction level of each job for the demand maker.

SOLUTION:

Step1: Find <AHR> of the processing time of each job as define in step 1 of the algorithm

Job	1	2	3	4	5
A_1	61/6	57/6	50/6	67/6	74/6
A_2	74/6	69/6	113/6	52/6	49/6

Step 2: Add transportation time as define in step 2 of the algorithm

Job	1	2	3	4	5
A_1’	91/6	75/6	74/6	79/6	110/6
A_2’	104/6	87/6	137/6	64/6	85/6

Step 3: The Non decreasing sequence in which jobs are processed is 3-2-4-1-5.

Step 4: Processing time for each job is as follows

Job	M1		M2	
	Time In	Time Out	Time In	Time Out
3	0	74/6	74/6	211/6
2	74/6	149/6	211/6	298/6
4	149/6	228/6	298/6	362/6
1	228/6	319/6	362/6	466/6
5	319/6	429/6	466/6	551/6

Hence P_1=466/6, P_2=298/6, P_3=211/6, P_4=362/6, P_5=551/6.

Step 5: Using the formula as per step 5 in algorithm, K=4.49.

Step 6: The list of due time in fuzzy environment as step 6 of algorithm is as follows:

d_{11}=35.92	d_{12}=40.41	d_{13} =49.39	d_{14}=58.37
d_{21}= 35.92	d_{22}=40.41	d_{23}=44.9	d_{24}=49.39
d_{31}=26.94	d_{32}=35.92	d_{33}=40.41	d_{34}=44.9

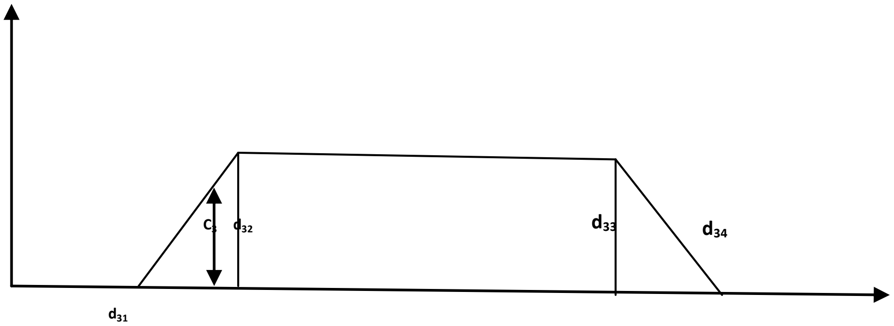
d_{41} =40.41	d_{42} =44.9	d_{43}=53.88	d_{44}=62.86
d_{51}=44.9	d_{52}=53.88	d_{53}=58.37	d_{54}=62.86

Step 7: The completion time of jobs as per step 7 of algorithm are as follows:

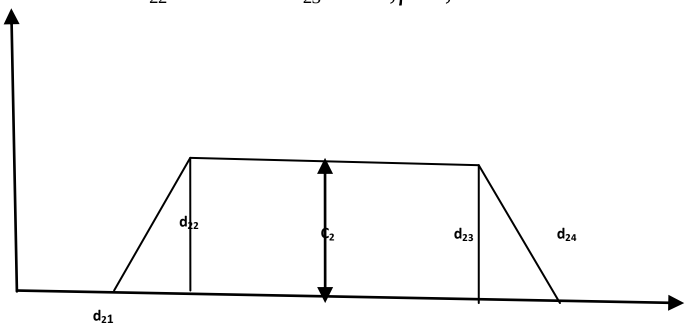
C_3=31.16, C_2=42.66, C_4=60.33, C_1=50.83, C_5=69.83

Step 8: **ANALYTICAL STUDY:**

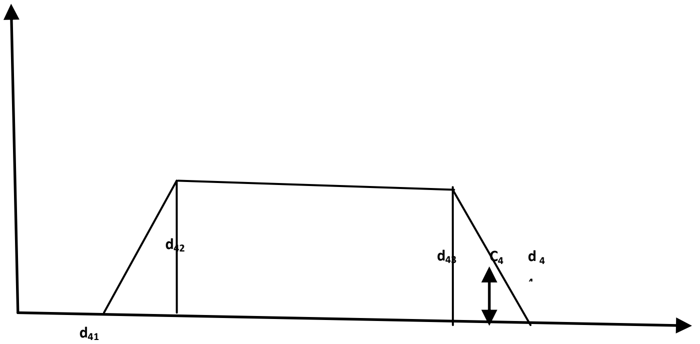
(a) C_3=31.16 lies between d_{31}=26.94 & d_{32}=35.92, μ=.88, satisfaction level is .88



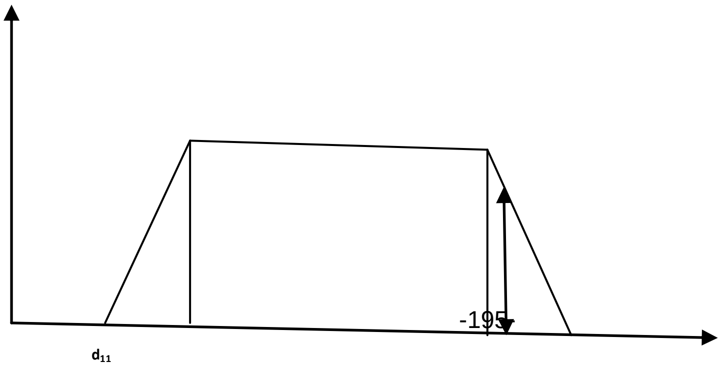
(b) C_2 =42.66 lies between d_{22}=40.41 & d_{23}=44.9,μ=1, satisfaction level is 1.



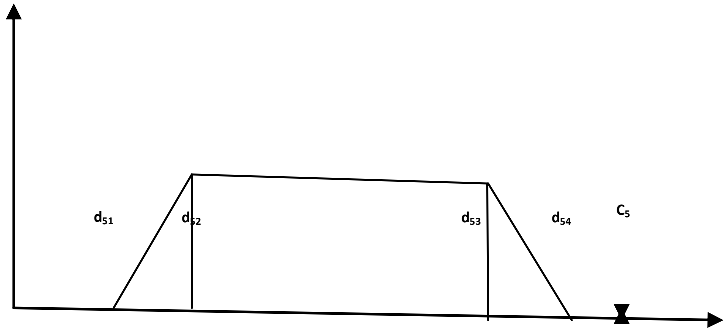
(c) C_4=60.33 lies between d_{43} =53.88 and d_{44} =62.86, μ= .28,Satisfaction level is .28



(d) C_1=50.80 lies between d_{13}=49.39 and d_{14}=58.37,μ= .84,Satisfaction level is .84



(e) $C_5 = 69.83$, $d_{54} = 62.86$, $C_5 > d_{54}$, $\mu = 0$, Satisfaction level is zero.



CONCLUDING REMARKS AND FURTHER RESEARCH AREA:

In this model we have shown that the satisfaction level of demand maker depends upon due date and comparative study of completion time and due time of jobs has been made quantitatively as well as graphically e.g. completion time of third job is 31.16 which lies between 1st & 2nd due dates of third job where satisfaction level of demand maker is 88 % and at this stage tardiness cost is minimum. Similarly other cases for each job have been discussed. The study can further be extended to 3 or m machines. We have considered only the lateness of the jobs not the earliness of the jobs. Generally lateness factor is supposed to be the sign of dissatisfaction and in certain company, a penalty is imposed after crossing the time of due date. Fuzzy due dates can be justified being flexible in the time variation.

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d12 d13 C1 d14