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THE COMPLETE SOLUTION PROCEDURE FOR THE FUZZY EOQ INVENTORY MODEL WITH LINEAR AND FIXED BACK ORDER COST

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ABSTRACT

The nature of the inventory problem consists of repeatedly placing and receiving orders of given sizes at set intervals. In this paper, we discuss the inventory problem with fuzzy backorder. Yager's ranking method for fuzzy numbers is utilized to find the inventory policy in terms of the fuzzy total cost. Finally a numerical example is given to illustrate the model.

Keywords : Fuzzy number, inventory, backorder, optimization.

INTRODUCTION

Within the context of traditional inventory models, the pattern of demands is either deterministic or uncertain. In practice, the latter corresponds more to the real-world environment. To solve these inventory problems with uncertain demands, the classical inventory models usually describe the demands as certain probability distributions and then solve them. However, some times, demands may be fuzzy, and more suitably described by linguistic term rather than probability distributions. If the traditional inventory theories can be extended to fuzzy senses, the traditional inventory models would have wider applications.

Usually, inventory systems are characterized by several parameters such as cost coefficients, demands etc. Accordingly, most of the inventory problems under fuzzy environment can be addressed by fuzzying these parameters. For instance, Park [8] discussed the EOQ model with fuzzy cost coefficients. Ishii and Konno [3], Petrovic et. al. [9], and Kao and Hsu [4] investigated the Newsboy inventory model with fuzzy cost coefficients and demands respectively. Roy and Maiti [10] developed a fuzzy EOQ model with a constraint of fuzzy storage capacity. Chang [1] construct a fuzzy EOQ model with fuzzy defective rate and fuzzy demand.

Yao and Chiang [13] compare the EOQ model with fuzzy demand and fuzzy holding cost in different solution methods. Kao and Hsu [5] find the lot size-reorder point model with fuzzy demand. Besides, there is another kind of studies which fuzzes the decision variables of inventory models. For example: Yao and Lee [15] developed the EOQ model with fuzzy ordering quantities; Chang and Yao [2] investigated the EOQ model with fuzzy order point; Wen-Kai K. Hsu and Jun-Wen Chen [11] studied Fuzzy EOQ model with stock out. Madhu & Deepa [7] developed an EOQ model for deteriorating items having exponential declining rate of demand under inflation & shortage. Kun-Jen Chung, Leopoldo Eduardo Cárdenas-Barrón [6] compare the complete solution procedure for the EOQ and EPQ inventory models with linear and fixed backorder costs. Recently W. Ritha etal. [14] fuzzified EOQ Model with one time discount offer allowing back.

In this paper, we discuss the inventory problem with fuzzy back order. The decision variables are the ordering quantity Q and the back order quantity S. The approach of this paper is to find the optimal order quantity Q^* with the minimum cost determined from Yager's ranking method.

The rest of this paper is organized as follows; in section 2, the preliminaries are given. In section 3, the total inventory cost of the problem constructed from the α-cut of the back order. In section 4, optimal ordering quantity is derived using Yager's ranking method [12]. Finally, a numerical example is given to illustrate the model.

2. PRELIMINARIES

2.1. Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$. In the pair $\{(x, \mu_{\tilde{A}}(x))\}$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2. α - Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set or α - cut. $A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha$

2.3. Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,

(iii) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,

(iv) $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,

(v) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(vi) $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$, where a_1, a_2, a_3 and a_4 are real numbers.

2.4. Trapezoidal Fuzzy Number

The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, where $a_1 < a_2 < a_3 < a_4$ and defined on R is called the trapezoidal fuzzy number, if the membership function $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \ x < a_1 \text{ or } x > a_4 \\ \dfrac{(x - a_1)}{(a_2 - a_1)} & ; \ a_1 \le x < a_2 \\ 1 & ; \ a_2 \le x < a_3 \\ \dfrac{(x - a_4)}{(a_3 - a_4)} & ; \ a_3 \le x \le a_4 \end{cases}$$

2.5. Yagers' Ranking Method

If the α cut of any fuzzy number $\tilde{A}$ is $[A_L\alpha), A_g(\alpha)]$, then its ranking index $I(\tilde{A})$ is $\frac{1}{2}\int_0^1 \left[A_L(\alpha) + A_g(\alpha)\right] d\alpha$.

3. TOTAL INVENTORY COST:

Notations Used:

D - Demand rate in units /unit of time

K - Ordering cost/ order/set up

h - Holing cost / unit/unit of time

p - Linear back order cost

π - Fixed back order cost

Q - Size of order quantity

S - size of the back order quantity

Q* - optimal value of Q

TC (Q, S) - Total annual cost function of Q and S in $ / year

The total cost function is

$$TC(Q,S) = \frac{KD}{Q} + \frac{h(Q-S)^2}{2Q} + \frac{pS^2}{2Q} + \frac{\pi DS}{Q}$$

$$\Rightarrow \frac{KD}{Q} + \frac{hQ}{2} - hS + \frac{hS^2}{2Q} + \frac{pS^2}{2Q} + \frac{\pi DS}{Q} \qquad \ldots (1)$$

The objective is to find the optimal order quantity which minimize the total cost

The necessary conditions for minimum

$$\frac{\partial TC(Q,S)}{\partial Q} = 0 \Rightarrow -\frac{KD}{Q^2} + \frac{h}{2} - \frac{hS^2}{2Q^2} - \frac{pS^2}{2Q^2} - \frac{\pi DS}{Q^2}$$

Differentiate (1) partially w.r.to S, we obtain,

$$\frac{\partial TC(Q,S)}{\partial S} = 0 \Rightarrow \quad -h + \frac{hS}{Q} + \frac{pS}{Q} + \frac{\pi D}{Q} = 0$$

we get, $S = \frac{hQ - \pi D}{h + p}$

Substitute S in (1), Hence the optimal order quantity is

$$Q^* = \sqrt{\frac{2KD(h+p) - \pi^2 D^2}{hp}}$$

4. THE EOQ MODEL WITH BACK ORDER AND FUZZY DEMANDS

Let $\tilde{D}$ be a normal fuzzy number with parameters $\tilde{D}$ = (l, m, n, u), then the membership function of $\tilde{D}$ can be defined by a left shape function L(x) and a right shape function R(x) as:

$$\mu_{\tilde{D}}(x) = \begin{cases} L(x), & l \le x \le m \\ 1, & m \le x \le n \\ R(x), & n \le x \le u \end{cases}$$

The above equation can also be described by the terms of α-level cut of $\tilde{D}$ as:

$$D(\alpha) = \left[\min . \mu_{\tilde{D}}^{-1}(\alpha),\ \max . \mu_{\tilde{D}}^{-1}(\alpha)\right]$$

$$= \left[L^{-1}(\alpha),\ R^{-1}(\alpha)\right],\ 0 \le \alpha \le 1$$

First, we discussed the EOQ model with fuzzy demand, according to the extension principle; the model can be described by terms of α as

$$\tilde{T}(\alpha) = \left\{TC\left[Q \mid D = L^{-1}(\alpha)\right],\ TC\left[Q \mid D = R^{-1}(\alpha)\right]\right\},\ 0 \le \alpha \le 1$$

Where $TC(Q) = \frac{KD}{Q} + \frac{hQ}{2}$

Since the annual cost function $\tilde{T}(\alpha_D)$ is a fuzzy number, we can compare $\tilde{T}(\alpha_D)$ of different Q by using some ranking methods to find the optimal solution Q* with a minimal total cost, of which, not every method is applicable to rank $\tilde{T}(\alpha_D)$ of all possible Q. The method proposed by Yager [12], does not need to know the explicit form of the membership functions, and can thus be applied here.

The Yager's ranking index ranks the fuzzy numbers by an area measurement defined as

$$I(\tilde{T}) = \frac{I_L(\tilde{T})}{2} + \frac{I_R(\tilde{T})}{2}$$

where $I_L(\tilde{T})$ represents the area bounded by the left shape function of $\tilde{T}(\alpha_D)$, the x axis, the y axis and the horizontal line $\mu_{\tilde{T}} = 1$ and $I_R(\tilde{T})$ represents the area bounded by the right shape function of $\tilde{T}(\alpha_D)$, the x axis, the y axis and the horizontal line $\mu_{\tilde{T}} = 1$.

The Yager's ranking index of $\tilde{T}(\alpha_K, \alpha_D)$ thus can be calculated as

$$I(\tilde{T}) = \frac{1}{4}\left\{\int_0^1 TC\left[Q,S \mid (K,D) = L_K^{-1}(\alpha_K)\, L_D^{-1}(\alpha_D)\right] d\alpha + \int_0^1 TC\left[Q,S \mid (K,D) = R_K^{-1}(\alpha_K) R_D^{-1}(\alpha_D)\right] d\alpha\right\}$$

Let $K_1(\alpha_K,\alpha_D)=\frac{1}{4}\left\{\int_0^1 L_K^{-1}(\alpha_K)\, L_D^{-1}(\alpha_D)d\alpha + \int_0^1 R_K^{-1}(\alpha_K)R_D^{-1}(\alpha_D)d\alpha\right\}$

Now $$I(\tilde{T}) = \frac{K_1(\alpha_K,\alpha_D)}{Q} + \frac{hQ}{2}$$

Taking the partial derivative of $\tilde{T}(\alpha_D)$ with respect to Q and setting to zero, the necessary condition of optimal solution of $\tilde{T}(\alpha_D)$ can be found as

$$Q^* = \sqrt{\frac{2K_1(\alpha_K,\alpha_D)}{h}}$$

$$\tilde{T}^*(\alpha) = \left\{TC\left[Q^* \mid D = L^{-1}(\alpha)\right], TC\left[Q^* \mid D = R^{-1}(\alpha)\right]\right\}, 0 \le \alpha \le 1$$

Now if, back order cost is permitted, the model can be described by terms of α as

$$\tilde{T}_\pi(\alpha) = \frac{1}{4}\left\{TC_\pi\left[Q,S \mid (K,D) = L_\pi^{-1}(\alpha_\pi)\, L_D^{-1}(\alpha_D)\right] + TC_\pi\left[Q,S \mid (K,D) = R_\pi^{-1}(\alpha_\pi)R_D^{-1}(\alpha_D)\right]\right\}$$

$0 \le \alpha \le 1$

The Yager's index of $\tilde{T}_\pi(\alpha)$ then can be derived as

$$I(\tilde{T}_\pi) = \frac{1}{4}\left\{\int_0^1 TC\left[Q,S \mid (\pi,D) = L_\pi^{-1}(\alpha_\pi)\, L_D^{-1}(\alpha_D)\right] d\alpha + \int_0^1 TC\left[Q,S \mid (\pi,D) = R_\pi^{-1}(\alpha_\pi)R_D^{-1}(\alpha_D)\right] d\alpha\right\}$$

Let $K_4(\alpha_\pi,\alpha_D)=\frac{1}{4}\left\{\int_0^1 L_\pi^{-1}(\alpha_\pi)\, L_D^{-1}(\alpha_D)d\alpha + \int_0^1 R_\pi^{-1}(\alpha_\pi)R_D^{-1}(\alpha_D)d\alpha\right\}$

The necessary conditions for $I(\tilde{T}_\pi)$ has attain the minimum on $I'_Q(\tilde{T}_\pi) = 0$ which can be calculated as follows.

$$I(\tilde{T}) = \frac{K_1(\alpha_K,\alpha_D)}{Q} + \frac{hQ}{2} + \frac{SK_4(\alpha_\pi,\alpha_D)}{Q}$$

$$\frac{\partial I(\tilde{T})}{\partial Q} = 0 \Rightarrow \frac{-K_1(\alpha_K,\alpha_D)}{Q^2} + \frac{h}{2} - \frac{hS^2}{2Q^2} - \frac{pS^2}{2Q^2} - \frac{SK_4(\alpha_\pi,\alpha_D)}{Q^2} = 0$$

$$Q^2 = \frac{1}{h}\left[2K_1(\alpha_K,\alpha_D) + (h+p)S^2 + 2SK_4(\alpha_\pi,\alpha_D)\right]$$

$$Q = \sqrt{\frac{1}{h}\left[2K_1(\alpha_K,\alpha_D) + (h+p)S^2 + 2SK_4(\alpha_\pi,\alpha_D)\right]} \quad - (2)$$

The sufficient conditions for the $I(\tilde{T}_\pi)$ to attain the minimum are $I''_Q(\tilde{T}_\pi) > 0$.

Because of $K_1(\alpha_K, \alpha_D) > 0$ and $K_4(\alpha_\pi,\alpha_D) > 0$, the sufficient conditions are clearly held from the above equations. The optimal solutions Q^* that can be found from equation.(3.1.3) and the optimal annual cost can be calculated as,

$$\tilde{T}_\pi^*(\alpha) = \left\{TC_\pi\left[Q^*, S \mid (\pi,D) = L_\pi^{-1}(\alpha_\pi)L_D^{-1}(\alpha_D)\right], TC_p\left[Q^*, S \mid (\pi,D) = R_\pi^{-1}(\alpha_\pi)R_D^{-1}(\alpha_D)\right]\right\},$$

$0 \le \alpha \le 1$

To show the characteristics of proposed models a trapezoidal fuzzy demand is employed. Let $\tilde{D}$ be the trapezoidal fuzzy demands with parameters:

$\tilde{D} = [l, m, n, u]$

It is easy to find that $K_1(\alpha) = \dfrac{l+m+n+u}{4}$

For the EOQ model with $\tilde{D}$, we have

$$Q^* = \sqrt{\frac{2K}{h}\left[\frac{1}{4}(l+m+n+u)\right]}$$

If $\tilde{D}$ is a symmetrical fuzzy number then

$u - n = m - l$

That is., $u + l = m + n$

Let $D_0 = \dfrac{m+n}{2}$, the mean of $\tilde{D}$, then

$$Q^* = \sqrt{\frac{2K}{h}\left[\frac{1}{4}(m+n)\right]} = \sqrt{\frac{2K}{h}D_0}$$

Q^* is the conventional EOQ with crisp demands, $\dfrac{m+n}{2}$.

This result implies that no matter what the spreads of fuzzy demands, as long as the fuzzy demands are symmetric with the same mean, the Q^* will be the same and equal to the conventional EOQ with the mean of fuzzy demands. The fuzzy number of annual cost can be calculated as

$$\tilde{T}^*(\alpha) = \left\{TC\left[Q^* | D = l + \alpha(m - l)\right],\ TC\left[Q^* | D = u - \alpha(u - n)\right]\right\}, 0 \le \alpha \le 1$$

The above equation shows that the annual cost will also be a trapezoidal fuzzy number and with parameter as

$$TC\left(Q^*\right) = \left[\frac{Kl}{Q^*} + \frac{hQ^*}{2}, \frac{Km}{Q^*} + \frac{hQ^*}{2}, \frac{Kn}{Q^*} + \frac{hQ^*}{2}, \frac{Ku}{Q^*} + \frac{hQ^*}{2}\right]$$

This implies that the shape of membership function of $TC\left(Q^*\right)$ is the same as the $\tilde{D}$, but with a different scale. Accordingly, the spread of $TC\left(Q^*\right)$ will vary according to the spread of $\tilde{D}$.

$$TC(Q,S) = \frac{KD}{Q} + \frac{hQ}{2} - 2hS + \frac{hS^2}{2Q} + \frac{pS^2}{2Q} + \frac{\pi DS}{Q}$$

Let $\tilde{K}, \tilde{D}, \tilde{h}, \tilde{p}, \tilde{\pi}$ be fuzzy trapezoidal numbers and they are defined as follows. [That is., they are described by the α-cuts]

$$\tilde{K}(\alpha_K) = \left[\min . \mu_{\tilde{K}}^{-1}(\alpha_K),\ \max . \mu_{\tilde{K}}^{-1}(\alpha_K)\right]$$

$$= \left[L_{\tilde{K}}^{-1}(\alpha_K),\ R_{\tilde{K}}^{-1}(\alpha_K)\right], 0 \le \alpha_K \le 1$$

$$\tilde{D}(\alpha_D) = \left[\min . \mu_{\tilde{D}}^{-1}(\alpha_D),\ \max . \mu_{\tilde{D}}^{-1}(\alpha_D)\right]$$

$$= \left[L_{\tilde{D}}^{-1}(\alpha_D),\ R_{\tilde{D}}^{-1}(\alpha_D)\right], 0 \le \alpha_D \le 1 \quad \text{and}$$

$$\tilde{h}(\alpha_h) = \left[\min . \mu_{\tilde{h}}^{-1}(\alpha_h),\ \max . \mu_{\tilde{h}}^{-1}(\alpha_h)\right]$$

$$= \left[L_h^{-1}(\alpha_h),\ R_h^{-1}(\alpha_h)\right],\ 0 \le \alpha_h \le 1$$

$$\tilde{p}(\alpha_p) = \left[\min . \mu_{\tilde{p}}^{-1}(\alpha_{\tilde{p}}),\ \max . \mu_{\tilde{p}}^{-1}(\alpha_{\tilde{p}})\right]$$

$$= \left[L_{\tilde{p}}^{-1}(\alpha_p),\ R_{\tilde{p}}^{-1}(\alpha_p)\right],\quad 0 \le \alpha_h \le 1$$

$$\tilde{\pi}(\alpha_\pi) = \left[\min .\ \mu_{\tilde{\pi}}^{-1}(\alpha_{\tilde{\pi}}),\ \max .\ \mu_{\tilde{\pi}}^{-1}(\alpha_{\tilde{\pi}})\right]$$

$$= \left[L_{\tilde{\pi}}^{-1}(\alpha_{\tilde{\pi}}),\ R_{\tilde{\pi}}^{-1}(\alpha_{\tilde{\pi}})\right],\quad 0 \le \alpha_{\tilde{\pi}} \le 1$$

Yager's ranking index can be derived as

$$I(\tilde{T}) = \frac{K_1(\alpha_K,\alpha_D)}{Q} + K_2(\alpha_h)\frac{Q}{2} - K_2(\alpha_h)S + \frac{K_2(\alpha_h)S^2}{2Q} + \frac{K_3(\alpha_p)S^2}{2Q} + \frac{K_4(\alpha_\pi,\alpha_D)S}{Q}$$

Where

$$K_1(\alpha_K,\alpha_D)=\frac{1}{4}\left\{\int_0^1 L_K^{-1}(\alpha_K)\, L_D^{-1}(\alpha_D)d\alpha + \int_0^1 R_K^{-1}(\alpha_K)R_D^{-1}(\alpha_D)d\alpha\right\}$$

$$K_2(\alpha_h)=\frac{1}{2}\left[\int_0^1\left[L_h^{-1}(\alpha_h)\right]d\alpha_h+\int_0^1\left[R_h^{-1}(\alpha_h)\right]d\alpha_h\right]$$

$$K_3(\alpha_p)=\frac{1}{2}\left[\int_0^1 L_p^{-1}(\alpha_p)d\alpha_p+\int_0^1 R_p^{-1}(\alpha_p)d\alpha_p\right]$$

$$K_4(\alpha_\pi,\alpha_D)=\frac{1}{4}\left\{\int_0^1 L_\pi^{-1}(\alpha_\pi)\, L_D^{-1}(\alpha_D)d\alpha + \int_0^1 R_\pi^{-1}(\alpha_\pi)R_D^{-1}(\alpha_D)d\alpha\right\}$$

And the optimal solution can be obtained as

$$Q^* = \sqrt{\frac{2K_1(\alpha_K,\alpha_D)\left[K_2(\alpha_h) + K_3(\alpha_p)\right] - K_4^2(\alpha_\pi,\alpha_D)}{\left[K_2(\alpha_h).K_3(\alpha_p)\right]}} \qquad \ldots (3)$$

Numerical Example:

As an illustration, consider a fuzzy inventory problem with a trapezoidal demand, ordering cost, linear back order, holding cost and fixed back order cost we have,

$\tilde{D}$ = (180,190,210,220); $\tilde{K}$ = (30,40,60,70); $\tilde{p}$ = (8,9,11,12); $\tilde{h}$ = (1.6,1.8,2.2,2.4);

$\tilde{\pi}$ = (1, 1.5, 2.5, 3)

Substituting these values in (3), we obtain Q^*= 63 units.

5. CONCLUSION:

This paper provides a methodology for constructing the fuzzy total inventory cost when the linear and fixed back orders are fuzzy. This method is applicable to other inventory problems like inventory problems with shortages lost sales and shortages lead time.

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