

## APPLICATION OF A STOCHASTIC QUEUE MODEL IN A RESTAURANT - A CASE STUDY

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### ABSTRACT:

*This study aims to show that queuing theory satisfies the stochastic model when applied to a practical and real time scenario. We obtained the data from a famous restaurant A.P. Residency at Ambala City. The various performance measures as utilization rate, waiting time queue length and the probability of potential customers to balk based on the data have been studied using Little's formulae. The study in one way is helpful to increase the quality of service in the Restaurant by anticipating if there are many customers in the queue; on the other hand, the restaurant can set a target profit that should be achieved on daily basis. In our numerical we have shown mathematically that the service time is needed to be improved in preventing customers balking and go to another competitive restaurant. The study gives more significant than the Simulation Models.*

**Key Words:** *Simulation models, Little's theorem, Poisson distribution, Utilization factor.*

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### INTRODUCTION:

Several parameters are responsible in grading a restaurant whether it deserves up to the desired level or not, whether it is of required standard or sub-standard, good or a bad one, maintains quality or sub-quality. Food quality, cleanliness, restaurant layout, spacious designing and setting etc. are some of the important parameters. These parameters, when managed carefully, attract plenty of customers. Besides these parameters, there is also another factor that needs to be taken in account especially when the restaurant has already succeeding in attracting customers. The factor is the customer waiting time. Human behavior plays an important role in such situation. Some customers do not like to wait. Therefore, they balk away from the system; some customers wait for a while but after certain period they lose their patience and renege away from the system which makes a loss to the system. We have to find the probability of balking customers in certain states such as empty or full.

Waiting lines are common phenomenon in restaurants especially during lunch and dinner time. On Saturday and Sunday a huge congestion of customers has been observed in most of the familiar restaurants of the big cities. Restaurants would avoid losing their customers due to a long wait on the line. Some restaurants initially provide more waiting chairs than they actually need to put them in the safe side and reducing the chairs as the time goes on safe space. However, waiting chairs alone would not solve a problem when customers withdrawn and go to the competitor's door, the service time may need to be improved. This shows a need of quantitative model for the restaurant management to understand the situation better.

Various researchers have already used queuing theory to model the restaurant operation. Most of these researchers have applied **simulation approach** in restaurant operation. Bromn & Kulick(2002); Farahm and Mortinez(1996). Curin, Vosko, etal (2005) made an attempt in reducing cycle time in a busy fast food restaurant. Kharwat (1991) attempt was to increase the throughput and efficiency through computer simulation in a fast food restaurant. Rust (2008) estimated cycle time and cost using little's formulae. Whyte & Starks (1996) investigated a decision tool for Restaurant managers. Dharmabirya & Erwin (2011) studied numerical model for a restaurant at Sonayan city, Jakarta. In our study, we presented the application of queuing theory in stochastic environment in a real case situation using mathematical formulae by obtaining data from a famous restaurant A.P. Residency at Ambala City.

If the queue is very long then the customers do not enough patience to wait for service and will seek for other outlets i.e. they balk from the service centre. In such a situation, the servicing centre may incur in loss. To solve such cases, the study of queuing theory is imperative. An application of a simple queue Model's has been applied to predict fluctuating demand and to enable one to provide adequate service for the customers with tolerable waiting. The main objective of our study is to reduce the waiting line and maximize the profit. For this purpose the performance measures like the expected queue length, the probability that the servers are idle and the probability the server's busy time are needed.

**LITTLE'S FORMULA:**

One of the most powerful relationships in queuing theory was developed by D.C. Little (1966). Little related the steady state mean system sizes to the steady state average customer waiting times.

Let  $T_q \rightarrow$  Time a customer spends waiting in queue prior to entering service &  $T_s$  is total time a customer spends in the system ( $T_s = T_q + S$  where  $S \rightarrow$  service time  $T$ ,  $T_q$  &  $S$  are random variables)

$\therefore W_q = E(T_q)$  &  $W_s = E(T_s)$

By Little's formula

$$L_s = \lambda W_s \text{ \& \ } L_q = \lambda W_q$$

Thus it is necessary to find only one of the four expected value measures, in view of Little's formula & the fact that  $E(T_s) = E(T_q) + E(S)$  or equivalently  $W = W_q + \frac{1}{\mu}$ ;  $\mu \rightarrow$  Mean service rate

Little formula, global & local stochastic balance are the examples of conservation laws. The general idea of conservation is that the expected change of a state function is zero over any finite or infinitesimal span of time picked at random in the steady state. Little's result says that the expected change of aggregate waiting time of customers in system is zero during a randomly chosen finite time interval which implies that the time needed to clear out the system is unchanged, on average over time intervals. The fact that  $\rho = 1 - p_0$  for all G/G/1 queues implies that expected net change in the number in a system over a random time interval is zero since it is equivalent to  $\lambda = \mu(1-p_0)$

In  $L_s = \lambda W_s$  where  $\lambda$  is average customer arrival rate and  $W_s$  is average waiting time in system for a customer.

For a restaurant where  $\lambda$  doubles but the customers still spend the same amount of time in restaurant i.e.  $W_s$ , the average queue length customers in restaurant will be double. We can derive the following relationship by Little's formulae

1.  $L_s$  increase if  $\lambda$  or  $W_s$  increases
2.  $\lambda$  increases if  $L_s$  increases or  $W_s$  decreases
3.  $W_s$  increases if  $L_s$  increases or  $\lambda$  decreases

Rust (2008) applied Little's formula in quantifying the maximum achievable operational improvements and in estimating the performance change when the system is modified.

Using G/G/1 model we find some formulae which are used in our study as given,

The probability that no customer in queue i.e.  $P_0 = 1 - \rho$

$$P_n = (1 - \rho) (\rho)^n$$

$$L_s = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda}, \quad L_q = L_s \times \rho = \frac{\rho^2}{1 - \rho} = \frac{\rho \lambda}{\mu - \lambda}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu - \lambda}, \quad W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$$

**OBSERVATIONS AND DATA'S COLLECTION :**

We collected the data regarding the average number of customers arrived in a particular week of a month from A.P. Residency through personal interview with the restaurant manager as well as data collections through observations time to time at the restaurant as shown in table-1.

**Table-1**

Week/Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1 <sup>st</sup>	300	350	325	390	557	771	854
2 <sup>nd</sup>	325	380	435	370	500	798	895
3 <sup>rd</sup>	318	368	460	545	638	800	950
4 <sup>th</sup>	390	400	470	510	670	820	945

**CALCULATION:-**

We conducted the research study at dinner time. There are average 450 people coming to restaurant in 3 hours 30 minutes time window of dinner time.

We derive arrival rate  $\lambda = \frac{450}{210} = 2.14$  customers/min.

We observed & discussed with manager of the restaurant that each customers spend 40 min. on average in restaurant ( $W_s$ ), the queue length  $L_q$  is about 35 people on average & waiting time is nearly 15 minutes, using  $W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu - \lambda}$  the observed actual waiting time is not very much different when compared to the theoretical waiting time.

$$W_q = \frac{35}{2.14} = 16 \text{ minutes}$$

Now, calculating average no. of customers in restaurant using little's formula

$$L_s = \lambda W_s = 2.14 \times 40 \text{ min.} = 85.6 \text{ customers}$$

Now, we derive utilization rate & service rate using,  $L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda}$

$$\mu = \frac{\lambda(1+L_s)}{L_s} = \frac{2.14(1+85.6)}{85.6} = 2.165 \text{ cpm.}$$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{2.14}{2.165} = .988$$

With very high utilization rate of .988 during dinner time, the probability of zero customers in restaurant is very small as  $P_0 = 1 - \rho = 1 - .988 = .012$

Probability of living n customer can be obtained using general formula

$$P_n = (1 - .988) (.988)^n = .012 \times (.988)^n$$

Assume that potential customers will start balking when they find more than 10 customers already in queue. Assume that maximum queue length of tolerance is 35 customers. As the capacity of restaurant when fully occupied is 100 customers, we calculate the probability of 10 people in queue or the probability when there are 110 = (100+10) in system as follows

Probability of customers going away =  $P(>10 \text{ people in queue}) = P(\text{more than } 110) \text{ in restaurant}$

$$P_{110-135} = \sum_{n=110}^{135} P_n = \sum_{n=110}^{135} P_n = \sum_{n=110}^{135} (1 - .988) (.988)^n$$

$$= .012 \sum_{n=110}^{135} (.988)^n = .012 (.988)^{110} \frac{[1 - (.988)^{26}]}{1 - .988}$$

$$= .265 \times .804 = .213 \times 100 = 21.3\%$$

$$V = \frac{\rho}{(1-\rho)^2} = \frac{.988}{(.012)^2} = 6842$$

Average length of non empty queue (time to time) is,

$$L_n = \frac{\mu}{\mu - \lambda} = \frac{2.165}{2.165 - 2.14} = 86.6$$

Average waiting time of non empty queue (time to time) is,

$$W_n = \frac{1}{\mu - \lambda} = 40$$

On changing waiting time in the system we can summarized the value of other parameters as shown in the table 2 given below,

**Table-2**

$W_s$	$L_s$	$\mu$	$\rho$	$P_n$	$V$	$L_n$	$W_n$
20	42.8	2.19	.977	7.3	1846	20	43.8
18	38.5	2.195	.974	5.3	1440	18.1	39.9
16	34.2	2.20	.972	4.3	1239	16.6	36.6
14	29.9	2.21	.968	2.7	945.3	14.2	31.5

**CONCLUDING REMARKS:**

1. As clear from the table-2 the utilization is directly proportional with mean no. of customers. It means that no. of customers increase as utilization rate increases.
2. The utilization rate at restaurant is very high at .988. This however is only utilization rate during lunch and dinner time on Saturday and Sunday. On other week day the utilization rate is almost half of it. This is because other days the no. of visitors almost half as shown in table-1. The no of waiters remains same whether it is peak hours or off peak hours.
3. In case customers waiting time is lower or less than 20 minutes as shown in table-2, the no. of customers that are able to be served per minute will increase. When service rate ( $\mu$ ) is higher, the utilization will be lower which implies that the probability of customers going away from the system i.e. balking decreases.
4. The study may be helpful to increase the quality of service of the Restaurant, by anticipating if there are many customers are in waiting line and all the seats are filled.
5. The result of the study may become the basis to evaluate the current system and improve the next system also a target profit can be selected.
6. Our mathematical study is simpler than the creation of simulation model.

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