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GEOMETRICAL ANALYSIS OF PATH TRACED BY COUPLER POINT OF AN EIGHT-LINK MECHANISM

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ABSTRACT:
The paper focuses on path geometry of a coupler point in an eight-link mechanism having three fixed pivots and four ternary links. Kinematic analysis is performed by solving three vector-loop equations. Displacement equations are six coupled non-linear algebraic equations containing six unknown variables and can be solved by a numerical technique. Method of kinematic coefficient is used for obtaining the geometry of coupler curve i.e. radius of curvature and centre of curvature. The formulation of expressions is made possible by inversion of mechanism. First and second order kinematic coefficients are obtained by differentiating the position equation with respect to an independent variable. This paper shows a suitable technique to decouple the 6x6 matrix into three 2x2 matrices in order to derive relations for first and second order kinematic coefficients. The method shown in this paper can be used to find the geometry of all coupler points of the mechanism

Keywords: ***Vector-loop equations; Inversion of mechanism; Kinematic coefficients; Radius of curvature; Centre of curvature.***

1. INTRODUCTION

Since the four-bar mechanism is the simplest mechanism; most of the researches are directed towards analysis and synthesis of this mechanism. An exhaustive study of kinematics in planar four-bar mechanism has been made by J.A. Hrones and G.L. Nelson [1]. Arthur and Sendor [2] presented graphical method for linkages. Freudenstein [3] developed analytical approach for synthesis of planar mechanisms. For mechanisms with more than four links, G.R. Pennok and Ali Israr [4] performed kinematic analysis and synthesis of six-link mechanism in which output link was an overturning clutch. In case of input-crank mechanisms [5], crank rotates at uniform speed and for its complete rotation, motion of the output link is determined. Study of kinematics begins with the displacement analysis, which is done by formation of vector-loop equations. Number of vector-loop equations depends upon the number of respective links in the

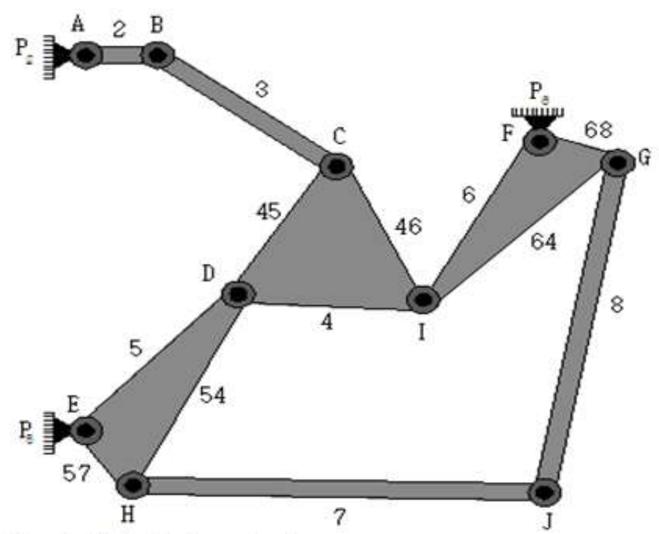

Fig. 1. Eight-link mechanism

mechanism. The vector-loop approach for solving the output position of four-bar linkage was described in detail by Halls [6]. Two vector-loops were suggested by Pennok [4] for the analysis of six-link mechanism. For position a closed form solution was presented by A.K. Dhingra and D. Kohli [7]. Soni [8] examined coupler curves and their possible applications for the synthesis of six-link mechanisms. A geometric approach to determine the centers of curvature in network mechanisms, based on the idea of linkage reduction, was presented by Dijksman [9]. As most of the literature is found to be having the research work involving four- and six-link mechanisms, it is felt that an eight link mechanism should also be analyzed to observe the shape of one of the coupler points. Keeping the same in mind this paper has been presented based on the study of a coupler curve.

2. DESCRIPTION OF AN EIGHT-LINK MECHANISM

An eight-link input crank mechanism has been shown in Fig. 1. It consists of four ternary links 1, 4, 5 and 6 and four binary links 2, 3, 7 and 8. The mechanism is shown to have the link 1 as fixed link, pivoted points being at P_2, P_5 and P_6. Four vector-loops with respective notation are shown in Fig. 2, out of which three loops can be used for displacement analysis. The X and Y components of each vector-loop provides two algebraic equations resulting in six algebraic equations in all. These algebraic equations can be solved by a numerical method such as Newton Raphson method. The kinematic analysis further deals with the investigation of coupler curve. An analytical method is presented, in which method of kinematic coefficient is used to find the geometry of curve. The first order kinematic coefficients of various links are obtained by differentiating the displacement equation with respect to an independent variable. The second derivative of displacement equation with respect to independent variable provides second order kinematic coefficients.

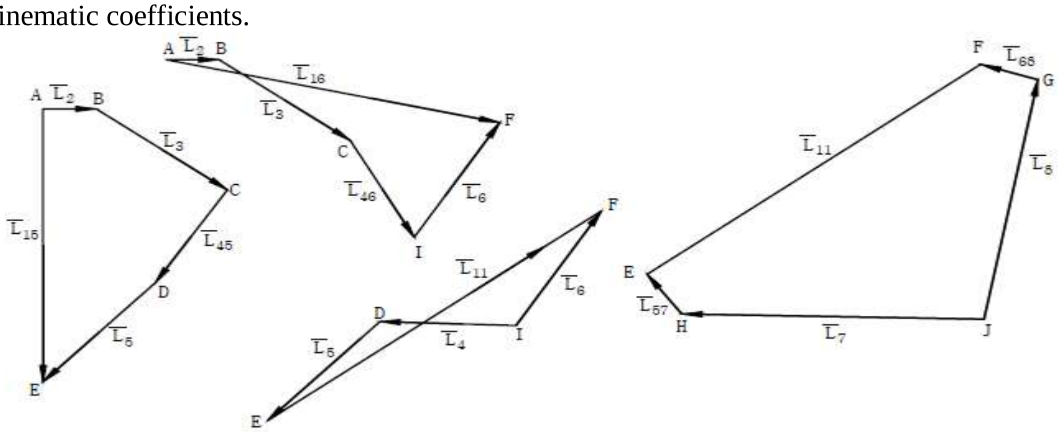

Fig. 2 – The vector-loops for eight link mechanism

The values of kinematic coefficients are constant at a particular position of input link. A relative motion between various links can be determined by comparing these coefficients [4]. The six algebraic equations can be decoupled into three two-coupled equations to simplify the solution. It is made possible by inversion of mechanism. The technique can be adopted for the analysis of radius of curvature and center of curvature for all the coupler points.

3. DISPLACEMENT ANALYSIS

An eight-link mechanism is shown in Fig. 1, in which P_2, P_5 and P_6 are three ground pivots. Four ternary links are represented by 1, 4, 5 and 6. Link 2 is the input link, which is assumed to be rotating with constant angular velocity. Figure 2 shows the following vector-loops-\

- One four bar loop EDIF
- Three five bar loops ABCDE, ABCIF and EFGJH

Three vector-loops ABCDE, EDIF and EFGJH are considered for displacement analysis these three vector-loops contain all the variables. The above mentioned loops provide three vector-loops equations, which can be expressed as

$$\overline{L}_2 + \overline{L}_3 + \overline{L}_{45} + \overline{L}_5 - \overline{L}_{15} = 0 \tag{1}$$

$$-\overline{L}_6 + \overline{L}_4 + \overline{L}_5 + \overline{L}_{11} = 0 \tag{2}$$

and

$$\overline{L}_7 + \overline{L}_{57} + \overline{L}_{11} - \overline{L}_{68} - \overline{L}_8 = 0 \tag{3}$$

The X and Y components of (1), (2) and (3) yields

$$l_2 Cos\theta_2 + l_3 Cos\theta_3 + l_{45} Cos\theta_{45} + l_5 Cos\theta_5 - l_{15} Cos\theta_{15} = 0 \tag{4a}$$

$$l_2 Sin\theta_2 + l_3 Sin\theta_3 + l_{45} Sin\theta_{45} + l_5 Sin\theta_5 - l_{15} Sin\theta_{15} = 0 \tag{4b}$$

$$-l_6 Cos\theta_6 + l_4 Cos\theta_4 + l_5 Cos\theta_5 + l_{11} Cos\theta_{11} = 0 \tag{4c}$$

$$-l_6 Sin\theta_6 + l_4 Sin\theta_4 + l_5 Sin\theta_5 + l_{11} Sin\theta_{11} = 0 \tag{4d}$$

$$l_7 Cos\theta_7 + l_{57} Cos\theta_{57} + l_{11} Cos\theta_{11} - l_{68} Cos\theta_{68} - l_8 Cos\theta_8 = 0 \tag{4e}$$

$$l_7 Sin\theta_7 + l_{57} Sin\theta_{57} + l_{11} Sin\theta_{11} - l_{68} Sin\theta_{68} - l_8 Sin\theta_8 = 0 \tag{4f}$$

θ_2 is the angle of input crank. $\theta_3, \theta_4, \theta_5, \theta_6, \theta_7$ and θ_8 are six unknown variables which are to be determined. The constraints between the two vectors attached to the ternary links are

$$\theta_{45} = \theta_4 + \alpha \tag{5a}$$

$$\theta_{57} = \theta_5 - \beta \tag{5b}$$

$$\theta_{68} = \theta_6 + \gamma \tag{5c}$$

Where α is the angle between sides CD and CI, β is the angle between sides DE and EH and γ is the angle between sides IF and FG. The solution of the Eqs, (4) can be obtained by any method, such as Newton-Raphson method. Differentiating (4) with respect to input variable θ_2 yields

$$-l_2 Sin\theta_2 - l_3 Sin\theta_3 C_3' - l_{45} Sin\theta_{45} C_4' - l_5 Sin\theta_5 C_5' = 0 \tag{6a}$$

$$l_2 Cos\theta_2 + l_3 Cos\theta_3 C_3' + l_{45} Cos\theta_{45} C_4' + l_5 Cos\theta_5 C_5' = 0 \tag{6b}$$

$$l_6 Sin\theta_6 C_6' - l_4 Sin\theta_4 C_4' - l_5 Sin\theta_5 C_5' = 0 \tag{6c}$$

$$-l_6 Cos\theta_6 C_6' + l_4 Cos\theta_4 C_4' + l_5 Cos\theta_5 C_5' = 0 \tag{6d}$$

$$-l_7 Sin\theta_7 C_7' - l_{57} Sin\theta_{57} C_5' + l_{68} Sin\theta_{68} C_6' + l_8 Sin\theta_8 C_8' = 0 \tag{6e}$$

$$l_7 Cos\theta_7 C_7' + l_{57} Cos\theta_{57} C_5' - l_{68} Cos\theta_{68} C_6' - l_8 Cos\theta_8 C_8' = 0 \tag{6f}$$

where $C_n^{'}$ is the first order kinematic coefficient.

$$C_n' = \frac{d\theta_n}{d\theta_2} \qquad \{n = 3, 4, 5, 6, 7 \text{ and } 8\} \tag{7}$$

Differentiating (6) with respect to independent variable θ_2 yields

$$-l_2Cos\theta_2 - l_3Cos\theta_3C_3'^2 - l_3Sin\theta_3C_3'' - l_{46}Cos\theta_{46}C_4'^2 - l_{46}Sin\theta_{46}C_4'' - l_5Cos\theta_5C_5'^2 - l_5Sin\theta_5C_5'' = 0 \quad (8a)$$

$$-l_2Sin\theta_2 - l_3Sin\theta_3C_3'^2 + l_3Cos\theta_3C_3'' - l_{46}Sin\theta_{46}C_4'^2 + l_{46}Cos\theta_{46}C_4'' - l_5Sin\theta_5C_5'^2 + l_5Cos_5C_5'' = 0 \quad (8b)$$

$$l_6Cos\theta_6C_6'^2 + l_6Sin\theta_6C_6'' - l_4Cos\theta_4C_4'^2 - l_4Sin\theta_4C_4'' - l_5Cos\theta_5C_5'^2 - l_5Sin\theta_5C_5'' = 0 \quad (8c)$$

$$l_6Sin\theta_6C_6'^2 - l_6Cos\theta_6C_6'' - l_4Sin\theta_4C_4'^2 + l_4Cos\theta_4C_4'' - l_5Sin\theta_5C_5'^2 + l_5Cos\theta_5C_5'' = 0 \quad (8d)$$

$$-l_7Cos\theta_6C_7'^2 - l_7Sin\theta_7C_7'' - l_{57}Cos\theta_{57}C_5'^2 - l_{57}Sin\theta_{57}C_5'' + l_{68}Cos\theta_{68}C_6'^2 + l_{68}Sin\theta_{68}C_6'' + l_8Cos\theta_8C_6'^2 + l_8Sin\theta_8C_6'' = 0 \quad (8e)$$

$$-l_7Sin\theta_6C_7'^2 + l_7Cos\theta_7C_7'' - l_{57}Sin\theta_{57}C_5'^2 + l_{57}Cos\theta_{57}C_5'' + l_{68}Sin\theta_{68}C_6'^2 - l_{68}Cos\theta_{68}C_6'' + l_8Sin\theta_8C_6'^2 - l_8Cos\theta_8C_6'' = 0 \quad (8f)$$

Where C_n'' is the second order kinematic coefficient of the linkage

$$C_n'' = \frac{d^2\theta_n}{d\theta_2^2} \quad \{n = 3, 4, 5, 6, 7 \text{ and } 8\} \quad (9)$$

Though (6) can be solved to find out first order kinematic coefficients, these will not provide any relationship between various links. Equations (6a) and (6b) carry four terms with three kinematic coefficients so formulations of expressions for kinematic coefficients are not possible. (6c) and (6d) carry three terms with three kinematic coefficients, formulations of expression is possible only by inversion of mechanism. This paper consider θ_5 as an independent variable.

Differentiating (4) with respect to θ_5 yields

$$-l_2Sin\theta_2D_2' - l_3Sin\theta_3D_3' - l_{45}Sin\theta_{45}D_4' - l_5Sin\theta_5 = 0 \quad (10a)$$

$$l_2Cos\theta_2D_2' + l_3Cos\theta_3D_3' + l_{45}Cos\theta_{45}D_4' + l_5Cos\theta_5 = 0 \quad (10b)$$

$$l_6Sin\theta_6D_6' - l_4Sin\theta_4D_4' - l_5Sin\theta_5 = 0 \quad (10c)$$

$$-l_6Cos\theta_6D_6' + l_4Cos\theta_4D_4' + l_5Cos\theta_5 = 0 \quad (10d)$$

$$-l_7Sin\theta_7D_7' - l_{57}Sin\theta_{57} + l_{68}Sin\theta_{68}D_6' + l_8Sin\theta_8D_8' = 0 \quad (10e)$$

and

$$l_7Cos\theta_7D_7' + l_{57}Cos\theta_{57} - l_{68}Cos\theta_{68}D_6' - l_8Cos\theta_8D_8' = 0 \quad (10f)$$

where

$$D_n' = \frac{d\theta_n}{d\theta_5} \quad \{n = 3, 4, 5, 6, 7 \text{ and } 8\} \quad (11)$$

The matrix form of (10) yields

$$\begin{bmatrix} l_6Sin\theta_6 & -l_4Sin\theta_4 \\ -l_6Cos\theta_6 & l_4Cos\theta_4 \end{bmatrix} \begin{bmatrix} D_6' \\ D_4' \end{bmatrix} = \begin{bmatrix} l_5Sin\theta_5 \\ -l_5Cos\theta_5 \end{bmatrix} \quad (12a)$$

$$\begin{bmatrix} -l_2Sin\theta_2 & -l_3Sin\theta_3 \\ l_2Cos\theta_2 & l_3Cos\theta_3 \end{bmatrix} \begin{bmatrix} D_2' \\ D_3' \end{bmatrix} = \begin{bmatrix} l_{45}Sin\theta_{45}D_4' + l_5Sin\theta_5 \\ -l_{45}Cos\theta_{45}D_4' - l_5Cos\theta_5 \end{bmatrix} \quad (12b)$$

and

$$\begin{bmatrix} -l_7Sin\theta_7 & l_8Sin\theta_8 \\ l_7Cos\theta_7 & -l_8Cos\theta_8 \end{bmatrix} \begin{bmatrix} D_7' \\ D_8' \end{bmatrix} = \begin{bmatrix} l_{57}Sin\theta_{57} + l_{68}Sin\theta_{68}D_6' \\ -l_{57}Cos\theta_{57} - l_{68}Cos\theta_{68}D_6' \end{bmatrix} \quad (12c)$$

First order kinematic coefficient of (10) can be obtained by Cramer's rule and is written as

$$D_4' = \frac{l_5 Sin(\theta_5 - \theta_6)}{l_4 Sin(\theta_6 - \theta_4)} \tag{13a}$$

$$D_6' = \frac{l_5 Sin(\theta_5 - \theta_4)}{l_6 Sin(\theta_6 - \theta_4)} \tag{13b}$$

$$D_2' = \frac{l_{45} Sin(\theta_{45} - \theta_3) D_4' + l_5 Sin(\theta_5 - \theta_3)}{l_2 Sin(\theta_3 - \theta_2)} \tag{13c}$$

$$D_3' = \frac{l_{45} Sin(\theta_2 - \theta_{45}) D_4' + l_5 Sin(\theta_2 - \theta_5)}{l_2 Sin(\theta_3 - \theta_2)} \tag{13d}$$

$$D_7' = \frac{l_{57} Sin(\theta_8 - \theta_{57}) + l_{68} Sin(\theta_{68} - \theta_8) D_6'}{l_7 Sin(\theta_7 - \theta_8)} \tag{13e}$$

and

$$D_8' = \frac{l_{57} Sin(\theta_7 - \theta_{57}) + l_{68} Sin(\theta_{68} - \theta_7) D_6'}{l_8 Sin(\theta_7 - \theta_8)} \tag{13f}$$

Differentiating (10) with respect to independent variable θ_5 yields

$$-l_2 Cos\theta_2 D_2'^2 - l_2 Sin\theta_2 D_2'' - l_3 Cos\theta_3 D_3'^2 - l_3 Sin\theta_3 D_3'' - l_{45} Cos\theta_{45} D_4'^2 + l_{45} Sin\theta_{45} D_4'' - l_5 Cos\theta_5 = 0 \tag{14a}$$

$$-l_2 Sin\theta_2 D_2'^2 + l_2 Cos\theta_2 D_2'' - l_3 Sin\theta_3 D_3'^2 + l_3 Cos\theta_3 D_3'' - l_{45} Sin\theta_{45} D_4'^2 + l_{45} Cos\theta_{45} D_4'' - l_5 Sin\theta_5 = 0 \tag{14b}$$

$$l_6 Cos\theta_6 D_6'^2 + l_6 Sin\theta_6 D_6'' - l_4 Cos\theta_4 D_4'^2 - l_4 Sin\theta_4 D_2'' - l_5 Cos\theta_5 = 0 \tag{14c}$$

$$l_6 Sin\theta_6 D_6'^2 - l_6 Cos\theta_6 D_6'' - l_4 Sin\theta_4 D_4'^2 + l_4 Cos\theta_4 D_2'' + l_5 Sin\theta_5 = 0 \tag{14d}$$

$$-l_7 Cos\theta_7 D_7'^2 - l_7 Sin\theta_7 D_7'' - l_{57} Cos\theta_{57} + l_{68} Cos\theta_{68} D_6'^2 + l_{68} Sin\theta_{68} D_6'' + l_8 Cos\theta_8 D_8'^2 + l_8 Sin\theta_8 D_8'' = 0 \tag{14e}$$

and

$$-l_7 Sin\theta_7 D_7'^2 + l_7 Cos\theta_7 D_7'' - l_{57} Sin\theta_{57} + l_{68} Sin\theta_{68} D_6'^2 - l_{68} Cos\theta_{68} D_6'' + l_8 Sin\theta_8 D_8'^2 - l_8 Cos\theta_8 D_8'' = 0 \tag{14f}$$

where $D_n'' = \dfrac{d^2\theta_n}{d\theta_5^2}$ {n = 3, 4, 5, 6, 7 and 8} (15)

The matrix form of (14) yields

$$\begin{bmatrix} -l_4 Sin\theta_4 & l_6 Sin\theta_6 \\ l_4 Cos\theta_4 & -l_6 Cos\theta_6 \end{bmatrix} \begin{bmatrix} D_4'' \\ D_6'' \end{bmatrix} = \begin{bmatrix} -l_6 Cos\theta_6 D_6'^2 + l_4 Cos\theta_4 D_4'^2 + l_5 Cos\theta_5 \\ -l_6 Sin\theta_6 D_6'^2 + l_4 Sin\theta_4 D_4'^2 + l_5 Sin\theta_5 \end{bmatrix} \tag{16a}$$

$$\begin{bmatrix} -l_2 Sin\theta_2 & -l_3 Sin\theta_3 \\ l_2 Cos\theta_2 & l_3 Cos\theta_3 \end{bmatrix} \begin{bmatrix} D_2'' \\ D_3'' \end{bmatrix} = \begin{bmatrix} l_2 Cos\theta_2 D_2'^2 + l_3 Cos\theta_3 D_3'^2 + l_{45} Cos\theta_{45} D_4'^2 + l_{45} Sin\theta_{45} D_4'' + l_5 Cos\theta_5 \\ l_2 Sin\theta_2 D_2'^2 + l_3 Sin\theta_3 D_3'^2 + l_{45} Sin\theta_{45} D_4'^2 - l_{45} Cos\theta_{45} D_4'' + l_5 Sin\theta_5 \end{bmatrix} \tag{16b}$$

$$\begin{bmatrix} -l_7 Sin\theta_7 & l_8 Sin\theta_8 \\ l_7 Cos\theta_7 & l_8 Cos\theta_8 \end{bmatrix} \begin{bmatrix} D_7'' \\ D_8'' \end{bmatrix} = \begin{bmatrix} l_7 Cos\theta_7 D_7'^2 + l_{57} Cos\theta_{57} - l_{68} Cos\theta_{68} D_6'^2 - l_{68} Sin\theta_{68} D_6'' - l_8 Cos\theta_8 D_8'^2 \\ l_7 Sin\theta_7 D_7'^2 + l_{57} Sin\theta_{57} - l_{68} Sin\theta_{68} D_6'^2 + l_{68} Cos\theta_{68} D_6'' - l_8 Sin\theta_8 D_8'^2 \end{bmatrix} \tag{16c}$$

Second order kinematic coefficient of (14) can be obtained by Cramer's rule and is written as

$$D_4'' = \frac{l_6 D_6'^2 - l_4 Cos(\theta_4 - \theta_6) D_4'^2 - l_5 Cos(\theta_5 - \theta_6)}{l_4 Sin(\theta_4 - \theta_6)} \tag{17a}$$

$$D_6'' = \frac{-l_4 D_4'^2 + l_6 Cos\left(\theta_4 - \theta_6\right) D_6'^2 - l_5 Cos\left(\theta_4 - \theta_5\right)}{l_6 Sin\left(\theta_4 - \theta_6\right)} \tag{17b}$$

$$D_2'' = \frac{l_2 Cos\left(\theta_2 - \theta_3\right) D_2'^2 + l_3 D_3'^2 + l_{45} Cos\left(\theta_{45} - \theta_3\right) D_4'^2 + l_{45} Sin\left(\theta_{45} - \theta_3\right) D_4'' + l_5 Cos\left(\theta_5 - \theta_3\right)}{l_2 Sin\left(\theta_3 - \theta_2\right)} \tag{17c}$$

$$D_3'' = \frac{-l_2 D_2'^2 - l_3 Cos\left(\theta_2 - \theta_3\right) D_3'^2 - l_{45} Cos\left(\theta_2 - \theta_{45}\right) D_4'^2 + l_{45} Sin\left(\theta_2 - \theta_{45}\right) D_4'' - l_5 Cos\left(\theta_2 - \theta_5\right)}{l_3 Sin\left(\theta_3 - \theta_2\right)} \tag{17d}$$

$$D_7'' = \frac{-l_7 Cos\left(\theta_7 - \theta_8\right) D_7'^2 - l_{57} Cos\left(\theta_{57} - \theta_8\right) + l_{68} Cos\left(\theta_{68} - \theta_8\right) D_6'^2 + l_{68} Sin\left(\theta_{68} - \theta_8\right) D_6'' + l_8 D_8'^2}{l_7 Sin\left(\theta_7 - \theta_8\right)} \tag{17e}$$

$$D_8'' = \frac{-l_7 D_7'^2 - l_{57} Cos\left(\theta_{57} - \theta_7\right) + l_{68} Cos\left(\theta_{68} - \theta_7\right) D_6'^2 - l_{68} Sin\left(\theta_7 - \theta_{68}\right) D_6'' + l_8 Cos\left(\theta_7 - \theta_8\right) D_8'^2}{l_8 Sin\left(\theta_7 - \theta_8\right)} \tag{17f}$$

4. PATH GEOMETRY OF COUPLER POINT

As mentioned above method of kinematic coefficient is used to obtain radius of curvature and centre of curvature of coupler point. Vector-loop equation for of point J as shown in Fig. 3 is written as

$$\overline{L}_2 + \overline{L}_3 + \overline{L}_{46} + \overline{L}_{64} - \overline{L}_8 - \overline{L}_J = 0 \tag{18}$$

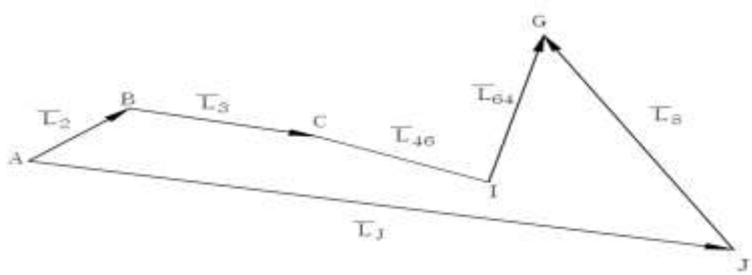

Fig. 3 – Vector-loop for coupler point J

The X and Y components of (18) can be written as

$$J_x = l_2 Cos\theta_2 + l_3 Cos\theta_3 + l_{46} Cos\theta_{46} + l_{64} Cos\theta_{64} - l_8 Cos\theta_8 \tag{19a}$$

and $$J_y = l_2 Sin\theta_2 + l_3 Sin\theta_3 + l_{46} Sin\theta_{46} + l_{64} Sin\theta_{64} - l_8 Sin\theta_8 \tag{19b}$$

Differentiating (19a) and (19b) with respect to input angle θ_2 yields

$$J_x' = -l_2 Sin\theta_2 - l_3 Sin\theta_3 D_3' - l_{46} Sin\theta_{46} D_4' - l_{64} Sin\theta_{64} D_6' + l_8 Sin\theta_8 D_8' \tag{20a}$$

and $$J_y' = -l_2 Cos\theta_2 + l_3 Cos\theta_3 D_3' + l_{46} Cos\theta_{46} D_4' + l_{64} Cos\theta_{64} D_6' - l_8 Cos\theta_8 D_8' \tag{20b}$$

Differentiating (20a) and (20b) with respect to input angle θ_2 yields

$$J_x'' = -l_2 Cos\theta_2 - l_3 Cos\theta_3 D_3'^2 - l_3 Cos\theta_3 D_3'' - l_{46} Cos\theta_{46} D_4'^2 - l_{46} Sin\theta_{46} D_4'' - l_{64} Cos\theta_{64} D_6'^2 - l_{64} Sin\theta_{64} D_6'' + R_8 Cos\theta_8 D_8'^2 + l_8 Sin\theta_8 D_8'' \tag{21a}$$

$$J_y'' = -l_2 Sin\theta_2 - l_3 Sin\theta_3 D_3'^2 + l_3 Cos\theta_3 D_3'' - l_{46} Sin\theta_{46} D_4'^2 + l_{46} Cos\theta_{46} D_4'' - l_{64} Sin\theta_{64} D_6'^2 + l_{64} Cos\theta_{64} D_6'' + l_8 Sin\theta_8 D_8'^2 - l_8 Cos\theta_8 D_8'' \tag{21b}$$

The first and second order kinematic coefficients for the linkage are known from kinematic analysis. The velocity and acceleration of point J can be written as

$$\overline{V}_J = (J'_x\hat{i} + J'_y\hat{j})\omega_2 \quad (22)$$

and $$\overline{A}_J = (J''_x\hat{i} + J''_y\hat{j})\omega_2^2 + (J'_x\hat{i} + J'_y\hat{j})\alpha_2 \quad (23)$$

The unit tangent and the unit normal vector to the path of point J can be expressed as

$$\hat{u}_t = \frac{J'_x\hat{i} + J'_y\hat{j}}{J'_{xy}} \quad (24a)$$

and $$\hat{u}_n = \hat{k} \times \hat{u}_t = \frac{-J'_y\hat{i} + J'_x\hat{j}}{J'_{xy}} \quad (24b)$$

where $$J'_{xy} = \sqrt{J'^2_x + J'^2_y} \quad (25)$$

The radius of curvature of path traced by point J can be expressed as

$$r_J = \frac{V_J^2}{A_J^n} \quad (26)$$

Where the normal acceleration of point J can be expressed as

$$A_J^n = \overline{A}_J.\overline{u}_n \quad (27)$$

Substituting (23) and (24b) into (27) and performing the dot product, the normal acceleration of point J can be expresses as

$$A_J^n = \frac{\left(J'_xJ''_y - J'_yJ''_x\right)\omega^2}{J'_{xy}} \quad (28)$$

Substituting (25) and (28) into (26) the radius of curvature of point J can be expresses as

$$r_J = \frac{J'^2_{xy}}{J'_xJ''_y - J'_yJ''_x} \quad (29)$$

The Cartesian coordinates of the centre of curvature of the path traced by point J can be written as

$$C_{xx} = J_x + r_J(u_n)_x \quad (30a)$$

and $$C_{yy} = J_y + r_J(u_n)_y \quad (30b)$$

Substituting (24b) into (30), the Cartesian coordinates of centre of curvature of the path traced by point J can be expresses as

$$C_{xx} = J_x - r_J\left[\frac{J'_y}{J'_{xy}}\right] \quad (31a)$$

and $$C_{yy} = J_y + r_J\left[\frac{J'_x}{J'_{xy}}\right] \quad (31b)$$

5. NUMERICAL EXAMPLE

The constant lengths (meters) of an eight-link mechanism with respective notations (Fig.1) are shown in Table 1. Distance between fixed pivots $P_2P_5 = P_2P_6 = 1.24$ m. constraint angles $\angle GFI = \angle DEH = 103^0$ and $\angle FIG = \angle EDH = 18^0$. Input link is assumed to be rotating with uniform angular velocity. Angles between fixed pivots $\angle P_6P_2P_5 = 77^o$ and $\angle P_2P_5P_6 = 52^0$

respectively.

TABLE 1

l_2	AB	0.35
l_3	BC	1.1
l_4	DI	0.9
l_{45}	CD	0.9
l_5	DE	1.1
l_{57}	EH	0.4
l_6	IF	1.1
l_{68}	FG	0.4
l_7	HJ	2
l_8	GJ	2
l_{46}	CI	0.9
l_{64}	IG	1.25

Solution: Displacement analysis is performed by substituting given data in (4) and then solving by Newton Raphson method. Results obtained from (4) can be plotted for full rotation of the input crank.

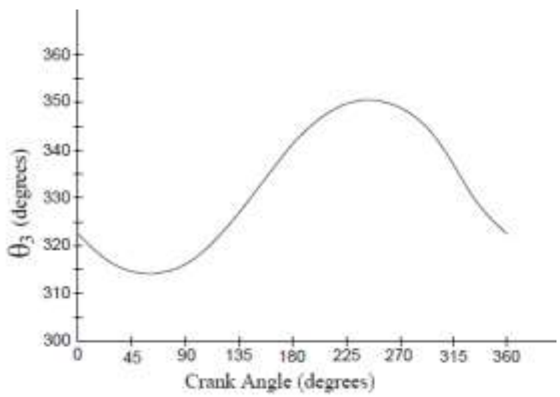

Fig. 4 Angular position of link 3 against input crank position

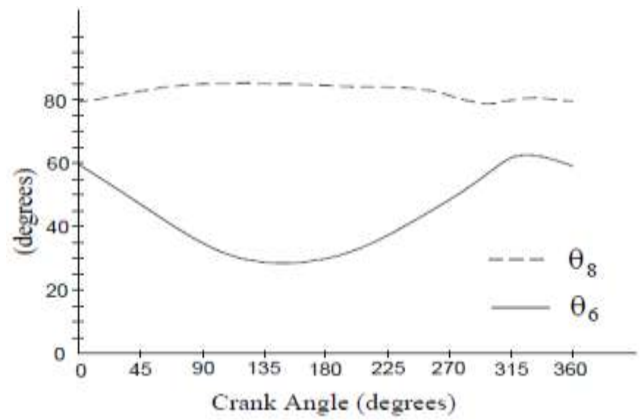

Fig. 6 Angular position of link 6 and 8 against input crank position

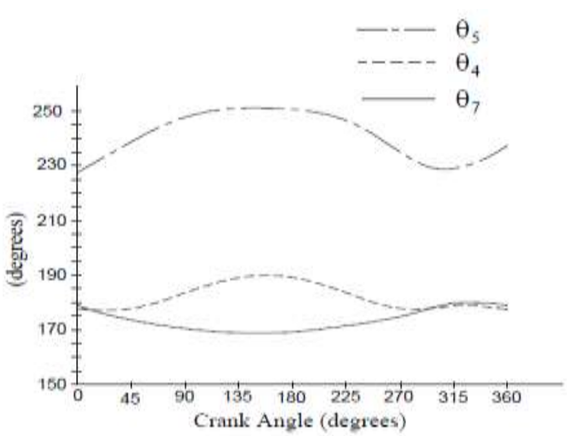

Fig. 5 Angular position of link 4, 5 and 7 against input crank position

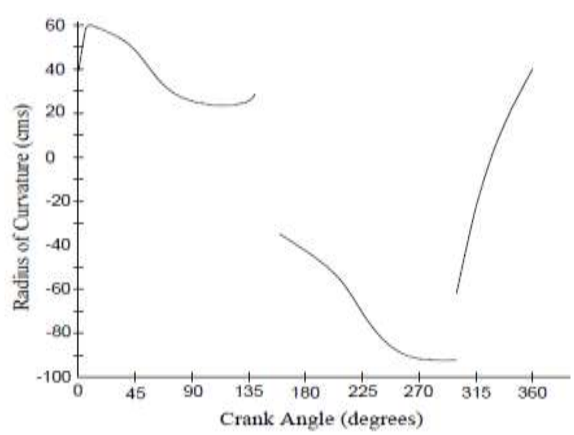

Fig. 7 The radius of curvature of coupler point against input crank position

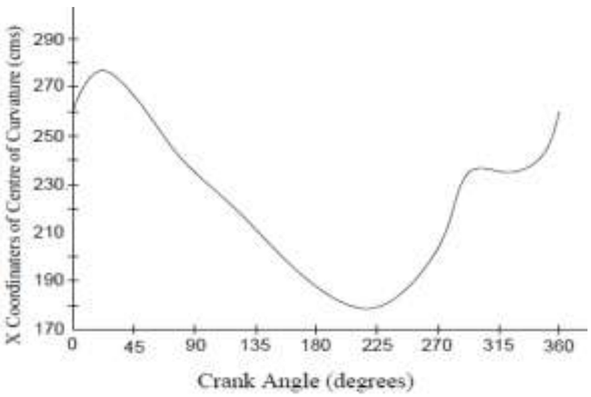

Fig. 8 The X coordinates of curvature of coupler point against input crank position

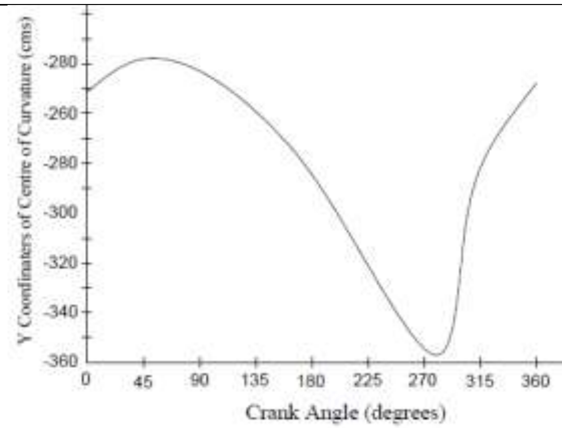

Fig. 9 The Y coordinates of curvature of coupler point against input crank position

Fig. 4 to Fig. 6 show angular positions of links 3, 4, 5, 6, 7 and 8 for $0^0 \le \theta_2 \le 360^0$. It can be noted that θ_3 oscillates from 314^0 to 350^0 i.e., a total angular variation of 36^0 for one complete rotation of input crank. θ_4 varies with total angular variation of 13^0 i.e. from 177^0 to 190^0 as shown in Fig. 5. Variation in θ_8 is from 78^0 to $85^0$and in θ_6 is from 223^0 to 251^0, as shown in Fig. 6. In a similar way Fig. 5 shows $\theta_7 = 169^0$ to 180^0 and $\theta_5 = 223^0$ to 251^0 for one rotation of crank.

By substituting the known values in (13) and (17), first and second order kinematic coefficients can be determined. Then substituting the results obtained from (4), (13) and (17) into (19) to (31) give the radius of curvature and centre of curvature of coupler point J. It can be noted from Fig. 7 that as the radius of curvature varies from 60 cm at crank position 20^0 to 29 cm at crank position 140^0, a discontinuity occurs. Radius of curvature varies from 35 cm and reaches up to 92.7cm at crank position of 300^0.

6. CONCLUSION

The paper presents a vector-loop technique for kinematic analysis of an eight-link mechanism in which the rotation of the input crank is converted into oscillation of the other links. The configuration consists of three loops resulting in three vector equations or six coupled algebraic equations. Although the equations are non-linear and carry more number of variables, a numerical technique is applied to generate the solution. The angular displacement of oscillating links is plotted against one complete rotation of the crank. The analysis also involves determination of radius of curvature and centre of curvature of coupler curve by method of kinematic coefficients. Geometrical relations for kinematic coefficients are made possible by inversion of the mechanism. A novel technique to decouple a 6 x 6 matrix into three 2 x 2 matrices is used to derive relations for kinematic coefficients. The variation in geometry of coupler curve for complete rotation of input crank is also investigated.

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