

CHANNEL ESTIMATION USING EXTENDED VERSION OF KALMAN FILTER FOR 2 X 2 MIMO SYSTEMS

*Anirudh Mudaliar

**Rakesh Mandal

***Sunil Kumar Vishwakarma

ABSTRACT

This paper proposes Robust Kalman Filter (RKF) for channel estimation in wireless network. Today various Multi-user Detection(MUD) are used for channel estimation. This paper compares the performance of RKF with popular MUD's like SIC and PIC and propose RKF to is beneficial. The paper briefly discusses the advantages of Multiple Input and Multiple Output (MIMO) systems with Orthogonal Frequency Division Multiple (OFDM) technique and tells why they are so popularly used. Then performance of SIC and PIC are discussed briefly. The need for Kalman Filter (KF) is then explained along with algorithm and performance of RKF and finally the performance comparison of all the above mentioned techniques is provided to show that RKF is beneficial for channel estimation in today's scenario.

Keywords- Discrete time system, Time-delay, KF, RKF, Kalman gain, SIC , PI

* Asstt. Professor, SSTC, Bhilai

**Asstt. Professor, SSTC, Bhilai

*** Student, ME, C V Raman University

1. INTRODUCTION

The channel estimation is the process of characterizing the effect of channel over the transmitted signal. The channel provides Multipath fading due to which ISI, ICI and Selective Frequency Fading occurs in general. For efficient working of wireless systems various techniques are available like OFDM, MIMO, RAKE receiver etc. Researchers have shown that of the available techniques MIMO assisted OFDM is more beneficial.

Multiple Input and Multiple Output (MIMO) Systems:

As indicated by the terminology, a MIMO system employs multiple transmitter and receiver antennas for delivering parallel data streams, Since the information is transmitted through different paths, a MIMO system is capable of exploiting both transmitter and receiver diversity, hence maintaining reliable communications.

Briefly, compared to single-input single-output (SISO) systems, the two most significant advantages of MIMO systems are as follows.

- A significant increase of both the system's capacity and spectral efficiency. The capacity of a wireless link increases linearly with the minimum of the number of transmitter or the receiver antennas. The data rate can be increased by spatial multiplexing without consuming more frequency resources and without increasing the total transmit power.
- Dramatic reduction of the effects of fading due to the increased diversity. This is particularly beneficial, when the different channels fade independently.

Orthogonal Frequency Division Multiplexing (OFDM) Technique:

OFDM is a multi carrier system. There are two main advantages of OFDM:

1. Reduces ISI and ICI
2. Selective Frequency Fading

MIMO-OFDM:

The quality of wireless link can be described by three parameters, namely transmission rate, transmission range and the transmission reliability. With the advent of MIMO assisted OFDM systems, all the three parameters may be simultaneously improved. Furthermore, recent research suggests that the implementation of MIMO-aided OFDM is more efficient than other techniques.

CHANNEL ESTIMATION USING SIC:

In SIC-aided SDMA-OFDM MUDs the following operations are carried out for all subcarriers.

- a) Arrange the detection order of users upon invoking an estimate of the total signal power received by each individual user on the different antenna elements and detect the highest-power user's signal using the above-mentioned MMSE detector, since it is the least contaminated by multiuser interference (MUI).
- b) Then re-generate the modulated signal of this user from his/her detected data and subtract it from the composite multiuser signal.
- c) Detect the next user according to the above procedure by means of MMSE combining, until the lowest-power signal is also detected, which has by now been "decontaminated" from the MUI.
- d) Since the lowest-power user's signal is detected last, a high diversity gain is achieved by the MMSE combiner, which, hence, mitigates not only the effects of MUI but also that of channel fades.
- e) It has to be noted, however that the SIC MUD is potentially prone to user-power classification errors as well as to inter-user error propagation, since in case of detection errors the wrong re-modulated signal is deducted from the composite multiuser signal. Hence, this technique is beneficial in near-far scenarios encountered in the absence of accurate power control.

CHANNEL ESTIMATION USING PIC:

By contrast, PIC dispenses with user-power classification all together, hence avoiding the above-mentioned potential mis-classification problem. It has benefits in more accurately power controlled scenarios. Its philosophy is briefly summarized as follows.

- a) First detection iteration for all subcarriers:

- On the basis of the composite received multiuser signal vector x generate an estimate of all user signals $y^{(i)}$ employing the above-mentioned MMSE detection;
- b) If channel encoding was used, decode, slice and channel encode again as well as re-modulate the each user's signal on an OFDM subcarrier basis.
- c) Second detection iteration for all subcarriers:
 - reconstruct the signal vectors $x^{(i)}$ of all users;
 - generate an estimate $y^{(k)}$ of all user signals by subtracting the signal vectors $x^{(i)}$, $i \neq k$ of all the other users followed by the above mentioned MMSE combining.
- d) Channel decode and slice the extracted user signals in order to generate $b^{(i)}$.
- e) Despite the absence of power-classification errors the PIC technique is also sensitive to inter-user error propagation.

Channel Estimation Using Kalman Filter:

The Kalman filter is a mathematical power tool that is playing an increasingly important role in computer graphics as we include sensing of the real world in our systems. The good news is you don't have to be a mathematical genius to understand and effectively use Kalman filters. This tutorial is designed to provide developers of graphical systems with a basic understanding of this important mathematical tool.

Finite time can be expressed as the following five steps:

- 1.) State Estimate Extrapolation(propagation)
- 2.) Covariance Estimate Extrapolation(propagation)
- 3.) Filter Estimate Update
- 4.) State Estimate Update
- 5.) Covariance Estimate "Update"

We use a KF to estimate the state $x_k \in \mathfrak{R}^n$ of a discrete time controlled system. The system is described by a linear stochastic difference equation.

$$x_{k+1} = Ax_k + Bw_k \quad (2.1)$$

$$y_k = Cx_k + v_k \quad (2.2)$$

where, $x_k \in \mathfrak{R}^n$ is the system state, $y_k \in \mathfrak{R}^m$ is the measured output, $w_k \in \mathfrak{R}^q$ is the process noise, $v_k \in \mathfrak{R}^p$ is the measurement noise. In the following v_k and w_k will be the process of finding the “best estimate” from noisy data amounts to “filtering out” the noise.

However a Kalman filter also doesn't just clean up the data measurements, but also *projects* these measurements onto the state estimate.

2. KALMAN FILTERING FOR LINEAR DISCRETE TIME SYSTEM

Kalman Filter (KF) is a numerical method used to track a time-varying signal in the presence of noise. It is the problem of estimating the instantaneous state of a linear system from a measurement of outputs that are linear combinations of the states but corrupted with Gaussian white noise. The resulting estimator is statically optimal with respect to a quadratic function of estimation error. From the mathematical point view, the KF is a set of equations that provides an efficient recursive computational solution of the linear estimation problem. The filter is very powerful in several aspects. The KF is an extremely effective and versatile procedure for combining noisy sensor outputs to estimate the state of the system with uncertain dynamics. When applied to a physical system, the observer or filter will be under the influence of two noise sources: (i) Process noise, (ii) Measurement noise. The estimate of the state is specified by its conditional probability density function. The purpose of a filter is to compute the state estimate, while an optimal filter minimizes the spread of the estimation error probability density. A recursive optimal filter propagates the conditional probability density function from one sampling instant to the next, keeping in view the system dynamics and inputs, and it incorporates measurements and measurement errors static in the estimate.

Regarded as zero mean, uncorrelated white noise sequence with covariance R_k and Q_k .

$$v_k = N(0, R_k) \quad (2.3)$$

$$w_k = N(0, Q_k). \quad (2.4)$$

The matrix $A \in \mathfrak{R}^{n \times n}$ in the difference Equation (2.1) is the dynamics matrix which relates the state at time step k to the state at time step $k+1$. The matrix $B \in \mathfrak{R}^{n \times 1}$ called noise matrix. The matrix $C \in \mathfrak{R}^{m \times n}$ in the measurement Equation (2.2) relates the state measurement y_k .

The KF algorithm can be seen as a form of feedback estimation. The set of the KF equation can be separated in two groups:

1. Time update equations
2. Measurement update equation

The time update equations project the current state and the error covariance estimates forward in the time to obtain a priori estimates for the next time step. The measurement update equations handle the feedback. In other words, it incorporates a new measurement into the a priori estimate to obtain a corrected a posteriori estimate. Therefore the time update equations are predictor equations, and the measurement update equations are corrector equations. That is, the KF is a predictor-corrector algorithm to provide a recursive solution to the discrete time linear system, as shown in Fig. 2.1.

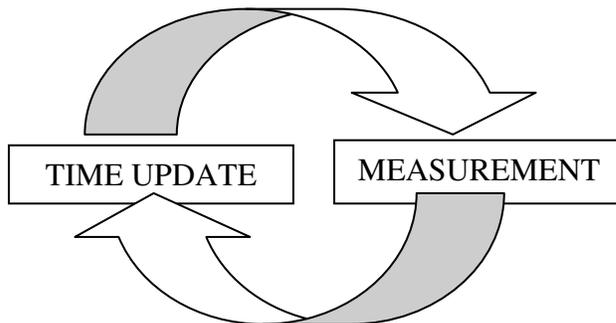


FIGURE 2.1: Discrete Kalman Filter Cycle

The time and measurement update equations are presented below:

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The time updates equations:

$$\hat{x}_{k+1}^- = A\hat{x}_k + Bu_k \quad (2.5)$$

$$P_{k+1}^- = AP_k A_k^T + Q_k \quad (2.6)$$

The measurements update equations:

$$K_f = P_k^- C_k^T (C_k P_k^- C_k^Y + R_k)^{-1} \quad (2.7)$$

$$\hat{x}_k = \hat{x}_k^- + K_f (y_k - C_k \hat{x}_k^-) \quad (2.8)$$

$$P_k = (I - K_f C_k) P_k^- \quad (2.9)$$

The time-update equation projects the state estimate and covariance from time step k to step $k+1$. To compute the Kalman Gain (KG) K_f is the first job in the measurement update equations. Then y_k is obtained by actual measurement of the system. Incorporating the actual measurement and the estimated one in equation (2.7), we generate a posterior estimate. The last step is to compute a posterior error covariance. This is the recursive operation of the KF. A complete picture of the operation of the KF is illustrated in Fig. 2.2, after each time and measurement update pair, the recursive algorithm is repeated with the previous a posterior estimates to predict the new a priori estimates. This recursive nature is the biggest advantage of the KF. This makes the practical implementation of the KF much easier and feasible than the implementation of the Wiener filter, because the Wiener filter obtains its estimates by using all of the precedent data directly. In contrast, the KF only uses the immediately previous data to predict the current states [9].

Robust Kalman Filter (RKF) :

The RKF is a robust version of the KF but with necessary modification to account for the parameter uncertainty. The RKF usually yields a suboptimal solution with respect to the nominal system but it guarantees an upper bound to the filtering error covariance in spite of large parameter uncertainties. The standard KF approach does not address the issue of robustness against large parameter uncertainty in the linear process model. The performance of a KF may be significantly degraded if the system having uncertainty. To avoid this problem RKF is used, the issue of robustness can be addressed by considering the optimal state estimation problem applied

to an uncertain dynamical system rather than a known linear system. In this chapter, we consider the problem of Robust Kalman Filtering for discrete-time systems with norm-bounded parameter uncertainty in both the state and output matrices [2]. The Algorithm for RKF is as follows:

System Equation

$$x_{k+1} = (A_k + \Delta A_k) x_k + B_k w_k$$

Measurement Equation

$$y = (C + \Delta C) x_k + v_k$$

$$\Delta A_k \text{ \& } \Delta C_k$$

These uncertainties are assumed to be the following structure

$$\begin{bmatrix} \Delta A_k \\ \Delta C_k \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E$$

Robust Kalman Filter Equation

$$\hat{x}_{k+1} = (A + \Delta A_{ek}) \hat{x}_k + K_f [y_k - (C + \Delta C_{ek}) \hat{x}_k]$$

Kalman gain

$$K_f = \left(A Q_k C^T + \frac{1}{\varepsilon_k} H_1 H_2^T \right) (R_{\varepsilon k} + C Q_k C^T)^{-1}$$

$$\Delta A_{ek} = \varepsilon_k A S_k E^T (I - \varepsilon_k E S_k E^T)^{-1} E$$

$$\Delta C = \varepsilon_k A C_k E^T (I - \varepsilon_k E S_k E^T)^{-1} E$$

$$A_f = (A + \Delta A_{ek})$$

Covariance of estimation error

$$P_{k+1} = A P_k A^T + \frac{1}{\varepsilon} H_1 H_1^T + B W B^T + A P_k E^T \left(\frac{1}{\varepsilon_k} I - E P_k E^T \right)^{-1} E P_k A^T$$

Covariance of process noise

$$Q_k^{-1} = S_k^{-1} - \varepsilon_k E^T E$$

Covariance of measurement noise

$$R_{\varepsilon k} = V + \varepsilon_k^{-1} H_2 H_2^T$$

Covariance of true state

$$S_{k+1} = A Q_k A^T + \frac{1}{\epsilon} H_1 H_1^T + B W B^T -$$

$$\left(A Q_k C^T + \frac{1}{\epsilon_k} H_1 H_2^T \right) \left(R_{\epsilon k} + C Q_k C^T \right)^{-1}$$

$$\left(A Q_k C^T + \frac{1}{\epsilon_k} H_1 H_2^T \right)$$

4. RESULTS & SIMULATIONS

4.1 Simulation Result of Kalman Filter

Consider the following discrete time system,

$$x_{k+1} = \begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix} x_k + \begin{bmatrix} -6 \\ 1 \end{bmatrix} w_k$$

$$y_k = [-100 \quad 10] x_k + v_k$$

Note that the above system is of the form of system with $P = S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $W = 1, V = 1$

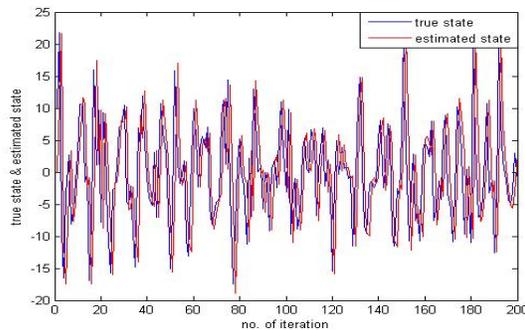


FIGURE 4.1: No. of Iteration vs. True State and Estimated State

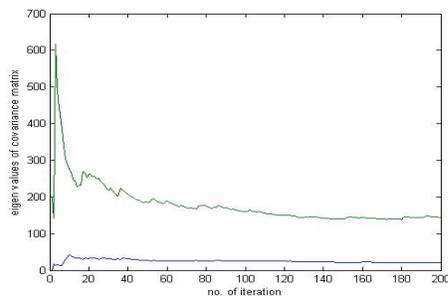


FIGURE 4.2: No. of Iteration vs. Eigen Value of Covariance Matrix.

4.2 COMPARISON OF MUD'S

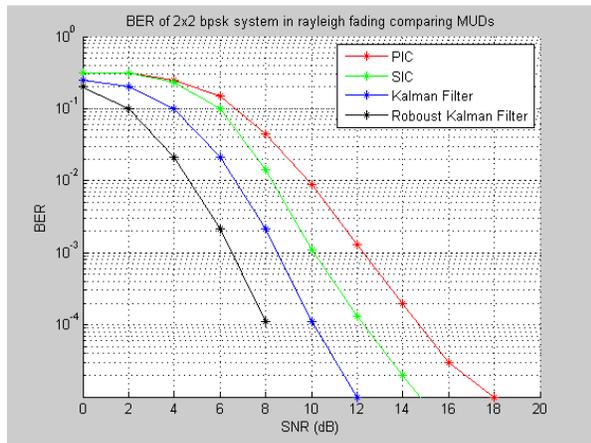


Figure 4.3 : BER vs SNR for 2x2 MIMO System

5. CONCLUSION

The simulation result of Kalman Filter and State Estimation using Kalman Filter in problem. Fig.4.3 shows the estimated states by KF and true state of a system which is considered. And we observe from the figure that estimated state is nearly equal to true state with some error. The estimation error increases in the system having uncertainty.

Finally after comparing the simulation results of SIC, PIC, KF and RKF, we conclude that performance of RKF is satisfactory as compared to others.

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