Letter to Editor

ON THE FORMAL–LOGICAL ANALYSIS OF THE FOUNDATIONS OF MATHEMATICS APPLIED TO PROBLEMS IN PHYSICS

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ABSTRACT :
Results of the critical analysis of the standard foundations of mathematics applied to problems in physics are discussed. The unity of formal logic and of rational dialectics is methodological basis of the analysis. The main result is as follows: the concept of "mathematical quantity" – central concept of mathematics – is meaningless, erroneous, and inadmissible one because it represents the following formal-logical and dialectical-materialistic errors: negation of the existence of the essential sign of a concept (i.e., negation the existence of the essence of the concept) and negation of the existence of measure of material object. The obtained results lead to the conclusion that the generally accepted foundations of mathematics should be reconsidered.
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1. As is well known, mathematical physics is the fundamental science of "the application of mathematics to problems in physics and the development of mathematical methods suitable for such applications and for the formulation of physical theories" (The *Journal of Mathematical Physics*). Mathematical physics arose from the needs of science and practice and has a long history of development. The important significance of this field of science is determined by the contribution of prominent scientists of past and present times. However, it does not mean that the problem of validity of the achievements of mathematical physics is now completely solved, or that the foundations of mathematics and physics are not in need of analysis within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. In my opinion, standard mathematical and physical theories can not be considered as scientific truth if there is no formal-logical and dialectical substantiation of it in science.

2. The correct methodological basis represents theoretical generalization of practice and, therefore, is the criterion of truth of theories. In this connection, the following questions arise: How can we apply formal-logical and dialectical laws to the analysis of the foundations of the special sciences? Do mathematical and physical theories obey the formal-logical and dialectical laws? In my opinion, the formal-logical law of identity and the dialectical-materialistic law of measure (i.e., the law of interrelation, of interdependence, of inter-conditionality of qualitative and quantitative aspects (determinacy) of a material object) can be used in mathematics and physics in the following wording: the left-hand side and the right-hand side of the mathematical (i.e., the quantitative) relationship must have identical qualitative determinacy. As first shown in my published works, the use of this statement leads to the following conclusion: the foundations of theoretical physics and of mathematics are not free from objections because the standard theories do not satisfy the criterion of truth.

3. In my opinion, a thorough understanding of the foundations of mathematics is impossible without a critical analysis of the concept of mathematical quantity – central concept of mathematics. A critical analysis of the concept of mathematical quantity leads to the following conclusion:

The concept of “mathematical quantity” is the result of the following mental operations: (i) abstraction of the “quantitative determinacy of physical quantity” from the “physical quantity” at that the “quantitative determinacy of physical quantity” is an independent object of thought; (ii) abstraction of the “amount (i.e., abstract number)” from the “quantitative determinacy of physical quantity” at that the “amount (i.e., abstract number)” is an independent object of thought. In this case, unnamed, abstract numbers are the only sign of the “mathematical quantity”. This sign is not an essential sign of the material objects. Therefore, the content of the concept of “mathematical quantity” is zero, and the volume of this concept is infinitely large.

Thus, the concept of mathematical quantity is meaningless, erroneous, and inadmissible concept in science because it represents the following formal-logical and dialectical-materialistic error: negation of the existence of the essential sign of the concept (i.e., negation the existence of the essence of the concept) and negation of the existence of measure of material object.

4. Mathematical theories can be applied to problems in physics if and only if the mathematical relationships are interpreted geometrically or physically. The interpretation is that the mathematical quantities in the standard relationship $y = f(x)$ are associated with geometric (metric) or physical quantities x_M and y_M characterizing the material object $\boldsymbol{M}$. Interpretation operation represents replacement $x \to x_M$, $y \to y_M$ in the relationship $y = f(x)$. In fact, the interpretation is expressed by the identities $x = x_M$, $y = y_M$. In this case, the relationship between the physical quantities has the form $y_M = f(x_M)$ and can be tested in practice. This relationship means that the interpretation leads to the restoration of the measure of the material object (i.e., the restoration of the unity of qualitative and quantitative determinacy of the material object). Pure mathematics (i.e., mathematics without restored measure) is, according to Einstein, useless science. Therefore, the interpretation is a criterion of truth in pure mathematics.

5. In my published works, analysis of the foundations of mathematics applied to problems in physics was proposed. It was first shown within the correct methodological basis that the foundations of differential and integral calculus, the foundations of vector calculus, the Pythagorean theorem, the foundations of trigonometry, and the foundations of the theory of negative numbers are not free from objection because these standard results contain the formal-logical and dialectical-materialistic errors. Removing these errors leads to the abolition of many standard theories. And the abolition of the standard theories turns science into helpless and barren knowledge.

In my opinion, the errors are an inevitable consequence of the inductive method of cognition.

CRITICAL ANALYSIS OF THE FOUNDATIONS OF THE THEORY OF NEGATIVE NUMBERS

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ABSTRACT :
Critical analysis of the foundations of the theory of negative numbers is proposed. The unity of formal logic and of rational dialectics is methodological basis of the analysis. It is shown that the foundations of the theory of negative numbers contradict practice and contain formal-logical errors. The main results are as follows: a) the concept "number sign" is inadmissible one because it represents a formal-logical error; b) all the numbers are neutral ones because the number "zero" is a neutral one; c) signs "plus" and "minus" are only symbols of mathematical operations. The obtained results are the sufficient reason for the following statement. The existence of logical errors in the theory of negative numbers determines the essence of the theory: the theory is a false one.

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INTRODUCTION

Recently, the progress of science, engineering, and technology has led to appearance of a new problem – the problem of rationalization of the fundamental sciences. Rationalization of sciences is impossible without rationalization of thinking and critical analysis of the foundations of sciences within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. Critical analysis of the sciences within the framework of the correct methodological basis shows [1-8] that the foundations of theoretical physics and of mathematics (for example, classical geometry, the Pythagorean theorem, differential and integral calculus, vector calculus, trigonometry) contain logical errors.

As is well known, the theory of negative numbers is an important part of mathematics [6-11] and of mathematical formalism of physics [12]. This theory is widely and successfully used in the natural sciences. The main result of this theory is the following statement: negative numbers and the concept "negative sign of number" have scientific and practical meaning (for example, $1 - 2 = 0 - 1 = -1$). However, it does not mean that the problem of validity of the theory is now completely solved, or that the foundations of the theory are not in need of formal-logical and dialectical analysis. In my view, the theory of negative numbers cannot be considered as scientific truth if there is no formal-logical and dialectical substantiation of it in science.

Understanding of the essence of the theory of negative numbers is impossible without critical analysis of the foundations of this theory. And a complete understanding of the foundations of this theory is possible only within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. However, the formal-logical analysis of this theory is absent in science. The purpose of the present work is to propose critical analysis of the foundations of the theory of negative numbers within the framework of the correct methodological basis.

1. GEOMETRICAL ANALYSIS OF THE CONCEPT "NEGATIVE NUMBER"

1. As is known, if the Cartesian coordinate system XOY on a plane is given, then the coordinate lines (scales) X and Y divide the plane into four quarters (I, II, III, IV), and the point of intersection of coordinate lines – point O – determines the origin of coordinates (i.e., the number "zero"). The origin of coordinates – the number "zero" – is on the coordinate scales and divides each scale into two parts: the scale of positive numbers and the scale of negative numbers. In this case, the number "zero" belongs to both the scale of positive numbers and the scale of negative numbers. The following formal-logical contradiction arises: the number "zero" is both the positive number and the negative number.

Standard mathematics asserts that: (a) zero belongs to the positive and negative scales; (b) zero is neither a positive number nor a negative number; (c) zero has no sign; zero is not characterized by a sign: zero is a "neutral number". In this case, the formal-logical contradiction is conserved.

The contradiction between the qualitative determinacy of the positive number, the qualitative determinacy of the negative number, and the qualitative determinacy of the neutral number has the form of the law of identity:

(positive number) = (negative number);

(positive number) = (neutral number);

(negative number) = (neutral number).

Then the following questions arise: How does one can eliminate this contradiction? Are negative numbers admissible ones in science and practice? Are there negative and positive numbers in science and practice? The answer to these questions is as follows. The contradiction is eliminated if and only if the law of absence of contradictions,

(positive number) ≠ (negative number),

(positive number) ≠ (neutral number),

(negative number) ≠ (neutral number),

is not violated. The only correct assertion follows from the law of absence of contradictions: if there exists a neutral number (i.e. the number "zero") on the numerical scale, then all the numbers on the numeric scale are neutral ones. Thus, neither positive numbers nor negative numbers do not exist on the numerical scale (i.e., they are not admissible numbers).

2. Let the material geometrical figure "square with identical sides $a\ meter$" be in the quarter I of the coordinate system XOY (in which all the numbers on scales have the dimension "$meter$") (Figure 1).

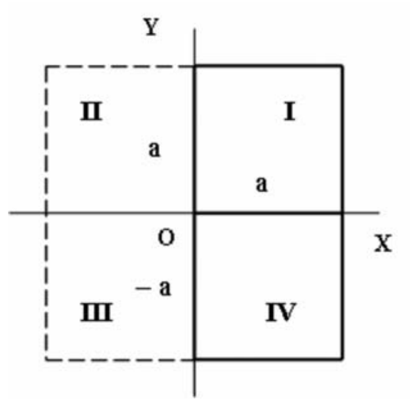

Figure 1. The position of the material geometrical figure "square with identical sides" in the quarters I and IV of the coordinate system.

If the figure "square with identical sides $a\ meter$" is situated in the quarters II and IV, then the sides "$a\ meter$" and "$-a\ meter$" are not identical ones: $a \neq -a$. In other words, the geometrical figure "square with identical sides $a\ meter$" turns into the geometrical figure "square with non-identical sides $a\ meter$ and $-a\ meter$" in the quarters II and IV of the coordinate system XOY. In this case, the correct mathematical relationship $a \neq -a$ is expressed by the formal-logical law of absence of contradictions:

(square with identical sides $a\ meter$) ≠

(square with non-identical sides $a\ meter$ and $-a\ meter$).

And the incorrect mathematical relationship $a = -a$ is expressed by the formal-logical law of identity:

(square with identical sides $a\ meter$) =

(square with non-identical sides $a\ meter$ and $-a\ meter$).

3. The material geometrical figure "square with identical sides $a\ meter$" has the area $S = a \times a = a^2$. The calculation of the area of this geometrical figure in the quarters I and IV of the coordinate system XOY leads to appearance of the concept "imaginary unit". Really, if $S_I \neq S_{IV}$, then

$$S_I = a \times a = a^2; \ \sqrt{S_I} = a;$$

$$S_{IV} = -a \times a = -a^2;$$

$$\sqrt{S_{IV}} = a\sqrt{-1} = ai; \quad i \equiv \sqrt{-1},$$

where S_I, S_{IV} and $\sqrt{S_I}$, $\sqrt{S_{IV}}$ are areas and sides of the figure in the quarters I and IV, respectively; i is imaginary unit. In this case, the following logical error appears:

$$\sqrt{S_{IV}} = a\sqrt{-1} = ai,$$

because

$$\sqrt{S_{IV}} \neq a, \ \sqrt{S_{IV}} \neq -a.$$

In other words, these relationships signify that S_{IV} represents the area of the square whose sides are equal to $\sqrt{S_{IV}} = a\sqrt{-1} = ai$. But $\sqrt{S_{IV}} = ai$ contradicts the condition that the sides of this square are equal to a and $-a$ in the expression $S_{IV} = -a \times a = -a^2$. Consequently, the concepts "negative number" and "imaginary unit" represent a formal-logical error in the case of $S_I \neq S_{IV}$.

Also, a logical error appears if $S_I \equiv S_{IV}$. Really, if $S_I \equiv S_{IV}$, then $1 \equiv -1$, $1 \equiv \sqrt{-1}$. In order to eliminate the logical error $1 \equiv -1$, one should introduce the concept of modulus of number: $|1| = |-1| \equiv 1$. (According to the standard mathematics, modulus of number is a unsigned number. The algebraic quantity of the number always has a sign: plus or minus). The use of the modulus sign signifies the movement of the geometric figure from the quarters II, III, and IV into the quarter I of the coordinate system XOY. In this case, the geometrical figure represents the "square with identical sides $a\ meter$" in the quarter I.

Thus, the geometrical analysis leads to the conclusion that the concepts "negative number" and "imaginary unit" represent a formal-logical error in all cases.

2. LOGICAL ANALYSIS OF THE CONCEPT "NUMBER SIGN"

1. As is well known, practice is a criterion of truth. From practical point of view, operations such as

$$\$1 - \$2 = \$0 - \$1 = -\$1,$$

$$1\,\text{kilogram} - 2\,\text{kilograms} =$$
$$0\,\text{kilogram} - 1\,\text{kilogram} = -1\,\text{kilogram},$$

$$1\,\text{meter} - 2\,\text{meters} =$$
$$0\,\text{meter} - 1\,\text{meter} = -1\,\text{meter},$$

$$1\,\text{second} - 2\,\text{seconds} =$$
$$0\,\text{second} - 1\,\text{second} = -1\,\text{second},$$

and the results of these operations are meaningless ones, wrong in essence. Interpretation of these operations does not represent a mathematical explanation, has no mathematical meaning. Really, the standard interpretation of negative numbers is that the quantity $-a$ is interpreted as modulus $|-a|$, and then one add an explanation which is not related to mathematics. In other words, the interpretation of negative numbers signifies a change of qualitative determinacy (meaning) of these numbers.

Since $a \neq -a$, $|a| = |-a|$ (where a is some number), positive and negative numbers have identical quantitative determinacy (i.e., $|a| = |-a|$) but non-identical qualitative determinacy (i.e., $a \neq -a$). Non-identity of qualitative determinacy is expressed by the formal-logical law of absence of contradiction:

(positive number) $\neq$ *(negative number);*

(positive number) $\neq$ *(unsigned number);*

(negative number) $\neq$ *(unsigned number).*

The following logical statements are true:

(a) positive numbers have identical quality (quantitative determinacy), and therefore they satisfy the formal-logical law of identity:

(positive number) = (positive numbers).

(If the number "zero" was a positive number, then the number "zero" would have to obey this law);

(b) negative numbers have identical quality (quantitative determinacy), and therefore they satisfy the formal-logical law of identity:

(negative numbers) = (negative numbers).

(If the number "zero" was a negative number, then the number "zero" would have to obey this law);

(c) the number "zero" is the unique (special, particular) number, and it satisfies the formal-logical law of identity:

(number "zero" not having a sign) =

(number "zero not having a sign).

(d) the number "zero" satisfies the formal-logical law of absence of contradiction:

(unsigned number) $\neq$ *(signed number).*

But the equations of a type such as

$$1\,\text{kilogram} - 2\,\text{kilograms} =$$
$$0\,\text{kilogram} - 1\,\text{kilogram} = -1\,\text{kilogram}$$

represent violation of the formal-logic law of absence of contradiction. Really, violation of the formal-logic law of absence contradiction is that the left-hand side and the right-hand side of such mathematical equations belong to different qualitative determinacy. In other words, the left-hand side contains positive numbers and neutral number "zero", and the right-hand side contains negative numbers:

(positive numbers and unsigned number "zero") =
(negative numbers).

This signifies that the mathematical equations containing positive and negative numbers and zero are inadmissible ones in science and practice. It follows that all the numbers are neutral numbers: the numbers have no signs because the number "zero" have no sign. If the number "zero" had a sign, then there would be both the positive and negative numbers.

2. From practical point of view, the number (figure) is a symbol designating some amount or absence of amount. Numbers can have dimensions (i.e., qualitative determinacy), but they can have no dimensions. The number "zero" is a symbol designating absence of amount. Mathematically, the essence of number "zero" is manifested in the following statements.

(a) The definition of zero is as follows:

$$a \equiv a\,,\; a - a \equiv 0\,,\; a = a + 0\,,$$

where a is a dimensional or dimensionless number. The definition of zero satisfies the formal-logical law of identity:

(number not having a sign) = (number not having a sign).

(b) The admissible operations on zero are as follows:

$$\frac{a-a}{a} = \frac{0}{a}, \quad \frac{0}{a} = 0\,; \quad \frac{a\,(a-a)}{a} = a\,0, \quad a\,0 = 0\,.$$

(c) The inadmissible operation on zero is as follows:

$$\frac{a-a}{0} = \frac{a}{0} - \frac{a}{0} = \frac{0}{0},$$

because the number "zero" does not designate some amount, i.e. the number "zero" designates absence of amount.

(d) Zero is a special (particular) number. Zero is not a part of any number a, zero is not divided into parts, zero is not composed of parts: $a = a + 0$, $0/a = 0$; $a/0$ is not a part of a; $0/0$ is not a part of 0. Zero is neither integer number nor fractional number; zero has no sign.

Therefore, firstly, the addition operation on zero and the subtraction operation on zero do not change amount: $0 \pm 0 = 0$; secondly, the multiplication operation on zero (i.e., $a \times 0 = 0$) and the division operation on zero by some number (i.e., $0/a = 0$) do not lead to change of zero; thirdly, the operations $0 - a$ and $a/0$ are inadmissible ones.

This signifies that zero is the beginning of amount counting out (i.e., the beginning of amount measuring). By definition, the concept "beginning of amount counting out" has the single sense: it is the designation of absence of amount. Therefore, the subtraction of numbers from zero (i.e., $0 - a$) and division of numbers by zero (i.e., $a/0$) are inadmissible operations. The appearance of negative numbers in standard mathematics is stipulated by the following logical error: the assumption that the number "zero" is composed of two parts (i.e., zero is divided into two parts): a and $-a$, i.e.

$$0 - a = -a\,, \quad a + (-a) = 0\,.$$

This assumption contradicts the definition of zero and the formal-logical law of absence of contradiction.

Thus, the formal-logical analysis of the concept “number sign” leads to the following conclusion: all the numbers are neutral ones; numbers have no signs; the concepts “positive number” and “negative number” represent a formal logical error.

3. DIALECTICAL ANALYSIS OF THE CONCEPT “SYMBOLS OF MATHEMATICAL OPERATIONS”

1. Movement is change in general. Movement is a change of the qualitative and quantitative determinacy of the object. If the qualitative determinacy of the object is not changed, then the movement of the object represents the process of transition of some states of the object into the other states of the object. The process of change is characterized by a direction. If one of the directions can be called a positive direction, then the opposite direction can be called a negative direction.

2. From practical point of view, mathematics is a science of calculations. In mathematics, the quantitative determinacy of the object (i.e., the state of the object) is characterized by a number, and a change in the quantitative determinacy of the object (i.e., the process of transition of some states into other states on condition that the qualitative determinacy of the object is not changed) is described by means of symbols of operations on quantities (numbers). The concepts “quantitative determinacy of the object (i.e., the state)” and “change of the quantitative determinacy of the object (i.e., process)” are not identical ones. Therefore, the identification of the concepts “state; number” and “change of state; mathematical operation” represents a formal-logical error (i.e., violation of the law of absence of contradiction). Mathematical operations are carried out by people. Therefore, mathematical formalism contains only quantities (numbers) and symbols of operations on quantities (numbers), but mathematical formalism do not contain movement (action).

3. The basic mathematical (quantitative) operations on quantities and numbers are as follows: addition operation (designated by the symbol “ + ”), subtraction operation (designated by the symbol “ – ”), multiplication operation (designated by the symbol “ × ”), division operation (designated by the symbol “ : ” or “ / ”). The quantitative relationship between quantities, symbols of operations on quantities, and result of operations is called mathematical equation. It is designated by the symbol “ = ”.

4. The addition operations and multiplication operations are actions which lead to an increase in the numerical value of the result of operations; subtraction operations and division operations are actions which lead to a decrease in the numerical value of the result of operations. Operations of increase of the numerical value (i.e., increase of amount) and operations of decrease of the numerical value (i.e., decrease of amount) are mutually opposite operations. If the direction of the operation of increase of amount may be called positive direction, then the direction of the operation of decrease of amount should be called negative direction. If some operation is called direct one, then the operation of inversion of direct operation is called inverse operation. For example, if the operations $a \times b$, $b \times a$, $a + b$, $b + a$ are called direct ones, then the operations $a : b$ (or a/b), $b : a$ (or b/a), $a - b$, $b - a$ are called inverse ones. Direct and inverse operations are called mutually opposite operations. In this connection, the following problem arises: How does one can express symbolically the inversion of the direction of operation?

5. The solution to this problem is as follows.

a) The symbols of mathematical operations have practical meaning and can be practically used only in combination with numbers and the designations of the numbers in letters: for example,

$$a + b = c,$$
$$a - b = d, \ a > b,$$
$$b - a = h, \ b > a,$$
$$a \times b = b \times a = k,$$
$$a/b = l, \ b/a = 1/l,$$

where the letters designate numbers. In other words, the symbol of the operation relates two quantities (numbers). Therefore, the symbol of the operation of inversion of direction should contain a letter (number) and the symbol of the mathematical operation.

b) The definition of operational form of operations and correspondence between the standard form of operations (left-hand side of relationships) and the operational form of operations (right-hand side of relationships) are as follows:

$$a + b \equiv \langle a+ \rangle b, \quad b + a \equiv \langle b+ \rangle a;$$
$$a - b \equiv \langle a- \rangle b, \quad b - a \equiv \langle b- \rangle a;$$
$$a \times b \equiv \langle a\times \rangle b, \quad b \times a \equiv \langle b\times \rangle a;$$
$$a/b \equiv \langle \ /b \rangle a, \quad b/a \equiv \langle \ /a \rangle b;$$
$$(a/b) \times (b/a) \equiv \langle \ /b \rangle a \times \langle \ /a \rangle b \equiv 1;$$
$$-1 \equiv \langle -1\times \rangle, \quad -a \equiv \langle -1\times \rangle a, \quad -b \equiv \langle -1\times \rangle b;$$
$$(-a) \times (-a) \equiv \langle -1\times \rangle \langle -1\times \rangle a^2 \equiv a^2;$$
$$(-b) \times (-b) \equiv \langle -1\times \rangle \langle -1\times \rangle b^2 \equiv b^2;$$
$$\langle -1\times \rangle \langle -1\times \rangle \equiv \langle 1\times \rangle, \quad \langle -1\times \rangle \langle 1\times \rangle \equiv \langle -1\times \rangle;$$
$$b - a \equiv \langle -1\times \rangle (a - b), \quad a - b \equiv \langle -1\times \rangle (b - a),$$

where expression in angle brackets $\langle \ \rangle$ designates an operator, $\langle -1\times \rangle$ is the operator of the inversion of direction of operation. Multiplication of operators represents successive fulfilment of operations: for example, the expression $\langle -1\times \rangle \langle -1\times \rangle \equiv \langle 1\times \rangle$ represents the inversion of the operation of inversion.

c) The establishing of correspondence between the standard form of the operations and the operational form of the operations is a necessary condition for understanding of the qualitative distinction between a number sign and a symbol of operation. If the understanding is achieved, it is possible to use standard mathematical designation. However, it is not allowed to ascribe sign "plus" or "minus" to quantities (numbers).

Thus, the dialectical analysis of the concepts "mathematical operation" and "symbol of mathematical operations" leads to the conclusion that the symbol of the mathematical operation can not be ascribed to a number. Number is not characterized by a symbol of mathematical operation, and, therefore, it has no sign. The concept "number sign" or the identification of the concepts "number sign" and "symbol of mathematical operation" represents a formal logical error.

4. DISCUSSION

1. As is well known, the concept of negative numbers appeared in ancient mathematics in the 7th century, and finally formed in the 19th century. The great mathematicians of antiquity were wise men because they understood that practice is criterion of truth. Therefore, they called negative numbers by “false”, “dummy”, “absurd", and “imaginary” numbers. “By the beginning of the 19th century Caspar Wessel (1745-1818) and Jean Argand (1768-1822) had produced different mathematical representations of 'imaginary' numbers, and around the same time Augustus De Morgan (1806-1871), George Peacock (1791-1858), William Hamilton (1805-1865), and others began to work on the 'logic' of arithmetic and algebra and a clearer definition of negative numbers, imaginary quantities, and the nature of the operations on them began to emerge . Negative numbers and imaginaries are now built into the mathematical models of the physical world of science, engineering and the commercial world. There are many applications of negative numbers today in banking, commodity markets, electrical engineering, and anywhere we use a frame of reference as in coordinate geometry, or relativity theory” (Encyclopedia). However, the concept “methodological basis of science” is not contained in mathematics until now.

2. The standard theory of negative numbers, first worded in the article *“Theory of Conjugate Functions, or Algebraic Couples; with a Preliminary and Elementary Essay on Algebra as the Science of Pure Time”* by William Rowan Hamilton, legalized the existence of negative numbers in mathematics and put an end to criticism of negative numbers. The scientists called positive and negative numbers, and number 0 by rational numbers and were satisfied that need for serious thinking about the true sense of negative numbers and of zero fell off. The stage of interpretation of negative numbers in science was begun. For example, the interpretation of some well-known negative numbers is as follows:

a) Number $-273{,}15\,^\circ C$ is the absolute zero of temperature, i.e. zero degrees Kelvin. Interpretation of this negative number is as follows: number $-273{,}15$ represents the modulus $\left|-273{,}15\right|$; sign “minus” signifies that number $\left|-273{,}15\right|$ is below zero; concepts “negative” and “below” are identical ones; the term “below” has no mathematical meaning.

b) Number $-1{,}602176565\times 10^{-19}\,Cl$ is the electron charge. Interpretation of this negative number is as follows: number $-1{,}602176565\times 10^{-19}$ represents the modulus $\left|-602176565\times 10^{-19}\right|$; number $\left|-602176565\times 10^{-19}\right|$ is quantity of charge (i.e., quantitative determinacy); sign “minus” signifies qualitative determinacy of the electron; concepts “minus sign” and “electron” are identical ones; the term “electron” has no mathematical meaning. Also, the term “proton” has no mathematical meaning if one identifies the concepts “plus sign” and “proton charge”.

c) Number $-13{,}7\ \mathit{milliard\ \ years}$ is the beginning of formation of the Universe. Interpretation of this negative number is as follows: number $-13{,}7$ represents the modulus $\left|-13{,}7\right|$; number $\left|-13{,}7\right|$ is quantitative determinacy; “minus sign” signifies qualitative determinacy of number $-13{,}7$; concepts “minus sign” and “beginning” are identical ones; the term “beginning” has no mathematical meaning.

Thus, qualitative determinacy of negative numbers is expressed by concepts which have no mathematical meaning:

(mathematical concept) = (non-mathematical concept).

Therefore, the interpretation of negative numbers represents a formal-logical error.

3. There is no logical definition of the concept “negative number” in science and practice. And definition such as “negative number represents the number which is not a positive number” is inadmissible one in formal logic because such definition represents “contradictious (negative) definition”. The correct definition should be “confirmatory (positive) definition”.

Positive and negative numbers and the number “zero” have different qualitative determinacy (even if these numbers have the same dimension). This signifies that the scale of positive numbers and the scale of negative numbers cannot have common point O (i.e., the number 0) in the Cartesian coordinate system XOY. Therefore, the existence of the coordinate system XOY represents a formal-logical error.

From a practical point of view, all the numbers (having dimension or not) are always a result of measurement (or comparison). Negative numbers do not represent a measuring result or a consequence of the existence of positive numbers. This signifies that the set of negative numbers is not a supplement (expansion, extension) of the set of positive integers because positive and negative numbers have different qualitative determinacy. (In other words, if the existence of negative numbers would be cause of the existence of positive numbers, then one could be built negative numbers on the basis of positive numbers). Consequently, the existence of negative numbers is not consistent with practice and is not confirmed by practice.

Negative numbers are inadmissible ones: they should exist neither in science nor in practice. All the numbers obtained in measurements and having the same dimension are characterized by identical qualitative determinacy. Number “zero” is a neutral number. Consequently, all the numbers represent neutral numbers (i.e., the numbers which have no sign “plus’ or “minus”), and the concept “number sign” is inadmissible one. Sign “plus” and “minus” are only symbols of mathematical operations.

4. The theory of negative numbers is not unique erroneous theory in mathematics. As shown in the works [6-28], differential and integral calculus, the Pythagorean theorem, vector calculus, and trigonometry are erroneous theories too. Therefore, today mathematics stands in front of the dilemma: either to recognize the existence of formal-logical errors or to continue movement on the wrong track.

CONCLUSION

Thus, the results of the critical analysis of the theory of negative numbers within the framework of correct methodological basis – the unity of formal logic and of rational dialectics – are as follows:

1. negative numbers are inadmissible ones in science because they represent a formal-logical error;
2. the concept “number sign” is inadmissible one because it represents a formal-logical error;
3. all the numbers are neutral ones because the number “zero” is a neutral one;
4. signs “plus” and “minus” are only symbols of mathematical operations;
5. the operational form of mathematical operations furnishes the clue to understanding of the operation of inversion of operation.

The obtained results are the sufficient reason for the following statement: the essence of the theory of negative numbers is that the theory is a false one.

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FORECASTING THE FREE CORTISOL LEVELS AFTER AWAKENING BASED ON HIGH-ORDER FUZZY LOGICAL RELATIONSHIP

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ABSTRACT :
People usually use many methods to predict the weather, the temperature, the stock index, the enrollments, the earthquake, the economy, etc. A growing body of data suggests that a significantly enhanced salivary cortisol response to waking may indicate an enduring tendency to abnormal cortisol regulation. Based on these forecasting results, our objective was to apply the response test to a population already known to have long-term hypothalamo–pituitary–adrenocortical (HPA) axis dysregulation. We hypothesized that the free cortisol response to waking, believed to be genetically influenced, would be elevated in a significant percent age of cases, regard less of the afternoon Dexamethasone Suppression Test (DST) value based on high-order fuzzy logical relationships. First, the proposed method fuzzifies the historical data into fuzzy sets to form high-order fuzzy logical relationships. Then, it calculates the value of the variable between the subscripts of adjacent fuzzy sets appearing in the antecedents of high-order fuzzy logical relationships. Then, it lets the high-order fuzzy logical relationships with the same variable value form a high-order fuzzy logical relationship group. Finally, it chooses a high-order fuzzy logical relationships group to forecast the free cortisol response to walking and the short day time profile.
Keywords: ***Fuzzy Time Series, Fuzzy Logical Relationship, Fuzzy Logical Relationship Groups, Mean Square Error, glucocorticoids, salivary cortisol, bipolar disorder, lithium, Dexamethasone Suppression Test, DST. 2000 Mathematics Subject Classification: Primary 90B22, Secondary 90B05; 60K30***

1. INTRODUCTION

A growing body of literature points to hypothalamo–pituitary–adrenocortical (HPA) axis dysregulation as a critical factor in the development of mood disorders. Long-term enhanced cortisol secretion may have important health ramifications in addition to its contribution to mood syndromes. The free cortisol response to waking is a promising series of salivary tests that may provide a useful and non-invasive measure of HPA functioning in high-risk studies. The small sample size limits generalizability of our findings. Because interrupted sleep may interfere with the waking cortisol rise, we may have underestimated the proportion of our population with enhanced cortisol secretion. Highly cooperative participants are required[4].

Song and Chissom[2], [3], [5] proposed the time invariant fuzzy time series model and the time-variant fuzzy time series model to deal with the forecasting problems in which the historical data are represented by linguistic values. Because Song and Chissom's method used the max-min operations to forecast the enrollments of the University of Alabama, they take a lot of computation time to deal with max-min composition operations. Chen[7], [11] presented a method to forecast the enrollments of the University of Alabama by using a simple fuzzy time series forecasting model.

Huarng[8], [10] pointed out that the different lengths of intervals in the universe of discourse can affect the forecasting result and a proper choice of the length of each interval can greatly improve the forecasting accuracy rate. Also Huarng presented an average-based length method and a distribution-based length method to deal with forecasting problems based on the intervals with different lengths. Recently Senthil & Harif [12] developed a fuzzy modelof extended Hausdroff distance to test the hypothesis that high job demands & low job control are associated with elevated free cortisol levels.

In this paper, we present a new method to forecast the A growing body of data suggests that a significantly enhanced salivary cortisol response to waking may indicate an enduring tendency to abnormal cortisol regulation. Based on these forecasting results, our objective was to apply the response test to a population already known to have long-term hypothalamo–pituitary–adrenocortical (HPA) axis dysregulation. We hypothesized that the free cortisol response to waking, believed to be genetically influenced, would be elevated in a significant percent age of cases, regard less of the afternoon Dexamethasone Suppression Test (DST) value based on high-order fuzzy logical relationships. First, the proposed method fuzzifies the historical data into fuzzy sets to form high-order fuzzy logical relationships. Then, it calculates the value of the variable between the subscripts of adjacent fuzzy sets appearing in the antecedents of high-order fuzzy logical relationships. Then, it lets the high-order fuzzy logical relationships with the same variable value form a high-order fuzzy logical relationship group.

2. PRELIMINARIES

The concepts of fuzzy time series are presented by Song and Chissom, where the values in a fuzzy time series are represented by fuzzy sets (Zadeh, 1965)[1]. Let D be the universe of discourse, where $D = \{d_i\}_{i=1}^{n}$. A fuzzy set A_i in the universe of discourse D is defined as follows:

$A_i = \sum_{i=1}^{n} \frac{f_{A_i}(d_i)}{d_i}$, Where f_{A_i} is the membership function of the fuzzy set A_i, $f_{A_i} : D \to [0,1]$, $f_{A_i}(d_j)$ is the degree of membership of d_j in the fuzzy set A_i, $f_{A_i}(d_j) \in [0,1]$ and $1 \le j \le n$.

Recently, interest has turned to more refined testing and the probability that HPA dysregulation may even predate the onset of clinical illness [9]. Preliminary data suggest that this dysregulation may be concentrated within the families of individuals with mood disorders [6], suggesting the hypothesis that early abnormalities in cortisol regulation may confer a risk for the future development of mood disorders. To understand the temporal relation between HPA dysregulation and the onset of bipolar disorder (BD), it is essential to have a reliable and non-invasive test that can be repeatedly administered prospectively and is acceptable to high-risk populations. Promising candidates for such a test include the salivary free cortisol response to waking and the short day time profile, a test that adds afternoon and evening measurements to the waking values[9].

Let $Y(t)\,(t = ...,0,1,2.....)$ be the universe of discourse in which fuzzy sets $f_i(t)\,(i = 1,2......)$ are defined in the universe of discourse $Y(t)$. Assume that $F(t)$ is a collection of $f_i(t)\,(i = 1,2......)$, then $F(t)$ is called a fuzzy time series of $Y(t)\,(t = ...,0,1,2.....)$.

Assume that there is a fuzzy relationship $R(t-1,t)$, such that $F(t) = F(t-1)\, o\, R(t-1,t)$, where the symbol "$o$" represents the max-min composition operator, then $F(t)$ is called caused by $F(t-1)$.

Let $F(t-1) = A_i$ and let $F(t) = A_j$, where A_i and A_j are fuzzy sets, then the fuzzy logical relationship (FLR) between $F(t-1)$ and $F(t)$ can be denoted by $A_i \to A_j$, where A_i and A_j are called the left-hand side(LHS) and the right hand side (RHS) of the fuzzy logical relationship, respectively.

Fuzzy logical relationships having the same left-hand side can be grouped into a fuzzy logical relationship group(FLRG). For example, assume that the following fuzzy logical relationships exist:

$$A_i \rightarrow A_{ja},$$
$$A_i \rightarrow A_{jb},$$
$$A_i \rightarrow A_{jc},$$
$$.$$
$$.$$
$$.$$
$$A_i \rightarrow A_{jm},$$

Then these fuzzy logical relationships can be grouped into a fuzzy logical relationship group, shown as follows:
$A_i \rightarrow A_{ja}, A_{jb}, A_{jc}, ...A_{jm}$.

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1), F(t-1), ..., and F(t-n)$, then the fuzzy logical relationship between them can be represented by the "nth-order fuzzy logical relationship", shown as follows:
$F(t-n), ..., F(t-2), F(t-1) \rightarrow F(t)$.

If $F(t-n) = A_{in}, ..., F(t-2) = A_{i2}, F(t-1) = A_{i1}$ *and* $F(t) = A_j$, where $A_{in}, ..., A_{i2}, A_{i1}$ *and* A_j are fuzzy sets, then the nth-order fuzzy logical relationship can be represented by $A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_j$,

Where $A_{in}, ..., A_{i2}, and A_{i1}$ are called the antecedent fuzzy sets of the nth-order fuzzy logical relationship; "$A_{in}, ..., A_{i2}, A_{i1}$" and "$A_j$" are called the left hand-side and the right-hand side of the nth-order fuzzy logical relationship, respectively.

If there are the nth-order fuzzy logical relationships having the same left-hand side, shown as follows:

$$A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_{ja},$$
$$A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_{jb},$$
$$A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_{jc},$$
$$.$$
$$.$$
$$.$$
$$A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_{jm},$$

then these nth-order fuzzy logical relationships form a nth-order fuzzy logical relationships group, shown as follows: $A_{in}, ..., A_{i2}, A_{i1} \rightarrow A_{ja}, A_{jb}, A_{jc}, ...A_{jm}$.

3. A NEW FORECASTING METHOD BASED ON HIGH-ORDER FUZZY LOGICAL RELATIONSHIPS

In this section, we present a new forecasting method based on high-order fuzzy logical relationships. The proposed method is now presented as follows:

Step1: Define the universe of discourse D, $D = [B_{\min} - B_1, B_{\max} + B_2]$ into intervals of equal length, where $B_{\min}$ and $B_{\max}$ are the minimum value and the maximum value of the historical data, respectively, and B_1 and B_2 are two proper positive real values to divide the universe of discourse D into n intervals $d_1, d_2, ..., d_n$ of equal length.

Step2: Define the linguistic terms A_i represented by fuzzy sets, shown as follows:

$$A_1 = \frac{1}{d_1} + \frac{0.5}{d_2} + \frac{0}{d_3} + \frac{0}{d_4} + \dots\dots\dots + \frac{0}{d_{n-2}} + \frac{0}{d_{n-1}} + \frac{0}{d_n},$$
$$A_2 = \frac{0.5}{d_1} + \frac{1}{d_2} + \frac{0.5}{d_3} + \frac{0}{d_4} + \dots\dots\dots + \frac{0}{d_{n-2}} + \frac{0}{d_{n-1}} + \frac{0}{d_n},$$
$$A_3 = \frac{0}{d_1} + \frac{0.5}{d_2} + \frac{1}{d_3} + \frac{0.5}{d_4} + \dots\dots\dots + \frac{0}{d_{n-2}} + \frac{0}{d_{n-1}} + \frac{0}{d_n},$$
$$A_{n-1} = \frac{0}{d_1} + \frac{0}{d_2} + \frac{0}{d_3} + \frac{0}{d_4} + \dots\dots\dots + \frac{0.5}{d_{n-2}} + \frac{1}{d_{n-1}} + \frac{0.5}{d_n},$$
$$A_n = \frac{0}{d_1} + \frac{0}{d_2} + \frac{0}{d_3} + \frac{0}{d_4} + \dots\dots\dots + \frac{0}{d_{n-2}} + \frac{0.5}{d_{n-1}} + \frac{1}{d_n},$$

Where $A_1, A_2, \dots\dots and A_n$ are linguistic terms represented by fuzzy sets.

Step3: Fuzzify each historical datum into a fuzzy set defined in **Step2.** If the historical datum belongs to d_i and the maximum membership value of A_i occurs at d_i , then the historical datum is fuzzified into A_i, where $1 \le i \le n$.

Step4: Construct the nth-order fuzzy logical relationships from the fuzzified historical datum of the training data set.

Step5: Transform each nth-order fuzzy logical relationship $"A_{X_1}, A_{X2}, A_{X3}, \dots\dots, A_{Xj}, \dots\dots, A_{X_n} \to A_{Xr}"$ into the following form:

$$A_{X_1}, A_{X_{1+V(X_1)}}, A_{X_{1+V(X_1)+V(X_2)}}, \dots\dots\dots A_{X_{1+V(X_1)+V(X_2)+\dots\dots+V(X_i)+\dots\dots}}, A_{X_{1+V(X_1)+V(X_2)+\dots\dots+V(X_i)+\dots\dots+V(X_m)}}$$

$\to A_{X_{1+V(X_1)+V(X_2)+\dots\dots+V(X_i)+\dots\dots+V(X_m)+V(X_n)}}$, where $V(X_1), V(X_2), \dots\dots, and V(X_n)$ are integers.

Step6: Let the transformed nth-order fuzzy logical relationships obtained in **Step5** having the same left-hand side form a nth-order fuzzy logical relationship group. For example, let us consider the following transformed third-order fuzzy logical relationships:

$$A_{a_1}, A_{a_{1+V(a_1)}}, A_{a_{1+V(a_1)+V(a_2)}} \to A_{a_{1+V(a_1)+V(a_2)+V(a_3)}},$$
$$A_{b_1}, A_{b_{1+V(b_1)}}, A_{b_{1+V(b_1)+V(b_2)}} \to A_{b_{1+V(b_1)+V(b_2)+V(b_3)}},$$

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$$A_{k_1}, A_{k_{1+V(k_1)}}, A_{k_{1+V(k_1)+V(k_2)}} \to A_{k_{1+V(k_1)+V(k_2)+V(k_3)}},$$

Where $V(a_1)=V(b_1)=\dots\dots\dots=V(k_1)$ and $V(a_2)=V(b_2)=\dots\dots\dots=V(k_2)$, then these third- order fuzzy logical relationships can be grouped into a transformed third-order fuzzy logical relationships group, shown as follows:

$A_X, A_{X_{+V(Y_1)}}, A_{X_{1+V(Y_1)+V(Y_2)}} \to A_{X_{+V(Y_1)+V(Y_2)+V(a_3)}}, A_{X_{+V(Y_1)+V(Y_2)+V(b_3)}}, \dots\dots\dots, A_{X_{+V(Y_1)+V(Y_2)+V(k_3)}}$, where $X = a_1, b_1, \dots\dots k_1$, $V(Y_1)=V(a_1)=V(b_1)=\dots\dots\dots=V(k_1)$ and $V(Y_2)=V(a_2)=V(b_2)=\dots\dots\dots=V(k_2)$.

Step7: Choose a transformed nth-order fuzzy logical relationship group for prediction. Assume that $F(t-n)=A_{in}, F(t-(n-1))=A_{i(n-1)}, \dots\dots\dots F(t-2)=A_{i2}, and\ F(t-1)=A_{i1}$, and assume that we want to predict $F(t)$, where $A_{i1}, A_{i2}, \dots\dots\dots, and\ A_{in}$ are fuzzy sets. Based on the transformed nth-order fuzzy logical relationship groups obtained in **Step6**, choose the corresponding transformed nth-order fuzzy logical relationship group for prediction. If the chosen transformed nth-order fuzzy logical relationship group is:

$A_X, A_{X+V(Y_1)}, ..., A_{X1+V(Y_1)+V(Y_2)+..........+V(Y_{n-1})} \rightarrow A_{X+V(Y_1)+..........+V(Y_{n-1})+V(a_n)}, A_{X+V(Y_1)+.........+V(Y_{n-1})+V(b_n)}, ..., A_{X+V(Y_1)+V(Y_2)+.......+V(Y_{n-1})+V(k_n)}$

where $A_{in} = A_X, A_{i(n-1)} = A_{X+V(Y_1)}, ..., A_{i1} = A_{X+V(Y_1)+V(Y_2)+........+V(Y_{n-1})}$, then replace X by the subscript in of the fuzzy set A_{in} to get the derived fuzzy sets $A_{in+V(Y_1)+........+V(Y_{(n-1)})+V(a_n)}, A_{in+V(Y_1)+........+V(Y_{(n-1)})+V(b_n)}, ..., andA_{in+V(Y_1)+........+V(Y_{(n-1)})+V(k_n)}$ for prediction. Let $A_{j1} = A_{in+V(Y_1)+........+V(Y_{n-1})+V(a_n)}$, let $A_{j2} = A_{in+V(Y_1)+........+V(Y_{n-1})+V(b_n)......,}$ and let $A_{jk} = A_{in+V(Y_1)+........+V(Y_{n-1})+V(k_n)}$. Then, the forecasted variable $FVar$ is calculated as follows:

$$FVar = \frac{\sum_{i=1}^{k} m_{ji}}{k} - m_{i1.} \qquad (1)$$

Where the maximum membership values of $A_{i1}, A_{j1}, A_{j2}, ..., andA_{jk}$ occur at the intervals $d_{i1}, d_{j1}, d_{j2}, ..., and\ d_{jk}$, respectively, and $m_{i1}, m_{j1}, m_{j2}, ..., andm_{jk}$ are the midpoints of the intervals $d_{i1}, d_{j1}, d_{j2}, ..., and\ d_{jk}$, respectively. The forecasted value FV is calculated as follows:

$$FV = RV(t-1) + FVar \qquad (2)$$

Where $RV(t-1)$ is the real value on trading day t-1.

4. EXAMPLE

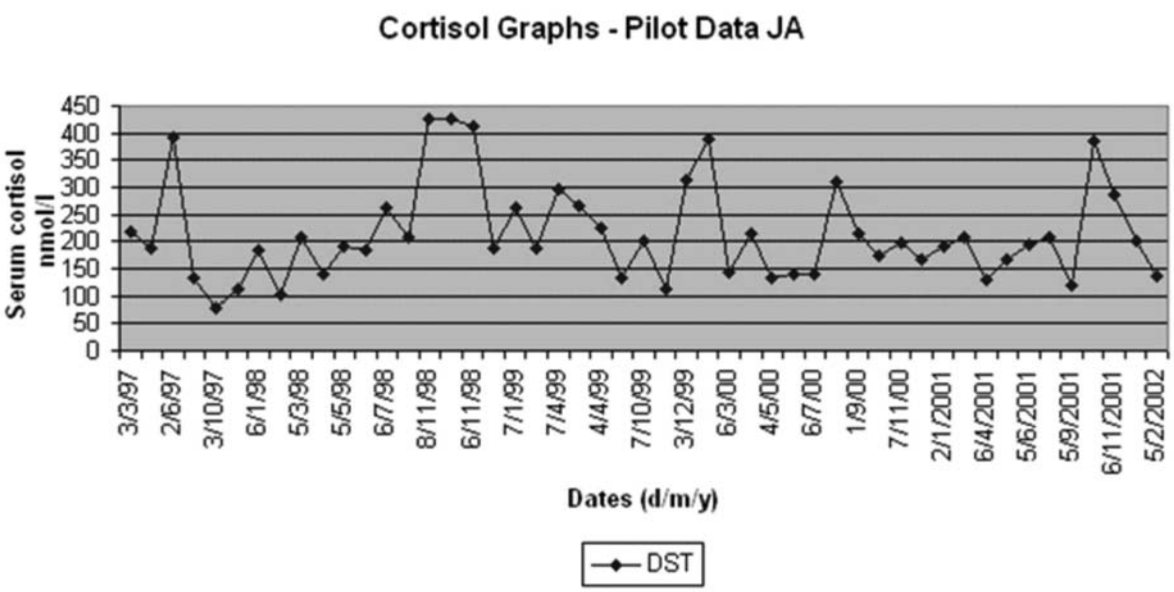

Figure 1: The Longitudinal Dexamethasone Suppression Test (DST) data on a fully remitted lithium responder for past 5 years who was asymptomatic and treated with lithium throughout

We apply the proposed method to forecast the Longitudinal Dexamethasone Suppression Test (DST)[10] data on a fully remitted lithium responder for past 5 years who was asymptomatic and treated with lithium throughout based on high order fuzzy logical relationships.

A_1 = [50, 100], A_2 = [100, 150], A_3 = [150, 200], A_4 = [200, 250], A_5 = [250, 300], A_6 = [300, 350], A_7 = [350, 400], A_8 = [400, 450]

Table 1: Fuzzified value and Fuzzy logical relationships for Medical data

S. No	Actual Value	Fuzzy set	Fuzzy logical relationships of first order	Fuzzy logical relationships of second order
1	225	A_4	$A_4 \to A_3$	$A_4, A_3 \to A_7$
2	190	A_3	$A_3 \to A_7$	$A_3, A_7 \to A_2$
3	395	A_7	$A_7 \to A_2$	$A_7, A_2 \to A_1$
4	140	A_2	$A_2 \to A_1$	$A_2, A_1 \to A_2$
5	90	A_1	$A_1 \to A_2$	$A_1, A_2 \to A_3$
6	120	A_2	$A_2 \to A_3$	$A_2, A_3 \to A_2$
7	180	A_3	$A_3 \to A_2$	$A_3, A_2 \to A_4$
8	110	A_2	$A_2 \to A_4$	$A_2, A_4 \to A_2$
9	210	A_4	$A_4 \to A_2$	$A_4, A_2 \to A_3$
10	145	A_2	$A_2 \to A_3$	$A_2, A_3 \to A_3$
11	190	A_3	$A_3 \to A_3$	$A_3, A_3 \to A_5$
12	185	A_3	$A_3 \to A_5$	$A_3, A_5 \to A_4$
13	260	A_5	$A_5 \to A_4$	$A_5, A_4 \to A_8$
14	210	A_4	$A_4 \to A_8$	$A_4, A_8 \to A_8$
15	430	A_8	$A_8 \to A_8$	$A_8, A_8 \to A_8$
16	430	A_8	$A_8 \to A_8$	$A_8, A_8 \to A_3$
17	420	A_8	$A_8 \to A_3$	$A_8, A_3 \to A_3$
18	190	A_3	$A_3 \to A_5$	$A_3, A_5 \to A_3$
19	260	A_5	$A_5 \to A_3$	$A_5, A_3 \to A_5$
20	190	A_3	$A_3 \to A_5$	$A_3, A_5 \to A_5$
21	295	A_5	$A_5 \to A_5$	$A_5, A_5 \to A_4$
22	270	A_5	$A_5 \to A_4$	$A_5, A_4 \to A_2$
23	230	A_4	$A_4 \to A_2$	$A_4, A_2 \to A_2$
24	140	A_2	$A_2 \to A_2$	$A_2, A_2 \to A_2$
25	199	A_2	$A_2 \to A_2$	$A_2, A_2 \to A_6$
26	120	A_2	$A_2 \to A_6$	$A_2, A_6 \to A_7$
27	315	A_6	$A_6 \to A_7$	$A_6, A_7 \to A_2$
28	390	A_7	$A_7 \to A_2$	$A_7, A_2 \to A_4$
29	145	A_2	$A_2 \to A_4$	$A_2, A_4 \to A_2$
30	210	A_4	$A_4 \to A_2$	$A_4, A_2 \to A_2$
31	135	A_2	$A_2 \to A_2$	$A_2, A_2 \to A_2$
32	140	A_2	$A_2 \to A_2$	$A_2, A_2 \to A_6$
33	140	A_2	$A_2 \to A_6$	$A_2, A_6 \to A_4$
34	310	A_6	$A_6 \to A_4$	$A_6, A_4 \to A_3$
35	210	A_4	$A_4 \to A_3$	$A_4, A_3 \to A_3$
36	180	A_3	$A_3 \to A_3$	$A_3, A_3 \to A_3$
37	195	A_3	$A_3 \to A_3$	$A_3, A_3 \to A_3$
38	175	A_3	$A_3 \to A_3$	$A_3, A_3 \to A_4$
39	190	A_3	$A_3 \to A_4$	$A_3, A_4 \to A_2$
40	210	A_4	$A_4 \to A_2$	$A_4, A_2 \to A_3$
41	135	A_2	$A_2 \to A_3$	$A_2, A_3 \to A_3$
42	175	A_3	$A_3 \to A_3$	$A_3, A_3 \to A_4$
43	195	A_3	$A_3 \to A_4$	$A_3, A_4 \to A_2$
44	210	A_4	$A_4 \to A_2$	$A_4, A_2 \to A_7$
45	120	A_2	$A_2 \to A_7$	$A_2, A_7 \to A_5$
46	385	A_7	$A_7 \to A_5$	$A_7, A_5 \to A_3$
47	290	A_5	$A_5 \to A_3$	$A_5, A_3 \to A_2$
48	195	A_3	$A_3 \to A_2$	-
49	140	A_2	-	-

Table 2: Transformed second order fuzzy logical relationship groups

Groups	Transformed second order fuzzy logical relationship
Group 1	$A_X, A_{X-5} \rightarrow A_{X-5-1}, A_{X-5+2}, A_{X-5+2}$
Group 2	$A_X, A_{X-2} \rightarrow A_{X-2+1}, A_{X-2+2}, A_{X-2+0}, A_{X-2+0}, A_{X-2-1}, A_{X-2+1}, A_{X-2+5}, A_{X-2-2}, A_{X-2-1}$
Group 3	$A_X, A_{X-1} \rightarrow A_{X-1+4}, A_{X-1+1}, A_{X-1+2}, A_{X-1+4}, A_{X-1-2}, A_{X-1+0}$
Group 4	$A_X, A_{X+0} \rightarrow A_{X+0+2}, A_{X+0+0}, A_{X+0-5}, A_{X+0-1}, A_{X+0+0}, A_{X+0+4}, A_{X+0+0}, A_{X+0+4}, A_{X+0+0}, A_{X+0+0}, A_{X+0+1}, A_{X+0+1}$
Group 5	$A_X, A_{X+1} \rightarrow A_{X+1+1}, A_{X+1-1}, A_{X+1+1}, A_{X+1-5}, A_{X+1+0}, A_{X+1+2}$
Group 6	$A_X, A_{X+2} \rightarrow A_{X+2-2}, A_{X+2-1}, A_{X+2-2}, A_{X+2+0}, A_{X+2-2}$
Group 7	$A_X, A_{X+4} \rightarrow A_{X+4+0}, A_{X+4+1}, A_{X+4-2}$
Group 8	$A_X, A_{X+5} \rightarrow A_{X+5-2}$

5. EXPERIMENTAL RESULTS

There was a significant difference between BD patients and our control subjects in the maximum percentage rise of salivary cortisol response to awakening. Those showing a waking response also had significantly higher mean cortisol values at 30 minutes after waking, compared with 509 normal subjects described in Wust's and others study [4]. Base line values at time zero, immediately upon waking, did not differ significantly between our sample and Wust's control subjects [4]. Patients and our 5 control subjects did not differ significantly in the percent age decline from the peak morning value to the evening values.

In this section we apply the proposed for forecasting the Longitudinal Dexamethasone Suppression Test (DST) data on a fully remitted lithium responder for past 5 years who was asymptomatic and treated with lithium throughout. We evaluate the performance of the proposed method using the root mean square error (RMSE), which is defined as follows:

$$\text{RMSE} = \frac{\left(\sqrt{\dfrac{|\,\text{forecasted value}_\text{i} - \mathit{acutual\ value}_i\,|^2}{n}}\right)}{\text{forecasted value}_\text{i}}$$

Where n denotes the number of dates needed to be forecasted, $\text{forecasted value}_\text{i}$ denotes the forecasted value on trading day i, $\mathit{acutual\ value}_i$ denotes the actual value on trading day i and $1 \le i \le n$.

The forecasted value and Root mean square error (RMSE) are presented below in table 3 :

Table 3: Forecasted Value and MSE

S. No	Actual Value	Forecasted Value	RMSE
1	225	-	-
2	190	-	-
3	395	270	0.0452
4	140	315	0.1785
5	90	10	0.1269
6	120	30	0.1071
7	180	130	0.0396
8	110	27	0.1077
9	210	120	0.0612
10	145	175	0.0295
11	190	184	0.0045
12	185	102	0.0640
13	260	210	0.0274
14	210	240	0.0204
15	430	305	0.0415
16	430	564	0.0445
17	420	378	0.0142
18	190	148	0.0315
19	260	360	0.0549
20	190	220	0.0225
21	295	278	0.0082
22	270	300	0.0158
23	230	145	0.0527
24	140	15	0.1275
25	199	193	0.0043
26	120	70	0.0595
27	315	265	0.0226
28	390	215	0.0641
29	145	24	0.1192
30	210	110	0.0680
31	135	165	0.0317
32	140	122	0.0183
33	140	90	0.0510
34	310	260	0.0230
35	210	35	0.1190
36	180	130	0.0396
37	195	70	0.0915
38	175	125	0.0408
39	190	140	0.0375
40	210	160	0.0340
41	135	52	0.0878
42	175	167	0.0065
43	195	112	0.0608
44	210	160	0.0340
45	120	37	0.0988
46	385	379	0.0022
47	290	290	0
48	195	145	0.0366
49	140	118	0.0224

It means that the proposed method gets a higher average forecasting accuracy rate than other existing methods to forecast the maximum percentage rise of salivary cortisol response to awakening. we can see that the proposed method get the smallest RMSE than Huarng's method and Huarng's and Yu's method for forecasting the enrollments of the University of Alabama.

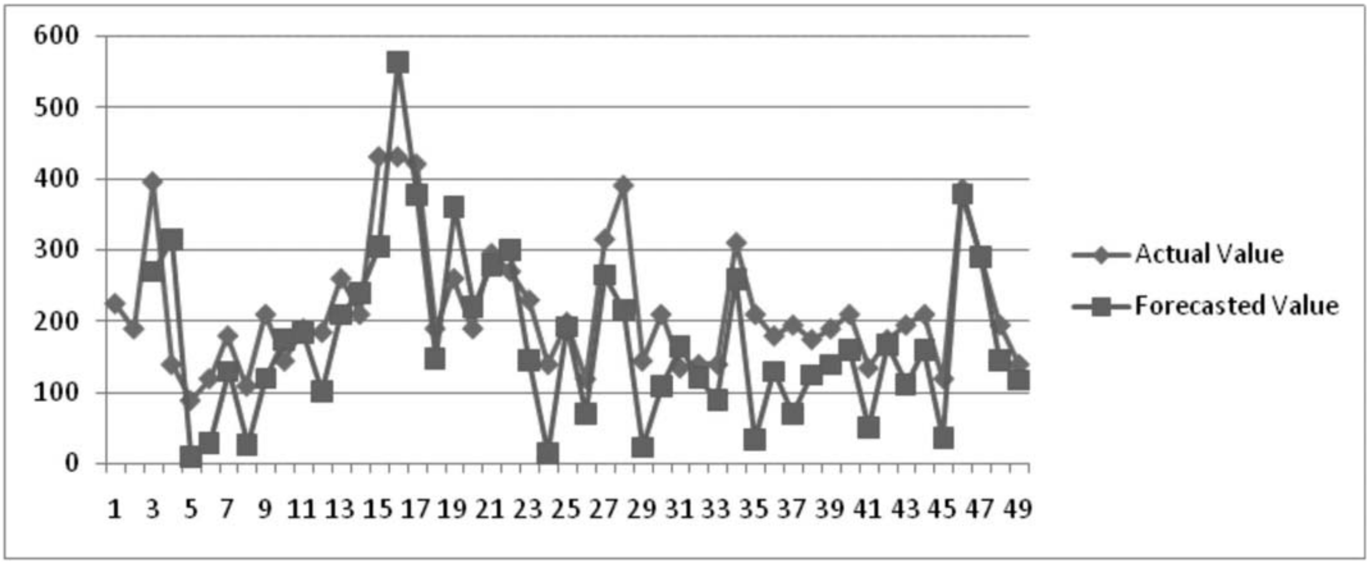

Figure 2: Actual value and Forecasted value for the Longitudinal Dexamethasone Suppression Test (DST)[10] data on a fully remitted lithium responder for past 5 years who was asymptomatic and treated with lithium throughout

CONCLUSION

In this paper, Our dysregulation, even when lithium-responsive BD patients are clinically well and their DSTs are observations support the hypothesis that the free cortisol response to waking can reflect relatively enduring HPA normal. Because the test is easy to administer, the free cortisol response to waking may hold promise as a marker in studies of high-risk families predisposed to, or at risk for, mood disorders, we have presented a new method for forecasting the Longitudinal Dexamethasone Suppression Test (DST) data on a fully remitted lithium responder for past 5 years who was asymptomatic and treated with lithium throughout based on high-order fuzzy logical relationships. The proposed method gets a higher forecasting rate than the existing methods.

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ELIMINATION OF PARAMETERS AND PRINCIPLE OF LEAST SQUARES: FITTING OF LINEAR CURVE TO AVERAGE MAXIMUM TEMPERATURE DATA IN THE CONTEXT OF ASSAM

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ABSTRACT :
The principle of least squares, innovated by the French mathematician Legendre, when applied to observed data in order to fit a mathematical curve yields normal equations. The parameters involved in the curve are estimated by solving the normal equations. The number of normal equations becomes larger when the number of parameters associated to the curve becomes larger. In this situation, the solution of the normal equations for estimating the parameters becomes more complicated. For this reason, one more convenient method has been search for computing the estimates of the parameters. The method has been developed by the stepwise application of the principle of least squares. The method innovated here consists of the elimination of parameters first and then the minimization of the sum of squares of the errors. In this paper, the method has been described with reference to the estimation of parameters of a linear curve based on observed data on monthly average maximum temperature at Guwahati.
Key Words : ***Linear curve, Least squares principle, Stepwise application, Monthly average temperature.***

1. INTRODUCTION:

The method of least squares, which is indispensible and is widely used method of curve fitting to numerical data, was first discovered by the French mathematician Legendre. The first proof of this method was given by the renowned statistician Adrian (1808) followed by its second proof given by the German Astronomer Gauss [(1809)]. Apart from this two proofs as many as eleven proofs were developed at different times by a number of mathematicians viz. Laplace (1810), Ivory (1825), Hagen (1837), Bassel (1838), Donkim (1844), John Herscel (1850), Crofton(1870) etc.. Though none of the thirteen proofs is perfectly satisfactory yet it has given new dimension in setting the subject in a new light. In the method of least squares, the parameters of a curve are estimated by solving the normal equations of the curve obtained by the principle of least squares. However, for a curve of higher degree polynomial, the estimation of parameters by solving the normal equations carries a complicated calculation as the number of normal equations becomes large which leads to think of searching for some simpler method of estimation of parameter. Recently, Chakrabarty (2014) developed a new method of fitting of a curve based on the application of the principle of least squares separately for each of parameters associated to the curve. In this study, an attempt has been made to discuss the method in the case of fitting of linear curve to observed data when the values of the independent variable are unequal intervals.

2. ESTIMATION OF PARAMETERS IN LINEAR CURVE:

Let the theoretical relationship between the dependent variable Y and the independent variable X be

$$Y = aX + b \tag{2.1}$$

Where 'a' and 'b' are the two parameters.

Let $Y_1, Y_2 \ldots\ldots, Y_n$ be n observations on Y corresponding to the observations $X_1, X_2, \ldots\ldots\ldots, X_n$ of X.

The objective here is to fit the curve given by (2.1) to the observed data on X and Y. Since the n pairs of observations

(X_1, Y_1), (X_2, Y_2), ……………. , (X_n, Y_n) may not lie on the curve (2.1), they satisfy the model

$y_i = ax_i + b + \xi_i$, $(i = 1, 2\ldots n)$ (2.2)

2.1 ESTIMATION OF PARAMETER: BY STEPWISE APPLICATION OF PRINCIPLES OF LEAST SQUARE AND BY SOLVING NORMAL EQUATION.

From (2.2), one can obtain the following n sets of (n – 1) equations in each set:

Set -1:

$$\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = a + \left(\frac{\xi_1 - \xi_2}{x_1 - x_2}\right)$$

$$\left(\frac{y_1 - y_3}{x_1 - x_3}\right) = a + \left(\frac{\xi_1 - \xi_3}{x_1 - x_3}\right)$$

$$\left(\frac{y_1 - y_4}{x_1 - x_4}\right) = a + \left(\frac{\xi_1 - \xi_4}{x_1 - x_4}\right)$$

$$\left(\frac{y_1 - y_n}{x_1 - x_n}\right) = a + \left(\frac{\xi_1 - \xi_n}{x_1 - x_n}\right)$$

Set -2:

$$\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = a + \left(\frac{\xi_2 - \xi_1}{x_2 - x_1}\right)$$

$$\left(\frac{y_2 - y_3}{x_2 - x_3}\right) = a + \left(\frac{\xi_2 - \xi_3}{x_2 - x_3}\right)$$

$$\left(\frac{y_2 - y_n}{x_2 - x_n}\right) = a + \left(\frac{\xi_2 - \xi_n}{x_2 - x_n}\right)$$

Set -*n*:

$$\left(\frac{y_n - y_1}{x_n - x_1}\right) = a + \left(\frac{\xi_n - \xi_1}{x_n - x_1}\right)$$

$$\left(\frac{y_n - y_2}{x_n - x_2}\right) = a + \left(\frac{\xi_n - \xi_2}{x_n - x_2}\right)$$

$$\left(\frac{y_n - y_{n-1}}{x_n - x_{n-1}}\right) = a + \left(\frac{\xi_n - \xi_{n-1}}{x_n - x_{n-1}}\right)$$

Now, Since $a_i = a + \zeta_i$, $i = 1,2,\ldots\ldots.n$

Therefore, the sum of squares of errors is

$$S = \sum_{i=1}^{n} \zeta_i^{\,2} = \sum_{i=1}^{n} (a_i - a)^2$$

Differentiating S w.r.t a and equating to zero we get

$$\therefore \qquad \frac{\delta S}{\delta a} = 0$$

Which yield

$$\sum_{i=1}^{n} (a_i - a)(-2) = 0$$

$$\Rightarrow \qquad \sum_{i=1}^{n} a_i = na$$

$$\hat{a} = \frac{\sum_{i=1}^{n} a_i}{n} \tag{2.3}$$

Using the value of (2.3) in (2.1) we get the value of 'b'

$$\hat{b} = \bar{y} - \hat{a}\bar{x} \tag{2.4}$$

Here, we have considered average of mean minimum and maximum temperature of five cities in the context of Assam as observed data to fit the following linear equation

Let the linear equation be

$Y_i = aX_i + b$ (i = 1, 2. …….n)

Where Y_i = Average of mean maximum Temperature.

X_i = Length of the day.

3. NUMERICAL PROBLEM: ON MAXIMUM TEMPERATURE

Ex: 3.1: Average of mean maximum temperature of Guwahati:

X_i	10.553	11.105	11.834	12.610	13.266	13.605	13.469	12.921	12.191	11.424	10.755	10.398
Y_i	23.536	26.226	29.972	30.883	31.363	31.768	31.995	32.470	31.720	30.383	27.731	24.724

Solution: The matrix $(C_{ij})_{12x12}$ where $C_{ij} = \frac{(y_i - y_j)}{(x_i - x_j)}$ has been obtained as

$$\begin{pmatrix}
0 & 4.873 & 5.024 & 3.572 & 2.885 & 2.697 & 2.901 & 3.773 & 4.996 & 7.861 & 20.767 & -7.665 \\
4.873 & 0 & 5.139 & 3.094 & 2.377 & 2.217 & 2.440 & 3.438 & 5.059 & 13.031 & -4.300 & 2.124 \\
5.024 & 5.139 & 0 & 1.174 & 0.971 & 1.014 & 1.237 & 2.298 & 4.896 & -1.002 & 2.077 & 3.654 \\
3.572 & 3.094 & 1.174 & 0 & 0.732 & 0.889 & 1.295 & 5.103 & -1.997 & 0.422 & 1.699 & 2.784 \\
2.885 & 2.377 & 0.971 & 0.732 & 0 & 1.195 & 3.113 & -3.209 & -0.332 & 0.532 & 1.446 & 2.315 \\
2.697 & 2.217 & 1.014 & 0.889 & 1.195 & 0 & -1.669 & -1.026 & 0.034 & 0.635 & 1.417 & 2.120 \\
2.901 & 2.440 & 1.237 & 1.295 & 3.113 & -1.669 & 0 & -0.867 & 0.215 & 0.788 & 1.571 & 2.368 \\
3.773 & 3.438 & 2.298 & 5.103 & -3.209 & -1.026 & -0.867 & 0 & 1.027 & 1.394 & 2.187 & 3.070 \\
4.996 & 5.059 & 4.896 & -1.998 & -0.332 & 0.034 & 0.215 & 1.027 & 0 & 1.743 & 2.778 & 3.902 \\
7.861 & 13.031 & -1.002 & 0.422 & 0.532 & 0.635 & 0.788 & 1.394 & 1.743 & 0 & 3.964 & 5.516 \\
20.767 & -4.300 & 2.077 & 1.699 & 1.446 & 1.417 & 1.571 & 2.188 & 2.278 & 3.964 & 0 & 8.423 \\
-7.665 & 2.124 & 3.654 & 2.784 & 2.315 & 2.196 & 2.368 & 3.070 & 3.902 & 5.516 & 8.423 & 0
\end{pmatrix}$$

The equation (2.3) which gives the estimate of the parameter '*a*' as shown below

$$\widehat{a}_{(STW)} = \frac{\sum_{i=1}^{n} a_i}{n}$$

$$= 2.394253722$$

The equation (2.4) gives

$$\widehat{b}_{(STW)} = \overline{y} - \widehat{a}\overline{x}$$

$$= 0.640401388$$

In this example, the normal equations for estimating *a* & *b* are

$$352.771 = 144.131a + 12b$$

$$\& \ 4271.610108 = 1746.108159\, a + 144.131\, b$$

Thus in this case,

$$\widehat{a}_{(NE)} = 2.306198622$$

$$\widehat{b}_{(NE)} = 1.698023869$$

Result :

$\widehat{a}_{(NE)}$ **= 2.306198622** $\widehat{a}_{(stw)}$ **= 2.394253722**

$\widehat{b}_{(NE)}$ **= 1.698023869** $\widehat{b}_{(stw)}$ **= 0.640401388**

Estimated value of temperature ($\widehat{y}$) by both the methods: Guwahati

Table: 3.1(a)

Length of Day (x)	Observed Temperature (y)	Estimated Temperature $\widehat{y}_{(STW)}$	Estimated Temperature $\widehat{y}_{(NE)}$	Estimates of Errors $\lvert\widehat{e}_{(NE)}\rvert = \lvert(y - \widehat{y}_{(NE)})\rvert$	Estimates of Errors $\lvert\widehat{e}_{(STW)}\rvert = \lvert(y - \widehat{y}_{(STW)})\rvert$
10.553	23.536	25.90696092	26.03533793	2.49933793	2.37096092
11.105	26.226	27.22858897	27.30835957	1.08235957	1.00258897
11.834	29.972	28.97399993	28.98957836	0.98242164	0.99800007
12.610	30.883	30.83194082	30.77918849	0.10381151	0.05105918
13.266	31.363	32.40257126	32.29205479	0.92905479	1.03957126
13.605	31.768	33.21422328	33.07385612	1.30585612	1.44622328
13.469	31.995	32.88860477	32.76021311	0.76521311	0.89360477
12.921	32.470	31.57655373	31.49641626	0.97358374	0.89344627
12.191	31.720	29.82874851	29.81289127	1.90710873	1.89125149
11.424	30.383	27.99235591	28.04403693	2.33896307	2.39064409
10.755	27.731	26.39060017	26.50119005	1.22980995	1.34039983
10.398	24.724	25.53585159	25.67787714	0.95387714	0.81185159
Total = 144.131	**Total = 352.771**	**Total = 352.7709999 ≅ 352.771**	**Total= 352.771**	**Sum of Absolute Deviation** $\sum\lvert\widehat{e}_{(NE)}\rvert$ **=15.0713973**	**Sum of Absolute Deviation** $\sum\lvert\widehat{e}_{(STW)}\rvert$ **= 15.12960172**

Absolute Mean Deviation ($\overline{e}_{(STW)}$ **) = 1.260800143**

Absolute Mean Deviation ($\overline{e}_{(NE)}$ **) = 1.255949775**

1. Test of significance for estimated temperature obtained by Stepwise Application of Principles of Least Squares (stw).

The null hypothesis to be tested is

H_0: There is no significant difference between the values of observed temperature and estimated temperature.

Under the null hypothesis H_0, the test statistic is

$$t = \frac{(\bar{y} - \bar{\hat{\hat{y}}}_{(STW)})}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$ with (n_1 + n_2 -2) d.f.

Where $$S^2 = \frac{1}{(n_1 + n_2 - 2)}\left[\sum_{i=1}^{12}(y_i - \bar{y})^2 + (\hat{y}_i - \bar{\hat{\hat{y}}}_{(STW)})^2\right]$$

And $n_1 = n_2 = 12$

Since $\bar{y} = \bar{\hat{\hat{y}}}_{(STW)}$ = 29.39758333

$$|t|_{cal} = 0 \quad \text{and} \quad t_{(tab,5\%,22d.f)} = 1.717$$

$$|t|_{cal} < t_{(tab,5\%,22d.f)}$$

Thus, the null hypothesis H_0 is accepted.

Accordingly, it can be concluded that the difference between observed temperature and the corresponding estimated temperature is insignificant.

2. Test of significance for estimated temperature obtained by solution of normal equations (NE).

The null hypothesis to be tested is

H_0: There is no significant difference between the values of observed temperature and estimated temperature.

Under the null hypothesis H_0, the test statistic is

$$t = \frac{(\bar{y} - \bar{\hat{\hat{y}}}_{(NE)})}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$ With (n_1 + n_2 -2) d.f.

Where $$S^2 = \frac{1}{(n_1 + n_2 - 2)}\left[\sum_{i=1}^{12}(y_i - \bar{y})^2 + (\hat{y}_i - \bar{\hat{\hat{y}}}_{(NE)})^2\right]$$

And $n_1 = n_2 = 12$

Since $\bar{y} = \bar{\hat{\hat{y}}}_{(NE)}$ = 29.39758333

$$|t|_{cal} = 0 \quad \text{and} \quad t_{(tab,5\%,22d.f)} = 1.717$$

$$|t|_{cal} < t_{(tab,5\%,22d.f)}$$

Thus, the null hypothesis H_0 is accepted.

Accordingly, it can be concluded that the difference between observed temperature and the corresponding estimated temperature is insignificant.

4. CONCLUSION

The method, developed here, is based on the principle of the elimination of parameters first and then the minimization of the sum of squares of the errors whiles the ordinary least squares is based on the principle of the minimization of the sum of squares of the errors first and then elimination of parameters.

It is to be noted that the number of steps of computations in estimating the parameters by the method introduced here is less than the number of steps in estimation of parameters by the solutions of the normal equations. This implies that the error that occurs due to approximation in computation is less in the former than in the later.

We, therefore, may conclude that stepwise application of principles of least squares method is a simpler method of obtaining least square estimates of parameters of linear curve than the method of solving the normal equations. The following tables **(Table-4.1(a))** show the values of *t* for the testing the significance of difference between the observed temperature and estimated temperature by both the method and comparison of their't' values

Table: 4.1(a)

Ex. No.	*Values of 't' in case of method of STW*	*Hypothesis*	*Significance /Insignificance*
1	$t_{cal} = 0 < t_{(tab,5\%.22d.f)} = 1.717$	H_0 Accepted	Insignificant
	Values of 't' in case of method of solution of NE	*Hypothesis*	*Significance /Insignificance*
2	$t_{cal} = 0 < t_{(tab,5\%.22d.f)} = 1.717$	H_0 Accepted	Insignificant
Comparison of 't' values of both the methods			
$t_{(STW)} = t_{(NE)}$			

From the above table, It is found that both the method are almost equal in estimating parameters associated with a linear equation in case of unequal interval of the independent variable. In this study, attempt has been made for the case of linear curve only. Other types of the curves are yet to be dealt

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CONTRA *vg*-OPEN AND CONTRA *vg*-CLOSED MAPPINGS

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ABSTRACT :
The aim of this paper is to introduce and study the concepts of contra vg-open and contra vg-closed mappings and the interrelationship between other contra-closed maps.
Keywords: ***vg-open set, vg-open map, vg-closed map, contra-closed map, contra-pre closed map, contra vg-open and contra vg-closed map.***
AMS Classification: ***54C10, 54C08, 54C05***

1. INTRODUCTION:

Mapping plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. During the years 2010 to 2014, S. Balasubramanian together with his research scholars defined and studied a variety of open, closed, almost open and almost closed mappings for *v*-open, rp-open *gpr*-closed and *vg*–closed sets as well contra-open and contra-closed mappings for semi-open, pre-open, rp-open, β-open and gpr-closed sets. Inspired with these concepts and its interesting properties the author of this paper tried to study a new variety of open and closed maps called contra *vg*-open and contra *vg*-closed maps. Throughout the paper *X*, *Y* means topological spaces (X, τ) and (Y,σ) on which no separation axioms are assured.

2. Preliminaries:

Definition 2.1: $A\subseteq X$ is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A\subseteq cl(int(A))$; $A\subseteq int(cl(int(A)))$; $A\subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A))\subseteq A$; $cl(int(cl(A)))\subseteq A$; $int(cl(int(A)))\subseteq A$]
b) *v*-open if there exists regular-open set U such that $U\subseteq A\subseteq cl(U)$.
c) *v*-dense in X if $v cl(A) = X$.
d) θ-closed if $A = Cl_\theta(A)$. The complement of a θ-closed set is said to be θ-open.
e) g-closed[rg-closed] if $cl(A)\subset U$[$rcl(A)\subset U$] whenever $A\subset U$ and U is open[r-open] in X.

f) g-open[rg-open] if its complement X - A is g-closed[rg-closed].

g) Zero[semi-zero] set of X if there exists a continuous [semi-continuous] function *f*: X $\rightarrow$ R such that A = $\{x \in X : f(x) = 0\}$. Its complement is called co-zero[co-semi-zero] set of X.

Definition 2.2: A function $f{:}X \rightarrow Y$ is said to be

a) continuous[resp:semi-continuous, r-continuous, *v*-continuous] if the inverse image of every open set is open [resp: semi open, regular open, *v*--open].

b) irresolute [resp: r-irresolute, *v*-irresolute] if the inverse image of every semi open [resp: regular open, *v*-open] set is semi open [resp: regular open, *v*-open].

c) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].

d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].

Definition 2.3: A function $f{:}\ X \rightarrow Y$ is said to be

a) contra closed[resp: contra semi-closed; contra pre-closed; contra α-closed; contra rα-closed; contra β-closed; contra g-closed; contra r*g*-closed; contra sg-closed; contra gs-closed; contra pg-closed; contra gp-closed;] if the image of every closed set in *X* is open[resp: semi-open; pre-open; α-open; rα-open; β-open; g-open; r*g*-open; sg-open; gs-open; pg-open; gp-open] in *Y*.

b) contra open[resp: contra semi-open; contra pre-open; contra α-open; contra rα-open; contra β-open; contra g-open; contra r*g*-open; contra sg-open; contra gs-open; contra pg-open; contra gp-open;] if the image of every open set in *X* is closed[resp: semi-closed; pre-closed; α-closed; rα-closed; β-closed; g-closed; r*g*-closed; sg-closed; gs-closed; pg-closed; gp-closed] in *Y*.

c) almost contra closed[resp: almost contra semi-closed; almost contra pre-closed; almost contra α-closed; almost contra rα-closed; almost contra β-closed; almost contra g-closed; almost contra r*g*-closed; almost contra sg-closed; almost contra gs-closed; almost contra pg-closed; almost contra gp-closed;] if the image of every closed set in *X* is open[resp: semi-open; pre-open; α-open; rα-open; β-open; g-open; r*g*-open; sg-open; gs-open; pg-open; gp-open] in *Y*.

d) almost contra open[resp: almost contra semi-open; almost contra pre-open; almost contra α-open; almost contra rα-open; almost contra β-open; almost contra g-open; almost contra r*g*-open; almost contra sg-open; almost contra gs-open; almost contra pg-open; almost contra gp-open;] if the image of every open set in *X* is closed[resp: semi-closed; pre-closed; α-closed; rα-closed; β-closed; g-closed; r*g*-closed; sg-closed; gs-closed; pg-closed; gp-closed] in *Y*.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

Remark 1: We have the following implication diagrams for closed sets.

g.closed gs.closed

↓ ↓

rαg.closed → rgα.closed → rg.closed → ***vg*.closed** ← sg.closed ← βg.closed

↑ ↑ ↑ ↑ ↑

→ → rα.closed → → *v*.closed →

↑ ↓ ↑ ↑

r.closed → π.closed → closed → α.closed → s.closed → β.closed
↓ ↓ ↓→→→
↓
πg.closed p.closed → ω.closed gα.closed
↓ ↓
gp.closed ← pg.closed rω.closed

§3. CONTRA *vg*-OPEN MAPPINGS:

Definition 3.1: A function $f: X \to Y$ is said to be contra *vg*-open if the image of every *r*-open set in X is *vg*-closed in Y.

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is contra *vg*-open, contra rg-open and contra rgα-open but not contra open, contra semi-open, contra pre-open, contra α-open, contra rα-open, contra *v*-open, contra π-open, contra β-open, contra g-open, contra sg-open, contra gs-open, contra pg-open, contra gp-open and contra βg-open .

Example 2: Let $X = Y = \{a, b, c, d\}$; $\tau = \{\varphi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f: X \to Y$ be defined $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then f is not contra *vg*-open.

Theorem 3.1: We have the following interrelation among the following contra open mappings

(i) c.g.open c.gs.open
↓ ↓
c.rαg.open → c.rgα.open → c.rg.open → c.***vg*.open** ← c.sg.open ← c.βg.open
↑ ↑ ↑ ↑ ↑
→→ c.rα.open → → c.*v*.open →
↑ ↓ ↑ ↑
c.r.open → c.π.open → c.open → c.α.open → c.s.open → c.β.open
↓ ↓ ↓
c.πg.open c.p.open → c.ω.open c.gα.open
↓ ↓
c.gp.open ← c.pg.open c.rω.open None is reversible

(ii) If *v*GC(Y) = RC(Y), then the reverse relations hold for all contra open maps.

c.g.open c.gs.open
↑↓ ↑↓
c.rαg.open ↔ c.rgα.open ↔ c.rg.open ↔ c.***vg*.open** ↔ c.sg.open ↔ c.βg.open
↑↓ ↑↓ ↑↓ ↑↓ ↑↓
↔ ↔ c.rα.open ↔ ↔ ↔ c.*v*.open ↔
↑↓ ↑↓ ↑↓ ↑↓
c.r.open ↔ c.π.open ↔ c.open ↔ c.α.open ↔ c.s.open ↔ c.β.open

Theorem 3.2:

(i) If (Y, σ) is discrete, then f is contra open of all types.

(ii) If f is contra open and g is vg-closed then gof is contra vg-open.

(iii) If f is open and g is contra vg-open then gof is contra vg-open.

Corollary 3.1: If f is contra open and g is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-] closed then gof is contra vg-open.

Corollary 3.2: If f is open[r-closed] and g is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-;c-π-] open then gof is contra vg-open.

Theorem 3.3: If f: X→ Y is contra vg-open, then $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Proof: Let A⊂X be r-open and f: X→ Y is vg-closed gives $f(\mathrm{cl}\{A\})$ is vg-closed in Y and $f(A) \subset f(\mathrm{cl}(A))$ which in turn gives $vg(\mathrm{cl}(f(A))) \subset vg\mathrm{cl}(f(\mathrm{cl}(A)))$- - - - - (1)

Since $f(\mathrm{cl}(A))$ is vg-closed in Y, $vg\mathrm{cl}(f(\mathrm{cl}(A))) = f(\mathrm{cl}(A))$ - - - - - - (2)

From (1) and (2) we have $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 3.3: If f: X→ Y is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-;c-π-] open, then $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Theorem 3.4: If $f:X\to Y$ is contra vg-open and $A\subseteq X$ is r-open, $f(A)$ is τ_{vg}-closed in Y.

Proof: Let A⊂X be r-open and f: X→ Y is vg-closed implies $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$ which in turn implies $vg(\mathrm{cl}(f(A))) \subset f(A)$, since $f(A) = f(\mathrm{cl}(A))$. But $f(A) \subset vg(\mathrm{cl}(f(A)))$. Combining we get $f(A) = vg(\mathrm{cl}(f(A)))$. Hence $f(A)$ is τ_{vg}-closed in Y.

Corollary 3.4: If $f:X\to Y$ is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-]open, then $f(A)$ is τ_{vg} closed in Y if A is r-open set in X.

Theorem 3.5: If $vg(\mathrm{cl}(f(A))) = \mathrm{rcl}(A)$ for every A⊂Y and X is discrete space, then the following are equivalent:

a) f: X→ Y is contra vg-open map

b) $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Proof: (a) ⇒ (b) follows from theorem 3.3

(b) ⇒ (a) Let A be any r-open set in X, then $f(A) = f(\mathrm{cl}(A)) \supset vg(\mathrm{cl}(f(A)))$ by hypothesis. We have $f(A) \subset vg(\mathrm{cl}(f(A)))$. Combining we get $f(A) = vg(\mathrm{cl}(f(A))) = \mathrm{r}(\mathrm{cl}(f(A)))$[by given condition] which implies $f(A)$ is r-closed and hence vg-closed. Thus f is contra vg-open.

Theorem 3.6: If $v(\mathrm{cl}(A)) = \mathrm{rcl}(A)$ for every A⊂Y and X is discrete space, then the following are equivalent:

a) f: X→ Y is contra vg-open map

b) $vg(\mathrm{cl}(f(A))) \subset f(\mathrm{cl}(A))$

Proof: (a) ⇒ (b) follows from theorem 3.3

(b) ⇒ (a) Let A be any r-open set in X, then $f(A) = f(\mathrm{cl}(A)) \supset vg(\mathrm{cl}(f(A)))$ by hypothesis. We have $f(A) \subset vg(\mathrm{cl}(f(A)))$. Combining we get $f(A) = vg(\mathrm{cl}(f(A))) = \mathrm{rcl}(f(A))$[by given condition] which implies $f(A)$ is r-closed and hence vg-closed. Thus f is contra vg-open.

Theorem 3.7: $f:X\to Y$ is contra vg-open iff for each subset S of Y and each $U\in RC(X, f^{-1}(S))$, there is an vg-open set V of Y such that $S\subseteq V$ and $f^{1}(V) \subseteq U$.

Proof: Assume $f:X\to Y$ is contra vg-open. Let $S \subseteq Y$ and $U\in RC(X, f^{-1}(S))$. Then X-U is r-open in X and $f(X\text{-}U)$ is vg-closed in Y as f is contra vg-open and $V=Y\text{-} f(X\text{-}U)$ is vg-open in Y. $f^{-1}(S) \subseteq U \Rightarrow S\subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-} f(X\text{-}U)) = f^{-1}(Y)\text{-} f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be r-closed in $X \Rightarrow F^c$ is r-open. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an *vg*-open set V of *Y*, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is *vg*-closed in *Y*. Therefore *f* is contra *vg*-open.

Remark 3: Composition of two contra *vg*-open maps is not contra *vg*-open in general.

Theorem 3.8: Let *X*, *Y*, *Z* be topological spaces and every *vg*-closed set is *r*-open in *Y*. Then the composition of two contra *vg*-open maps is contra *vg*-open.

Proof: (a) Let *f* and g be contra *vg*-open maps. Let A be any *r*-open set in $X \Rightarrow f(A)$ is *r*-open in *Y* (by assumption) $\Rightarrow g(f(A)) = g \circ f(A)$ is *vg*-closed in *Z*. Therefore *gof* is contra *vg*-open.

Corollary 3.5: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed set is open [*r*-open] in *Y*. Then the composition of two c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα-*; c-*rα-*; c-*α-; c-s-; c-p-*; c-*β-; c-v-; c-*π-; c-*r*-]open maps is contra *vg*-open.

Example 3: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f: X \to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g: Y \to Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then *g*, *f* and $g \circ f$ are contra *vg*-open.

Theorem 3.9: If $f: X \to Y$ is contra g-open[contra rg-open], $g: Y \to Z$ is *vg*-closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is contra *vg*-open.

Proof: (a) Let A be *r*-open in *X*. Then $f(A)$ is *g*-closed and so closed in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-closed in *Z* (since *g* is *vg*-closed). Hence *gof* is contra *vg*-open.

Corollary 3.6: If $f: X \to Y$ is contra *g*-open[contra *rg*-open], $g: Y \to Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; *r*-]closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is contra *vg*-open.

Theorem 3.10: If $f: X \to Y$ is g-open[rg-open], $g: Y \to Z$ is contra *vg*-open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is contra *vg*-open.

Proof: (a) Let A be *r*-open in *X*. Then $f(A)$ is *g*-open and so open in *Y* as *Y* is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is *vg*-closed in *Z* (since *g* is contra *vg*-open). Hence *gof* is contra *vg*-open.

Corollary 3.7: If $f: X \to Y$ is g-open[rg-open], $g: Y \to Z$ is c-*g*-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα-*; c-*rα-*; c-*α-; c-s-; c-p-*; c-*β-; c-*π-]open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is contra *vg*-open.

Theorem 3.11: If $f: X \to Y$ is c-*g*-open[c-*rg*-open], $g: Y \to Z$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;* π-]closed and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is contra *vg*-open.

Proof: Let A be *r*-open set in X, then $f(A)$ is *g*-closed in Y and so closed in *Y* (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is *gs*-closed in *Z*. Hence *gof* is contra *vg*-open [since every *gs*-closed set is *vg*-closed].

Theorem 3.12: If $f: X \to Y$, $g: Y \to Z$ be two mappings such that *gof* is contra *vg*-open then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is contra *vg*-open.

b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is contra *vg*-open.

Proof: For A *r*-open in *Y*, $f^{1}(A)$ open in $X \Rightarrow (g \circ f)(f^{1}(A)) = g(A)$ *vg*-closed in *Z*. Hence *g* is contra *vg*-open. Similarly one can prove the remaining parts and hence omitted.

Corollary 3.8: If $f: X \to Y$, $g: Y \to Z$ be two mappings such that *gof* is c-g-[c-*rg-; c-sg-; c-gs-; c-βg-; c-rαg-; c-rgα-*; c-*rα-*; c-*α-; c-s-; c-p-*; c-*β-; c-v-; c-*π-; c-*r*-]open then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is contra *vg*-open.

b) If f is g-continuous[rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is contra vg-open.

Theorem 3.13: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that gof is vg-closed then the following statements are true.

a) If f is contra-continuous [contra-r-continuous] and surjective then g is contra vg-open.

b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [resp: r-$T_{½}$] then g is contra vg-open.

Proof: For A r-open in Y, f^{1}(A) closed in $X \Rightarrow (g\ o\ f)(f^{1}(A)) = g(A)$ vg-closed in Z. Hence g is contra vg-open.

Corollary 3.9: If $f:X\to Y$, g:$Y\to Z$ be two mappings such that gof is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]closed then the following statements are true.

a) If f is contra-continuous [contra-r-continuous] and surjective then g is contra vg-open.

b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is contra vg-open.

Theorem 3.14: If X is vg-regular, f: X→ Y is r-closed, nearly-continuous, contra vg-open surjection and $\bar{A} = A$ for every vg-closed set in Y, then Y is vg-regular.

Proof: Let $p\in U\in vGO(Y)$. Then there exists a point x∈X such that f(x) = p as f is surjective. Since X is vg-regular and f is r-continuous there exists $V\in RO(X)$ such that $x\in V\subseteq \overline{V}\subseteq f^{1}(U)$ which implies $p\in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U$ → (1)

Since f is vg-closed, $f(\overline{V}) \subseteq U$, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ and $\overline{f(\overline{V})} = \overline{f(V)}$ →(2)

By (1) & (2) we have $p\in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is vg-open. Hence Y is vg-regular.

Corollary 3.10: If X is vg-regular, f: X→ Y is r-closed, nearly-continuous, contra vg-open surjection and $\bar{A} = A$ for every r-closed set in Y then Y is vg-regular.

Theorem 3.15: If $f:X\to Y$ is contra vg-open and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra vg-open.

Proof: Let F be an r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow f$(A) is vg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is contra vg-open.

Theorem 3.16: If $f:X\to Y$ is contra vg-open, X is $rT_{½}$ and A is rg-open set of X then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra vg-open.

Proof: Let F be a r-open set in A. Then F = A∩E for some r-open set E of X and so F is r-open in $X \Rightarrow f$(A) is vg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is contra vg-open.

Corollary 3.11: If $f:X\to Y$ is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-] open and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra vg-open.

Theorem 3.17: If $f_i:X_i\to Y_i$ be contra vg-open for i =1, 2. Let f: $X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is contra vg-open.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-closed set in $Y_1 x Y_2$. Hence f is contra vg-open.

Corollary 3.12: If $f_i:X_i\to Y_i$ be c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-]open for i =1, 2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is contra vg-open.

4. CONTRA *vg*-CLOSED MAPPINGS:

Definition 4.1: A function *f: X→Y* is said to be contra *vg*-closed if the image of every *r*-closed set in *X* is *vg*-open in *Y*.

Example 4: Let *X* = *Y* = {a, b, c}; τ = {φ, {a}, {b, c}, *X*}; σ = {φ, {a}, {b}, {a, b}, *Y*}. Let *f:X→Y* be defined *f*(a) = c, *f*(b) = a and *f*(c) = b. Then *f* is contra *vg*-closed, contra rg-closed and contra rgα-closed but not contra closed, contra semi-closed, contra pre-closed, contra α-closed, contra rα-closed, contra *v*-closed, contra π-closed, contra β-closed, contra g-closed, contra sg-closed, contra gs-closed, contra pg-closed, contra gp-closed and contra βg-closed.

Example 5: Let *X* = *Y* = {a, b, c, d}; τ = {φ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, *X*}. Let *f:X→Y* be defined *f*(a) = b, *f*(b) = a, *f*(c) = d and *f*(d) = c. Then *f* is not contra *vg*-closed.

Theorem 4.1: We have the following interrelation among the following contra closed mappings

(i) c.g.closed c.gs.closed

↓ ↓

c.rαg.closed → c.rgα.closed → c.rg.closed → c.***vg*.closed** ← c.sg.closed ← c.βg.closed

↑ ↑ ↑ ↑ ↑

→ → c.rα.closed → → c.*v*.closed → → →

↑ ↓↑ ↑

c.r.closed → c.π.closed → c.closed → c.α.closed → c.s.closed → c.β.closed

↓ ↓ ↓

c.πg.closed c.p.closed → c.ω.closed c.gα.closed

↓ ↓

c.gp.closed ← c.pg.closed c.rω.closed None is reversible

(ii) If *v*GO(Y) = RO(Y), then the reverse relations hold for all contra closed maps.

c.g.closed c.gs.closed

↑↓ ↑↓

c.rαg.closed ↔ c.rgα.closed ↔ c.rg.closed ↔ c.***vg*.closed** ↔ c.sg.closed ↔ c.βg.closed

↑↓ ↑↓ ↑↓ ↑↓ ↑↓

↔ ↔ c.rα.closed ↔ ↔ ↔ c.*v*.closed ↔

↑↓ ↑↓ ↑↓ ↑↓

c.r.closed ↔ c.π.closed ↔ c.closed ↔ c.α.closed ↔ c.s.closed ↔ c.β.closed

Theorem 4.2:

(i) If (Y, σ) is discrete, then *f* is contra closed of all types.

(ii) If *f* is contra closed and *g* is *vg*-open then *gof* is contra *vg*-closed.

(iii) If *f* is closed and *g* is contra *vg*-closed then *gof* is contra *vg*-closed.

Corollary 4.1: If *f* is contra closed and *g* is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; *r*-] open then *gof* is contra *vg*-closed.

Corollary 4.2: If f is closed[r-open] and g is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-;c-π-] closed then gof is contra vg-closed.

Theorem 4.3: If f: X→ Y is contra vg-closed, then $f(A^o) \subset vg(f(A))^o$

Proof: Let A⊆ X be r-closed and f: X→ Y is contra vg-closed gives $f(A^o)$ is vg-open in Y and $f(A^o) \subset f(A)$ which in turn gives $vg(f(A^o))^o \subset vg(f(A))^o$ - - - (1)

Since $f(A^o)$ is vg-open in Y, $vg(f(A^o))^o = f(A^o)$ - - - - - - - - - - -(2)

combining (1) and (2) we have $f(A^o) \subset vg(f(A))^o$ for every subset A of X.

Remark 4: Converse is not true in general.

Corollary 4.3: If f: X→ Y is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-;c-π-] closed, then $f(A^o) \subset vg(f(A))^o$

Theorem 4.4: If f:X→Y is contra vg-closed and A⊆X is r-closed, f(A) is τ_{vg}-open in Y.

Proo*f:* Let A⊂X be r-closed and f:X→Y is contra vg-closed ⇒ $f(A^o) \subset vg(f(A))^o \Rightarrow f(A) \subset vg(f(A))^o$, since $f(A) = f(A^o)$. But $vg(f(A))^o \subset f(A)$. Combining we get $f(A) = vg(f(A))^o$. Hence f(A) is τ_{vg}-open in Y.

Corollary 4.4: If f:X→Y is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-r-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-]closed, then f(A) is τ_{vg} open in Y if A is r-closed set in X.

Theorem 4.5: If $vg(A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:

a) f: X→ Y is contra vg-closed map

b) $f(A^o) \subset vg(f(A))^o$

Proo*f:* (a) ⇒(b) follows from theorem 4.3.

(b) ⇒ (a) Let A be any r-closed set in X, then $f(A) = f(A^o) \subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies f(A) is vg-open. Therefore f is contra vg-closed.

Theorem 4.6: If $v(A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:

a) f: X→ Y is contra vg-closed map

b) $f(A^o) \subset vg(f(A))^o$

Proo*f:* (a) ⇒(b) follows from theorem 4.3.

(b) ⇒ (a) Let A be any r-closed set in X, then $f(A) = f(A^o) \subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies f(A) is vg-open. Therefore f is contra vg-closed.

Theorem 4.7: f:X→Y is contra vg-closed iff for each subset S of Y and each U∈RO(X, f^{-1}(S)), there is an vg-closed set V of Y such that S⊆V and $f^1(V) \subseteq U$.

Proof: Assume f:X→Y is contra vg-closed. Let S ⊆ Y and U∈RO(X, f^{-1}(S)). Then X-U is r-closed in X and $f(X$-U) is vg-open in Y as f is contra vg-closed and V=Y- $f(X$-U) is vg-closed in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-} f(X\text{-}U)) = f^{-1}(Y)\text{-} f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be r-open in X ⇒ F^c is r-closed. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an vg-closed set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus f(F) is vg-open in Y. Therefore f is contra vg-closed.

Remark 5: Composition of two contra vg-closed maps is not contra vg-closed in general.

Theorem 4.8: Let X, Y, Z be topological spaces and every vg-open set is r-closed in Y. Then the composition of two contra vg-closed maps is contra vg-closed.

Proof: (a) Let f and g be contra vg-closed maps. Let A be any r-closed set in $X \Rightarrow f$(A) is r-closed in Y (by assumption) $\Rightarrow g(f(A)) = g\ o\ f(A)$ is vg-open in Z. Therefore gof is contra vg-closed.

Corollary 4.5: Let X, Y, Z be topological spaces and every g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; π-]open set is closed [r-closed] in Y. Then the composition of two c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-; c-r-]closed maps is contra vg-closed.

Example 6: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f{:}X\to Y$ be defined f(a) = c, f(b) = b and f(c) = a and $g{:}Y\to Z$ be defined g(a) = b, g(b) = a and g(c) = c, then g, f and $g\ o\ f$ are contra vg-closed.

Theorem 4.9: If $f{:}X\to Y$ is contra g-closed[contra rg-closed], g:$Y\to Z$ is vg-open and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra vg-closed.

Proof: (a) Let A be r-closed in X. Then f(A) is g-open and so open in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is vg-open in Z (since g is vg-open). Hence gof is contra vg-closed.

Corollary 4.6: If $f{:}X\to Y$ is contra g-closed[contra rg-closed], $g{:}Y\to Z$ is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]open and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra vg-open.

Theorem 4.10: If $f{:}X\to Y$ is g-closed[rg-closed], g:$Y\to Z$ is contra vg-closed and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra vg-closed.

Proof: (a) Let A be r-closed in X. Then f(A) is g-closed and so closed in Y as Y is $T_{1/2} \Rightarrow g(f(A)) = gof(A)$ is vg-open in Z (since g is contra vg-closed). Hence gof is contra vg-closed.

Corollary 4.7: If $f{:}X\to Y$ is g-closed[rg-closed], $g{:}Y\to Z$ is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-π-]closed and Y is $T_{1/2}$[r-$T_{1/2}$], then gof is contra vg-closed.

Proof: Let A be r-closed set in X, then f(A) is g-open in Y and so open in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is gs-open in Z. Hence gof is contra vg-closed [since every gs-open set is vg-open].

Theorem 4.11: If $f{:}X\to Y$ is c-g-closed[c-rg-closed], $g{:}Y\to Z$ is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; π-]open and Y is $T_{1/2}$[r-$T_{1/2}$], then gof is contra vg-closed.

Proof: Let A be r-closed set in X, then f(A) is g-open in Y and so open in Y (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is gs-open in Z. Hence gof is contra vg-closed [since every gs-open set is vg-open].

Theorem 4.12: If $f{:}X\to Y$, $g{:}Y\to Z$ be two mappings such that gof is contra vg-closed then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is contra vg-closed.

b) If f is g-continuous[resp: rg-continuous], surjective and X is $T_{1/2}$ [resp: r-$T_{1/2}$] then g is contra vg-closed.

Proof: For A r-closed in Y, f^{1}(A) closed in $X \Rightarrow (g\ o\ f)(f^{1}(A)) = g(A)$ vg-open in Z. Hence g is contra vg-closed. Similarly one can prove the remaining parts and hence omitted.

Corollary 4.8: If $f{:}X\to Y$, g:$Y\to Z$ be two mappings such that gof is c-g-[c-rg-; c-sg-; c-gs-; c-βg-; c-$r\alpha g$-; c-$rg\alpha$-; c-$r\alpha$-; c-α-; c-s-; c-p-; c-β-; c-v-; c-π-; c-r-]closed then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is contra vg-closed.

b) If f is g-continuous[rg-continuous], surjective and X is $T_{1/2}$ [r-$T_{1/2}$] then g is contra vg-closed.

Theorem 4.13: If $f{:}X\to Y$, $g{:}Y\to Z$ be two mappings such that gof is vg-open then the following statements are true.

a) If f is contra-continuous [contra-r-continuous] and surjective then g is contra vg-closed.

b) If f is contra-g-continuous[contra-rg-continuous], surjective and X is $T_{1/2}$ [resp: r-$T_{1/2}$] then g is contra vg-closed.

Proof: For A *r*-closed in *Y*, f^{-1}(A) open in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ *vg*-open in *Z*. Hence *g* is contra *vg*-closed.

Corollary 4.9: If $f{:}X\to Y$, $g{:}Y\to Z$ be two mappings such that *gof* is g-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; *r*-]open then the following statements are true.

a) If *f* is contra-continuous [contra-*r*-continuous] and surjective then *g* is contra *vg*-closed.

b) If *f* is contra-g-continuous[contra-rg-continuous], surjective and *X* is $T_{½}$ [r-$T_{½}$] then *g* is contra *vg*-closed.

Theorem 4.14: If *X* is *vg*-regular, $f{:}X\to Y$ is *r*-open, *r*-continuous, contra *vg*-closed surjective and $A^o = A$ for every *vg*-open set in *Y* then *Y* is *vg*-regular.

Proof: Let $p\in U\in v$GO(Y), ∃ a point $x\in X \ni f(x) = p$ by surjection. Since X is *vg*-regular and *f* is nearly-continuous, ∃ $V\in RC(X) \ni x\in V^o\subset V\subset f^{-1}(U)$ which implies $p\in f(V^o)\subset f(V) \subset U$-- - - - - - - - - (1)

for *f* is *vg*-open, $f(V^o)\subset U$ is *vg*-open. By hypothesis $f(V^o)^o = f(V^o)$ and $f(V^o)^o = \{f(V)\}^o$-- - - - - - -(2)

combining (1) and (2) $p\in f(V)^o\subset f(V) \subset U$ and $f(V)$ is *r*-closed. Hence Y is *vg*-regular.

Corollary 4.10: If *X* is *vg*-regular, $f{:}X\to Y$ is *r*-open, *r*-continuous, contra *vg*-closed, surjective and $A^o = A$ for every *r*-closed set in *Y* then *Y* is *vg*-regular.

Theorem 4.15: If $f{:}X\to Y$ is contra *vg*-closed and $A \in RC(X)$, then $f_A{:}(X,\tau(A))\to(Y,\sigma)$ is contra *vg*-closed.

Proof: Let F be an *r*-closed set in *A*. Then F = A∩E for some *r*-closed set E of *X* and so F is *r*-closed in $X \Rightarrow f(A)$ is *vg*-open in *Y*. But $f(F) = f_A(F)$. Therefore f_A is contra *vg*-closed.

Theorem 4.16: If $f{:}X\to Y$ is contra *vg*-closed, *X* is $rT_{½}$ and A is rg-closed set of *X* then $f_A{:}(X,\tau(A))\to(Y,\sigma)$ is contra *vg*-closed.

Proof: Let F be a *r*-closed set in *A*. Then F = A∩E for some *r*-closed set E of *X* and so F is *r*-closed in $X \Rightarrow f(A)$ is *vg*-open in *Y*. But $f(F) = f_A(F)$. Therefore f_A is contra *vg*-closed.

Corollary 4.11: If $f{:}X\to Y$ is c-g-[c-*rg-;* c-*sg-;* c-*gs-;* c-*βg-;* c-*rαg-;* c-*rgα-*; c-*r-*; c-*rα-*; c-*α-;* c-*s-;* c-*p-*; c-*β-;* c-*v-;* c-π-] closed and $A \in RC(X)$, then $f_A{:}(X,\tau(A))\to(Y,\sigma)$ is contra *vg*-closed.

Theorem 4.17: If $f_i{:}X_i\to Y_i$ be contra *vg*-closed for i =1, 2. Let $f{:}\ X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f{:}X_1xX_2\to Y_1xY_2$ is contra *vg*-closed.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is *r*-closed in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)\ x\ f_2(U_2)$ is *vg*-open set in Y_1xY_2. Hence *f* is contra *vg*-closed.

Corollary 4.12: If $f_i{:}X_i\to Y_i$ be c-g-[c-*rg-;* c-*sg-;* c-*gs-;* c-*βg-;* c-*rαg-;* c-*rgα-*; c-*r-*; c-*rα-*; c-*α-;* c-*s-;* c-*p-*; c-*β-;* c-*v-;* c-π-]closed for i =1, 2. Let $f{:}X_1xX_2\to Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f{:}X_1xX_2\to Y_1xY_2$ is contra *vg*-closed.

CONCLUSION:

In this paper the author introduced the concepts of contra *vg*-open mappings, contra *vg*-closed mappings, studied their basic properties and interrelationship between other such contra open and contra closed maps.

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OPTIMAL VACATION PERIOD WITH COST ANALYSIS OF AN INTERDEPENDENCE FUZZY QUEUE MODEL TO A COMMUNICATION SYSTEM

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ABSTRACT :
Mathematical study of queue model provides the basic frame work in communication system as there is a close resemblance between queue networks and communication systems. This paper deals with the application of an interdependence fuzzy queue model to a communication system which have the voice packetized statistical multiplexing. The objective of the paper is intended to review the state of affairs of analytical methods, queuing analytical techniques essential to modeling of communication system. By approximating the arrival of packets, transmission and the covariance between them under a bi-variate Poisson process, the performance of statistical multiplexing is measured. The steady state solution and system characteristics are derived. The model mechanism reduces the idle time of transmitter and enhances the system utilization capacity. A unified treatment of buffer storage over flow problem has been discussed as an application example in which we call attention to the analogy between buffer problem and waiting time.

The ratio of the cost associated with shifting the transmitter from voice buffer to data buffer and the cost associated with waiting time of voice packets is compared with idle time of transmitter in consideration of the dependence between arrival of messages and service transmission. The vacation period has been calculated with respect to cost ratio and change in interdependence between arrival of packets and number of transmission completion.
Keywords: ***Circuit switching, Packet switching, Communication Network, Fuzzy Arithmetic, Voice Transmission etc.***

1. Introduction:

Control of flow is of great interest in communication and production network application. Many researchers have examined this topic in context of queuing system and wealth of results has been published. Due to unpredictable nature of work demands placed on the resources (transmission lines) congestion occurs (depending upon workload level) in communication system, queues will be formed and delays introduced at critical resources. Hence, while evaluating the performance analysis of a communication system these queues are necessary to take into consideration. Of course, performance not the only measure technical issues in design of communication system but other issues such as routing and flow control, link capacity assignment, concentrator placement, allocation and distribution of data base are of great importance and can't be separated from performance analysis. In order to support efficient and flexible sharing of these resources, good techniques as message switching and packets switching multiplexing have been introduced, at the expense of additional complexity of system structure.

How to schedule and allocate resources effectively among competing requests is certainly in realm of congestion or queuing theory in a broader outlook. Queue Models have led to accurate analysis of Modern communication system. These models play significant role in modeling voice calls. The satellite communication system has been designed for voice traffic. With the advent of faxes and internet, the nature of traffic has changed dramatically. As a result, packet switched networks have gained importance over circuit switching networks. In all packet node communication system, network resources are managed by statistical multiplexing or dynamic memory allocation in which a communication channel is effectively divided into an arbitrary number of logical bit rate channels or data streams. The delay in packet switching can be reduced by utilizing the statistical multiplexing in communication system. The term multiplexing is used in connection with computer communication networks which employ some variant of time division multiplexing to share communication channels.

Various researchers studied the communication network as interconnected queues assuming arrivals and service patterns independent (Jenq (1984), Hoshida (1993), Srinivason Rao etal. (2000, 2006). But in many practical situations such as in computer communication applications the massage are transmitted between nodes in a networks, the service time depends upon (i) the length of message (ii) the line speed. Hence the service time of the same message at different nodes is dependent. Indeed not only the service time dependent but inter arrival time between two consecutive messages are also dependent. Srinivason Rao etal (2006) & Singh T.P. (2011) developed a queue model to interdependent communication system assuming arrival and transmission process at the node are correlated.

Since in communication system there is unpredictable and uncertain nature of demand at transmission lines, the fuzzy logic suites better comparative to random process. In the literature, customers inter arrival time and their service times are required to follow certain probability distributions with fixed parameters. However, in many real applications the parametric distribution may only be characterized subjectively in linguistic form i.e. the arrival and service are typically observed in everyday language, slow, average, fast, very fast etc. that can be best described by fuzzy set & logic. In this paper the arrival massages as well as the concerned activity in transmission and covariance between composite arrival and transmission completion have been assumed fuzzy in nature differs with the work done by Srinivason etal. The fuzzy concept in communication system was first introduced by T.P. Singh (see Singh T.P. & Kusum (2012)). The study was extended by Arti Tyagi & Singh T.P. (2012, 2013).

The present study is further an extended work of Arti Tyagi & Singh T.P. (2013) in which the trapezoidal fuzzy nature has been considered for interdependence communication system and a relation is derived between optimal vocational time & cost ration of system characteristic. The model is more significant and is relevant in real world situations. The model mechanism not only reduce the idle time of transmitter but enhance the capacity of channel utilization by approximating the arrival, packets transmission & covariance all considered fuzzy in nature under a bi-variate Poisson process and the various system characteristics are derived & analyzed for the model using fuzzy arithmetic. The main objective is to find out how the behavior of the buffer is characterized in terms of fuzzy process and buffer capacity.

4. Fuzzy Set:

Fuzzy logic extends Boolean logic to handle the expression of vague concepts. To express impression quantitatively a set membership function maps elements to real values between 0 & 1. The value indicates the degree to which an element belongs to a set. The degree is not describing probabilities that the item is in the set, but instead describes to what extent the item is in the set.

In the universe of discourse X, a fuzzy subset $\widetilde{A}$ on X is defined by the membership function $\mu_{\widetilde{A}}$ (X) Which maps each element x into X to a real number in the interval [0,1]. $\mu_{\widetilde{A}}$ (X) Denotes the grade or degree of membership and it is usually denoted by $\mu_{\widetilde{A}}(X) : X \to [0,1]$.

4.1 TRAPEZOIDAL FUZZY NUMBER:

$$\widetilde{A} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1 & a_2 \le x \le a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise} \end{cases}$$

4.2 TRAPEZOIDAL FUZZY NUMBER OPERATION:

Let $\widetilde{A} = (a_1,a_2,a_3, a_4)$ and $\tilde{B} = (b_1,b_2,b_3, b_4)$ be two trapezoidal fuzzy number, then the arithmetic operation on $\widetilde{A}$ and $\tilde{B}$ are given as follows :

Addition $\widetilde{A} + \tilde{B} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4]$

Subtraction $\widetilde{A} - \tilde{B} = [a_1-b_4, a_2-b_3, a_3-b_2, a_4- b_1]$

Multiplication $\widetilde{A} * \tilde{B} = [a_1b_2, a_2b_2, a_3b_3, a_4b_4]$

Division $\widetilde{A} / \tilde{B} = [a_1/b_4, a_2/b_3, a_3/b_2, a_4/ b_1]$

Provided $\widetilde{A}$ *and* $\tilde{B}$ are all non-zero positive numbers.

DEFUZZIFICATION OF TRAPEZOIDAL FUZZY NUMBER:

If $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then its associated crisp number is given by Chen's graded formula (1999) as follows:

$$A = \frac{a_1+2a_2+2a_3+a_4}{6}$$

5. Model Description & Notation:

Consider the arrival of packets and number of transmission are correlated. Both follows a bi-variate Poisson process having joint probability mass function based on the line of Milne (1974) & Srinivason Rao, etal.(2000) and Singh T.P. (2012). The capacity of buffer is assumed to be infinite and the number of packets arrival in any module is taken as a random variable x in fuzzy environment.

$\widetilde{\lambda_x}$: Arrival rate of message of size having x packets in fuzzy.

P(1,i)= Probability that i packets of voice are in buffer.

P (0,i) = Probability that i packets of data are in buffer.

$\widetilde{\epsilon_x}$: Covariance between arrival of packets and number of transmission completion in fuzzy.

$\tilde{\mu}$: Average transmission rate in fuzzy.

$\tilde{\rho}$ **:** Fuzzy busy time of the server = $\frac{\tilde{\lambda}}{\tilde{\mu}}$

$\widetilde{C_x}$: Probability that a batch of size x packets will arrive to buffer in fuzzy environment.

The composite arrival rate of packets $\tilde{\lambda} = \sum_x \widetilde{\lambda_x}$ and the covariance of the composite arrivals and transmission completions $\tilde{\varepsilon} = \sum_x \widetilde{\epsilon_x}$

The covariance is generated through bit dropping of flow control mechanism including the dependence between arrival of messages and service transmissions. Flow control is important because it is possible for a sending computer to transmit information at a faster rate than the destination computer can receive and process them. It can be distinguished from congestion control, which is used for controlling the flow of data when congestion has actually occurred. This situation can happen only if the receiving computer has a heavy traffic load than to sending computer, or if receiving computer has less processing power than the sending computer.

5.1. Postulates of the Model:

(1) The probability that there is no arrival and no service completion during a small interval of time Δt, when the system is in faster rate of arrival is $1- \left(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon}\right)\Delta t + O(\Delta t)$.

(2) The probability that there is one arrival & no service completion during a small interval of time Δt, when the system is in faster rate of arrivals, is $\left(\tilde{\lambda} - \tilde{\varepsilon}\right)\Delta t + O(\Delta t)$

(3) The probability that there is no arrival and in service completion during a small interval of time Δt, when the system is either in faster or slower rate of arrival is $(\tilde{\mu} - \tilde{\varepsilon})\ \Delta t + O(\Delta t)$

There are some additional assumptions given as.

(1) Assumed that data packets are stored for transmission and transmitted as and when transmitter is idle after voice transmission.

(2) Idle time of transmitter follows the negative exponential distribution with parameter $\frac{1}{\tilde{\eta}}$ .the differential difference equation of the fuzzy system is in steady state can be depicted as:

6. Steady State Equation:

The transition densities of the process are given by:

$$a_{ij} = \begin{cases} -(\tilde{\lambda} - \tilde{\varepsilon}) & i = j = 0 \\ -(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon}) & i = j = 1 \\ (\widetilde{\lambda -} \tilde{\varepsilon} + \tilde{\eta}) & i = 0, j = 1 \\ (\tilde{\mu} - \tilde{\varepsilon}) & i = 1, j = i + 1 \end{cases}$$

The Steady state Equations of Network which are written through matrix of densities are given by:

$(\tilde{\lambda} - \tilde{\varepsilon})\, P(0,0) = (\tilde{\mu} - \tilde{\varepsilon})\, P(1,1)$ i= 1,2,3………… (1)

$(\tilde{\lambda} - \tilde{\varepsilon} + \tilde{\eta})\, P(0,1) = (\tilde{\lambda} - \tilde{\varepsilon})\sum_{x=1}^{i} P(0, i-x)\, \widetilde{C_x}$ (2)

$(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon})\, P(1,1) = \tilde{\eta}\, P(0,1) + (\tilde{\mu} - \tilde{\varepsilon})\, P(1,2)$ (3)

$(\tilde{\lambda} + \tilde{\mu} - 2\tilde{\varepsilon})\, P(1,i) = \tilde{\eta}\, P(0,i) + (\tilde{\mu} - \tilde{\varepsilon})\, P(1,i+1) + (\tilde{\lambda} - \tilde{\varepsilon}) \sum_{x=1}^{i} P(0, i-x)\, \widetilde{C_x}$ (4)

i= 1, 2, 3…………

Probability that the Data/ voice Packets are under transmission is,

$P(K,.) = \sum_{i=k}^{\infty} P(K, i)$ (5)

7. Solution Methodology:

Consider p.d.f of P(K,i), K=1,2,3…………..

$P_K(\tilde{z}) = \sum_{i=k}^{\infty} P(K, i)\ \tilde{z}^{\,i}$ $|\tilde{z}| \leq 1$, for K=1,2,3………….. (6)

$\tilde{C}(\tilde{z}) = \sum_{x=1}^{\infty} \widetilde{C_x}\ \tilde{z}^{\,x}$ $|\tilde{z}| \leq 1$ (7)

Multiply (2) by $\tilde{z}^{\,i}$ & summing over i= 0, 1, 2……..

$(\tilde{\lambda} - \tilde{\varepsilon})\, P_0(\tilde{z}) + \tilde{\eta}\, P_0(\tilde{z}) = (\tilde{\lambda} - \tilde{\varepsilon})\, P_0(z)\, \tilde{C}(\tilde{z}) + \tilde{\eta}P(0,.)$

$(\tilde{\lambda} - \tilde{\varepsilon})\, [1 - \tilde{C}(\tilde{z})]\, P_0(\tilde{z}) + \tilde{\eta}\, P_0(\tilde{z}) = \tilde{\eta}P(0,.)$

$[(\tilde{\lambda} - \tilde{\varepsilon})[1 - \tilde{C}(\tilde{z})] + \tilde{\eta}]P_0(\tilde{z}) = \tilde{\eta}(0,.)$ (8)

Multiply (4) by $\tilde{z}^{\,i}$ & summing over i= 0,1,2…… with the help of (3) and (6)

$[(\tilde{\lambda} - \tilde{\varepsilon})\, Z[1 - \tilde{C}(\tilde{z})] - (\tilde{\mu} - \tilde{\varepsilon})(1 - \tilde{z})]P_1(\tilde{z}) = [(\tilde{\lambda} - \tilde{\varepsilon})[1 - \tilde{C}(\tilde{z}) + \tilde{\eta}(1 - \tilde{z})]\, P_0(z) + \tilde{\eta}P(0,.)(1 - \tilde{z})$ (9)

If we assume batch size $\tilde{c_x}$ is geometrically distributed

$\widetilde{C_x} = (1-\alpha)\, \alpha^{x-1}$ $0 < \alpha < 1$ & put in (6) (10)

After simplification,

$\tilde{C}(\tilde{z}) = \frac{(1-\alpha)\tilde{z}}{(1-\alpha\tilde{z})}$ (11)

$[\frac{(\tilde{\lambda}-\tilde{\varepsilon})\, \tilde{z}}{(\mathbf{1}-\boldsymbol{\alpha}\tilde{z})} - (\tilde{\mu} - \tilde{\varepsilon})]P_1(\tilde{z}) = -[\frac{(\tilde{\lambda}-\tilde{\varepsilon})\, \tilde{z}}{(\mathbf{1}-\boldsymbol{\alpha}\tilde{z})} - \tilde{\eta}]\, P_0(\tilde{z}) + \tilde{\eta}P(0,.)$ (12)

$P_1(\tilde{z}) = -[\frac{(\tilde{\lambda}-\tilde{\varepsilon}) + \eta(1-\alpha\tilde{z})}{(\tilde{\lambda}-\tilde{\varepsilon})\, \tilde{z} - (\tilde{\mu}-\tilde{\varepsilon})(1-\alpha\tilde{z})}]\, P_0(\tilde{z}) + \frac{\tilde{\eta}(1-\alpha\tilde{z})P(0,.)}{(\tilde{\lambda}-\tilde{\varepsilon})\, \tilde{z} - (\tilde{\mu}-\tilde{\varepsilon})(1-\alpha\tilde{z})}$ (13)

$P_0(\tilde{z}) = \frac{\tilde{\eta}(1-\alpha\tilde{z})P(0,.)}{(\tilde{\lambda}-\tilde{\varepsilon})(1-\tilde{z}) - \eta(1-\alpha\tilde{z})}$ (14)

Take $\tilde{z} = 1$,

$P_0(1) = P(0,.)$

$P_1(1) = \frac{(\tilde{\lambda}-\tilde{\varepsilon})P(0,.)}{(\tilde{\mu}-\tilde{\varepsilon})(\mathbf{1}-\boldsymbol{\alpha}) - (\tilde{\lambda}-\tilde{\varepsilon})}$ (15)

Using boundary condition,

$P_0(1) + P_1(1) = 1$

$P(0,.) = 1 - \frac{(\tilde{\lambda} - \tilde{\varepsilon})}{(\tilde{\mu} - \tilde{\varepsilon})(\mathbf{1-\alpha})}$

For $(\tilde{\lambda} - \tilde{\varepsilon}) < (\tilde{\mu} - \tilde{\varepsilon})(1-\alpha)$

$P(0,.) = 1 - \tilde{\rho}_0$, $\qquad \tilde{\rho}_0 \leq 1$

(16)

Where, $\tilde{\rho}_0 = \frac{(\tilde{\lambda} - \tilde{\varepsilon})}{(\tilde{\mu} - \tilde{\varepsilon})(\mathbf{1-\alpha})}$ (17)

The Probability that packets are under transmission is 1- $\tilde{\rho}_0$,

8. System Characteristics:

The Mean number of voice packets in network at any time, for this network is given by

$$\tilde{L} = \frac{\tilde{\rho}_0}{(1-\tilde{\rho}_0)(1-\alpha)} + \frac{(\tilde{\lambda} - \tilde{\varepsilon})}{\eta(1-\alpha)} \quad (18)$$

The Mean number of voice packets in buffer is,

$$\tilde{L}_q = \frac{\tilde{\rho}_0^{\,2}}{(1-\tilde{\rho}_0)(1-\alpha)} + \frac{\alpha\tilde{\rho}_0}{(1-\alpha)} + \frac{(\tilde{\lambda} - \tilde{\varepsilon})}{\tilde{\eta}(1-\alpha)} \quad (19)$$

$$\text{Variance} = \frac{\alpha\tilde{\rho}_0(1-\tilde{\rho}_0)+\tilde{\rho}_0}{(1-\tilde{\rho}_0)^2(1-\alpha)^2} + \frac{(1+\alpha)\tilde{\eta}}{\tilde{\eta}(1-\alpha)^2} + \frac{(\tilde{\lambda} - \tilde{\varepsilon})^2}{\tilde{\eta}^2(1-\alpha)^2} \quad (20)$$

The work further can be extended,

Assume when transmitter is utilized for data, the reward per unit time is R_2 along with a fixed set up cost C_0 associated with shifting the transmitter from voice buffer to data buffer. Let C be the cost associated with a waiting time of voice packet.

Maximize the profit for transmission, total profit of network.

$$R = R_2(1-\tilde{\rho}_0) - C_0\left(\frac{\tilde{\eta}}{\tilde{\lambda} - \tilde{\varepsilon} + \tilde{\eta}}\right)(1-\tilde{\rho}_0) - C\frac{(\tilde{\lambda} - \tilde{\varepsilon})}{\tilde{\eta}(1-\alpha)} \quad (21)$$

$C\frac{(\tilde{\lambda} - \tilde{\varepsilon})}{\tilde{\eta}(\mathbf{1-\alpha})}$ indicates the extra average waiting cost due to transmission of data.

Differentiate w.r.t $\tilde{\boldsymbol{\eta}}$ & equating to zero, the optimal value of $\frac{1}{\tilde{\boldsymbol{\eta}}}$ as,

$$\frac{1}{\tilde{\boldsymbol{\eta}}} = \frac{1}{(\tilde{\lambda} - \tilde{\varepsilon})}\sqrt{\frac{(1-\alpha)(1-\tilde{\rho}_0)C_0}{C}} - 1 \quad (22)$$

From (22) it is clear that optimal value of $\frac{1}{\tilde{\boldsymbol{\eta}}}$ (i.e the optimal vacation time) is practical valid if,

$$\frac{(1-\alpha)(1-\tilde{\rho}_0)C_0}{C} > 1 \quad (23)$$

9. Queuing systems are controlled using fuzzy control by emulating a skilled human operator at each decision epoch. The current state is observed and then an inference engine equipped with a fuzzy rule base fires an online decision to adjust the system behavior in order to guarantee that the system is optimal in some sense. The architecture of the fuzzy logic controllers depends on the features of each queuing system. The Universes of discourse for all fuzzy sets are continuous, and membership functions are chosen to be trapezoidal.

Table -1 shows the values of $\frac{1}{\tilde{\boldsymbol{\eta}}}$ by using Fuzzy Arithmetic Operation (4.2) on taking different values of cost ratio and fuzzy value of $\tilde{\varepsilon}$, for a given set of $\tilde{\lambda}$, $\tilde{\eta}$ taken in trapezoidal and given parameter α.

Table- 1 for $\tilde{\lambda}$ **=(3,4,5,6)**, $\tilde{\eta}$**= (9,8,7,6)**, α= .2

C_0/C	$\tilde{\varepsilon} \rightarrow$ (.1, .2, .25, 3)	(.26, 3,.34, .36)	(.38, .4, .42, .45)	(.5, .6, .7, .72)
10	(.141, .114, .09, .03)	(.15, .13, .105,.075)	(.16, .135, .115, .08)	(.18, .16, .13, .07)
15	(.21, .189, .176, .125)	(.22, .20, .19, .18)	(.23, .21, .20, .18)	(.26, .25, .22, .19)
20	(.27, .31, .30, .27)	(.28, .27, .26, .254)	(.29, .28, .279, .273)	(.33, .32, .30, .29)

After defuzzification the values of $\frac{1}{\tilde{\eta}}$ becomes

C_0/C	$\tilde{\varepsilon} \rightarrow$ (.1, .2, .25)	(.26, .28, .34)	(.36, .38, .44)	(.45, .47, .5)
10	.10	.11	.12	.14
15	.18	.19	.20	.23
20	.24	.26	.28	.31

Conclusion:

The Model has been applied for characterizing the packetized data or voice transmission as interdependent queue model in fuzzy environment with bulk arrivals. The performance of the model is measured through approximating fuzzified bi-varient Poisson processes. The ratio of the cost associated with shifting the transmitter from voice buffer to data buffer and the cost associated with waiting time of voice packets is compared with idle time of transmitter in consideration of the dependence between arrival of messages and service transmission. From the data table it is clear that as the cost ratio increases the optimal vacation period increases with increase in the dependence between arrival of messages and service transmission. We observe that the variability of number of voice packets are decreasing as the dependent parameter $\tilde{\varepsilon}$ is increasing for the given values of$\tilde{\lambda}$, $\tilde{\mu}$ and α. It is observed that these networks can reduce mean delay in transmission and burstness of buffer and the delay for transmission of voice packets can be reduced by increasing the dependence.

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STABILITY OF JUNGCK ITERATION PROCESS IN COMPLETE METRIC SPACES

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ABSTRACT :
The aim of this paper is to establish w^2-stablilty of Jungck iteration procedure for some contractive conditions. It is generalization of some well known results in the literature.
Keywords and Phrases: ***Fixed Point iteration procedures, T-stability, w^2-stability***
Subject Classification: ***47H 10, 54H25***

1. INTRODUCTION:

The theory of stability of iterative procedures plays an important role in the study of numerical computations. The revolution in the computational mathematics through computer programming has accelerated the increasing interest in the stability theory of iterative procedures. The study of stability of iterative procedures enjoys a celebrated place in applied sciences and engineering due to chaotic behavior of functions in discrete dynamics. We say that iterative procedure is stable if the actual sequence converges to a fixed point of the function, and then the approximate sequence also converges to the same fixed point. See Berinde [1,2] for details on various iteration processes.

Jungck iteration procedure introduced by Jungck [3] as follows:

For Jungck iteration procedure, suppose that (X,d) be a complete metric space, $Y \subseteq X$ and $S, T : Y \to X$, $T(Y) \subseteq S(Y)$. For any $x_0 \in Y$, $\{Sx_n\}_{n=0}^{\infty} \subset X$, be iteratively defined as follows:

$Sx_{n+1} = Tx_n$.

By taking S = I (identity mapping) and Y =X, Jungck iteration procedure becomes Picard iteration procedure. Jungck [16] proved that the maps S and T satisfying

$d(Tx, Ty) \leq k\, d(Sx, Sy), \quad 0 \leq k < 1$, (J)

for all $x, y \in X$ have a unique common fixed point in complex linear space X, provided that S and T are commuting, , $T(X) \subseteq S(X)$,S is continuous.

Definition 1.1 [1]: Let X be a Banach space, T a self map of X and assume that $x_{n+1} = f(T, x_n)$ defines some iteration procedure involving T. For example, if $f(T, x_n) = Tx_n$. Suppose that $\{x_n\}$ converges to a fixed point p of T. Let $\{y_n\}$ be an arbitrary sequence in X and define $\in_n = \|y_{n+1} - f(T, y_n)\|$ for n = 0, 1, 2,….. If $\lim \in_n = 0$ implies that $\lim_n y_n = p$, then the iteration process $x_{n+1} = f(T, x_n)$ is said to be T-stable.

Definition 1.2 Let (X,d) be a metric space and $\{Sx_n\}_{n=1}^{\infty} \subset X$ be a given sequence . Then $\{Sy_n\}_{n=1}^{\infty} \in X$ is an approximate sequence of $\{Sx_n\}_{n=1}^{\infty}$ if, for any $k \in N$, there exist $\eta = \eta(k)$ such that $d(Sx_n , Sy_n) \leq \eta$ for all $n \geq k$

Definition 1.3 Let (X, d) be a complete metric space and S, T: $Y \to X$ for an arbitrary set Y such that $T(Y) \subseteq S(Y)$, and S(Y) or T(Y) is a complete subspace of X. Assume that z is a coincidence point of T and S, that is, $Sz = Tz = x^*$. Let $\{Sx_n\}$ be an iteration procedure defined by $x_0 \in X$ and $Sx_{n+1} = Tx_n$, for all $n \geq 0$.

Suppose that $\{Sx_n\}$ converges to the common fixed point of S and T, say p. If for any approximate sequence $\{Sy_n\} \subset X$ of $\{Sx_n\}$

$$\lim_{n\to\infty} d(Sy_{n+1}, f(T, Sy_n)) = 0$$

implies that

$$\lim_{n\to\infty} Sy_n = p.$$

Then the iteration procedure is weak w^2 stable with respect to S and T.

Definition1.4 Two sequences $\{Sx_n\}_{n=0}^{\infty}$ and $\{Sy_n\}_{n=0}^{\infty}$ are equivalent sequences if $d(Sx_n, Sy_n) \to 0$ as $n \to \infty$.

Any equivalent sequence is an approximate sequence but reverse is not true.

Definition1.5 Let (X, d) be a complete metric space and S, T: $Y \to X$ for an arbitrary set Y such that $T(Y) \subseteq S(Y)$, and S(Y) or T(Y) is a complete subspace of X. Assume that z is a coincidence point of T and S, that is, Sz = Tz = x*. Let $\{Sx_n\}$ be an iteration procedure defined by $x_0 \in X$ and

$$Sx_{n+1} = Tx_n, \text{ for all } n \geq 0.$$

Suppose that $\{Sx_n\}$ converges to the common fixed point of S and T, say p. If for any equivalent sequence $\{Sy_n\} \subset X$ of $\{Sx_n\}$

$$\lim_{n\to\infty} d(Sy_{n+1}, f(T, Sy_n)) = 0$$

implies that

$$\lim_{n\to\infty} Sy_n = p.$$

Then the iteration procedure is weak w^2 stable with respect to S and T.

2. Main Results

Theorem 2.1. Let (X, d) be a complete metric space and S, T: $Y \to X$ for an arbitrary set Y such that the following condition holds

$$d(Tx, Ty) < \max\{d(Sx, Tx), d(Sy, Ty)\}.$$

and $T(Y) \subseteq S(Y)$, and S(Y) or T(Y) is a complete subspace of X. Let z be a coincidence point of T and S, that is, Sz = Tz = x* (say). Let $\{Sx_n\}_{n=0}^{\infty}$ be an iterative procedure defined by $x_0 \in X$ and $Sx_{n+1} = Tx_n$, for all $n \geq 0$ and the sequence $\{Sx_n\}$ converges to x* , the unique fixed point of T .Then, the Jungck iteration is w^2 - stable.

Proof. Consider $\{Sy_n\}_{n=0}^{\infty}$ to be an equivalent sequence of $\{Sx_n\}$. Then, according to w^2 - stability of S and T, if $\lim_{n\to\infty} d(Sy_{n+1}, Ty_n) = 0$ implies that $\lim_{n\to\infty} Sy_n = x^*$, then the Jungck iteration is w^2 -stable

In order to prove this, we suppose that $\lim_{n\to\infty} d(Sy_{n+1}, Ty_n) = 0$ Therefore $\forall \varepsilon > 0$, $\exists\ n_o = n(\varepsilon)$ such that $d(Sy_{n+1}, Ty_n) < \varepsilon$. $\forall n \geq n_o$.

So,

$$d(Sy_{n+1}, x^*) \leq d(Sy_{n+1}, Sx_{n+1}) + d(Sx_{n+1}, x^*) \leq d(Sy_{n+1}, Ty_n) + d(Ty_n, Tx_n) + d(Sx_{n+1}, x^*)$$

$$< d(Sy_{n+1}, Ty_n) + \max\{d(Sx_n, Tx_n), d(Sy_n, Ty_n)\} + d(Sx_{n+1}, x^*).$$

For the hypothesis, from $Sx_n \to x^*$, we have that $d(Sx_n, Tx_n) = d(Sx_n, Sx_{n+1})$

$$\leq d(Sx_n, x^*) + d(x^*, Sx_{n+1}) \to 0.$$

If $\max\{d(Sx_n, Tx_n), d(Sy_n, Ty_n)\} = d(Sx_n, Tx_n)$ by taking to the limit, we obtain that $d(Sy_{n+1}, x^*) \to 0$.

If $\max\{d(Sx_n, Tx_n), d(Sy_n, Ty_n)\} = d(Sy_n, Ty_n)$, we have that

$$d(Sy_n, Ty_n) \leq d(Sy_n, Sx_n) + d(Sx_n, Sx_{n+1}) + d(Sx_{n+1}, Sy_{n+1}) + d(Sy_{n+1}, Ty_n).$$

From definition of equivalent sequences we have that $\left(Sx_n, Sy_n\right) \to 0$ and by taking to the limit, we obtain that $d\left(Sy_{n+1}, x^*\right) \to 0.$

This shows that the Jungck iteration is w^2 -stable with respect to S and T.

Theorem 2.2. Let (X, d) be a complete metric space and S, T: Y$\to$ X for an arbitrary set Y such that the following condition holds

$$d\left(Tx,Ty\right) < \max\left\{d\left(Sx,Tx\right), d\left(Sy,Ty\right), d\left(Sx,Sy\right)\right\}.$$

and T(Y) $\subseteq$S(Y), and S(Y) or T(Y) is a complete subspace of X. Let z be a coincidence point of T and S, that is, Sz = Tz = x* (say). Let $\left\{Sx_n\right\}_{n=0}^{\infty}$ an iterative procedure defined by $x_0 \in$ X and $Sx_{n+1} = Tx_n$, for all n $\geq$ 0 and the sequence $\left\{Sx_n\right\}$ converges to x* , the unique fixed point of T .Then, the Jungck iteration is w^2 - stable w.r.t. S and T.

Proof. We follow the same assumption as in Theorem 2.1 by taking $\left\{Sy_n\right\}_{n=0}^{\infty}$ to be an equivalent sequence of $\left\{Sx_n\right\}$.

According to w^2 - stability of S and T if $\lim_{n\to\infty} d\left(Sy_{n+1}, Ty_n\right) = 0$ implies that $\lim_{n\to\infty} Sy_n = x^*$ then the Jungck iteration is w^2 - stable.

In theorem 2.1, this result holds if we consider max $\left\{d\left(Sx,Tx\right), d\left(Sy,Ty\right)\right\}$. In this case, there is a new situation, when max could be $d\left(Sx,Sy\right)$. Therefore, following the same steps, we get that $\max\left\{d\left(Sx_n,Tx_n\right), d\left(Sy_n,Ty_n\right), d\left(Sx_n,Sy_n\right)\right\} = d\left(Sx_n,Sy_n\right)$. From Definition of equivalent sequences, we get that $d\left(Sx_n,Sy_n\right) \to 0$ and by taking to the limit as it is show in the above theorem, we obtain the required result.

Theorem 2.3. Let (X, d) be a complete metric space and S, T: Y$\to$ X for an arbitrary set Y such that the following condition holds $d\left(Tx,Ty\right) < \max\left\{d\left(Sx,Tx\right), d\left(Sy,Ty\right), d\left(Sx,Sy\right), d\left(Sx,Ty\right), d\left(Sy,Tx\right)\right\}$ and T(Y) $\subseteq$ S(Y), and S(Y) or T(Y) is a complete subspace of X. Let z be a coincidence point of T and S, that is, Sz = Tz = x* (say). Let $\left\{Sx_n\right\}_{n=0}^{\infty}$ an iterative procedure defined by $x_0 \in$ X and $Sx_{n+1} = Tx_n$, for all n $\geq$ 0 and the sequence $\left\{Sx_n\right\}$ converges to x* , the unique fixed point of T .Then, the Jungck iteration is w^2 - stable w.r.t. S and T.

Proof. We follow the same assumption as in Theorem 2.2 where is show this result if we consider $\max\left\{d\left(Sx,Tx\right), d\left(Sy,Ty\right), d\left(Sx,Sy\right)\right\}$ in this case, there are new situations.

When max could be $d\left(Sx,Ty\right)$ or $d\left(Sy,Tx\right)$. Again, we follow the same steps.

If $\max\left\{d\left(Sx_n,Tx_n\right), d\left(Sy_n,Ty_n\right), d\left(Sx_n,Sy_n\right), d\left(Sx_n,Ty_n\right), d\left(Sy_n,Tx_n\right)\right\} = d\left(Sx_n,Ty_n\right)$. We have that $d\left(Sx_n,Ty_n\right) \leq d\left(Sx_n,Sy_n\right) + d\left(Sy_n,Ty_n\right)$. From Definition of equivalent sequences, we get that $d\left(Sx_n,Sy_n\right) \to$ 0 and the expression of $d\left(Sy_n,Ty_n\right)$ was treated in Theorem 2.1.

On the other hand, if $\max = d\left(Sy_n,Tx_n\right)$, then $d\left(Sy_n,Tx_n\right) \leq d\left(Sy_n,Sx_n\right) + d\left(Sx_n,Tx_n\right)$.

By taking to the limit in a same way as in above theorems, we obtain the required result.

Theorem 2.4. Let (X, d) be a complete metric space and S, T: Y$\to$ X for an arbitrary set Y such that the following condition holds

$$d\left(Tx,Ty\right) < \max\left\{d\left(Sx,Tx\right), d\left(Sy,Ty\right), d\left(Sx,Sy\right), \frac{d\left(Sx,Ty\right)+d\left(Sy,Tx\right)}{2}\right\}$$

and T(Y) $\subseteq$ S(Y), and S(Y) or T(Y) is a complete subspace of X. Let z be a coincidence point of T and S, that is, Sz = Tz = x* (say). Let $\{Sx_n\}_{n=0}^{\infty}$ an iterative procedure defined by $x_0 \in$ X and $Sx_{n+1} = Tx_n$, for all n $\geq$ 0 and the sequence $\{Sx_n\}$ converges to x* , the unique fixed point of T .Then, the Jungck iteration is w^2 - stable w.r.t. S and T.

Proof. We follow the same assumptions as in Theorem 2.3 where this result has been shown if we consider $\max\{d(Sx,Tx)d(Sy,Ty)d(Sx,Ty),d(Sy,Tx)\}$. In this case there is a new situation, when max could be $\frac{d(Sx,Ty)+d(Sy,Tx)}{2}$.Then following the same steps as in Theorem 2.3. We obtain that $d(Sx_n,Ty_n)\to 0$ and $d(Sy_n,Tx_n)\to 0$ so, by taking to the limit in the whole expression, we get the required result.

Theorem 2.5. Let (X, d) be a complete metric space and S, T: Y$\to$ X for an arbitrary set Y such that the (δ, L) contraction holds, i.e., if there exists $\delta \in (0,1)$ and L $\geq$ 0 such that

$$d(Tx,Ty)\leq \delta(Sx,Sy)+Ld(Sx,Tx).$$

and T(Y) $\subseteq$ S(Y), and S(Y) or T(Y) is a complete subspace of X. Let z be a coincidence point of T and S, that is, Sz = Tz = x* (say). Let $\{Sx_n\}_{n=0}^{\infty}$ an iterative procedure defined by $x_0 \in$ X and $Sx_{n+1} = Tx_n$, for all n $\geq$ 0 and the sequence $\{Sx_n\}$ converges to x* , the unique fixed point of T .Then, the Jungck iteration is w^2 - stable.

Proof. Let us consider $\{Sy_n\}_{n=0}^{\infty}$ to be an equivalent sequence of $\{Sx_n\}$. Then, by using w^2 - stability of S and T , we get that if $\lim_{n\to\infty} \mathrm{d}(Sy_{n+1},Ty_n)=0$ implies that $\lim_{n\to\infty} Sy_n =$ x*, then the Jungck iteration is w^2 -stable.

For the assertion of our result, let us assume that $\lim_{n\to\infty} \mathrm{d}(Sy_{n+1,}Ty_n)$=0 Therefore $\forall\varepsilon>0$, $\exists n_o=n(\varepsilon)$ such that d $(Sy_{n+1,}Ty_n)<\varepsilon$. $\forall n\geq n_{o.}$

Consider,

$$d(Sy_{n+1},x^*)\leq d(Sy_{n+1,}Sx_{n+1})+d(Sx_{n+1},x^*)$$
$$\leq d(Sy_{n+1,}Ty_n)+d(Ty_{n,}Tx_n)+\mathrm{d}(Sx_{n+1},x^*)$$
$$< d(Sy_{n+1,}Ty_n)+\{\delta d(Sx_{n,}Sy_n)+Ld(Sx_{n,}Tx_n)\}+d(Sx_{n+1},x^*)$$

Using $Sx_n \to x^*$ from the hypothesis, we get that

$$\mathrm{d}(Sx_{n,}Tx_n)=\mathrm{d}(Sx_n,Sx_{n+1})\leq \mathrm{d}(Sx_n,x^*)+d(Sx^*,Sx_{n+1})\to 0.$$

If $[\delta d(Sx_n,Sy_n)+Ld(Sx_n,Tx_n)]=\mathrm{d}(Sx_n,Tx_n)$, then by taking to the limit, we obtain that $d(Sy_{n+1},x^*)\to 0$.

If$[\delta d(Sx_n,Sy_n)+Ld(Sx_n,Tx_n)]=d(Sx_n,Sy_n)$, then by using definition of equivalent sequences, we get that $d(Sx_n, Sy_n)\to 0$ and by taking to the limit, we obtain that $d(Sy_{n+1},x^*)\to 0$.

This proves the result that the Jungck iteration is w^2 -stable with respect to S and T.

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A COMPARATIVE STUDY OF EFFECTIVENESS REGARDING EXISTING & PROPOSED ALGORITHM IN DISK SCHEDULING POLICIES ON CHANGING THE HEAD POSITION AND DISTANCE BETWEEN THE TRACK REQUEST

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ABSTRACT :
Disk Management & its Efficiency is one of the important & challenging issues of operating system before the researchers as it is required in all technical and software development. Various Scheduling Policies with varying degree of efficiency have been explored by the researchers so far. In this paper, we have made an effort to design some new scheduling policies on the basis of disk arm movement. Disk scheduling algorithms are used to reduce the total seek time of any request. Hence, to service a request, a disk system requires that the head should be moved to the desired track then a wait for latency & finally the transform of data. It has been observed that selecting an optimal disk scheduling algorithm, performance depends upon the initial head Position (IHP) on keeping the same track Request. Minimum seek Time has been calculated for different Policies & the conclusion has been drawn through graphical representation. In a nutshell, on one hand we have examined the efficiency of various scheduling algorithms & on the other hand, we explored some new algorithms on the basis of Disk Arm Movement & then a conclusion has been drawn.
Keywords : ***Disk Scheduling, Seek Time, Head Movement, Rotational Delay, FCFS, SSTF, SCAN, C-SCAN, LOOK, C-LOOK, NEW HEURISTIC.***

1. INTRODUCTION

Since the invention of movable Hard Disk, the I/O performance of the operating system has been a subject of research for improvement either by implementing the suitable intelligent scheduling of disk access or by exploring the efficient scheduling algorithm. In fact the disk is one of the important computer resources for the disk drive & these Computer resources are scheduled first before use. Disk scheduling technique is a process of allocating services to the request in well designed manner. This technique reduces the effect of starvation of the request, which degrades the performance of process. The main objective of disk scheduling algorithm is to reduce the seek time & Rotational delay for the set of request.

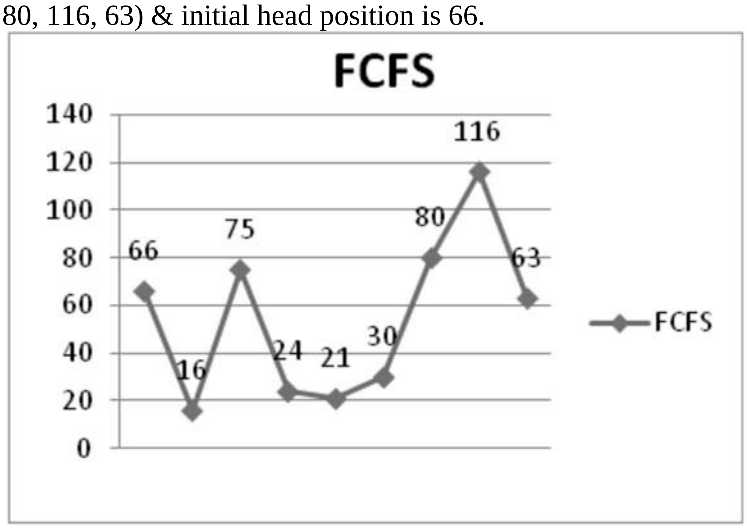
In Multiprogramming operating system, many processes may generate requests for reading & writing disk records. These processes sometimes make request faster as compared to the service provided by disk. In order to increase the performance, many researchers suggest enhancing the efficiency of the disk by reducing the average seek time. The performance of I/O operation can be improved. Disk scheduling needs careful examination & study of pending request to determine the most efficient way in order to service the request. A lot of Disk scheduling algorithms are available in the literature such as: -1) **FCFS 2) SSTF 3) SCAN 4) C-SCAN 5) LOOK 6) C-LOOK.** In this paper, we have explored some new algorithms. These new algorithms have been designed on the basis of changing in the head position. Finally we have compared the efficiency of existing scheduling policies with new other algorithms by keeping the same track request & on changing the head arm movement in each scheduling policies. We find that our results are more closer to optimality. The paper is organized as : **Section-1** contains the introductory part. **Section-2** contains Literature Survey.**Section-3** highlights the different Disk scheduling algorithms. **Section-4** proposes the new algorithms based on the change in Head position. In the last section concluding remarks as well as comparative study regarding the efficiency of scheduling policies have been discussed in detail.

2. RELATED WORK DONE

A lot of work has been done by various researchers to improve the disk performance. **Margo seltzer [1990]** discussed the scheduling techniques based on sectors transverse and rotational delay by movable Head system. **Jacobson [1995]** observed scheduling algorithm based on rotational position. **Javed & Khan [2000]** compared the performance of Four disk scheduling algorithms. **Suri & Mittal** explored shortest access time algorithm which is continuum between **FCFS & SSTF**. **C.Tsai et al., [2008]** served an efficient real time scheduling frame work. The work of **Muqaddas et al., [2009]** deserves credit in the sense that they made **S-Look** in a priority disk scheduling in offline & online environments. **Saha, Akhtar & Kasim[2013]** proposed a new disk scheduling algorithm to reduce the number of movement of Head, in which the authors have shown to maximize throughput for modern storage device. **Priya Hooda & Raheja [2014],** made an attempt to find a new approach on disk scheduling & consider the factor seek time & rotational delay both to schedule the disk. Recently **Sachin, Silky & T.P. Singh [2014]** measured the performance of **C-Look** algorithm in uncertain environments. We extend the work done by **Saha et.al [2013]** on changing the head position**.**

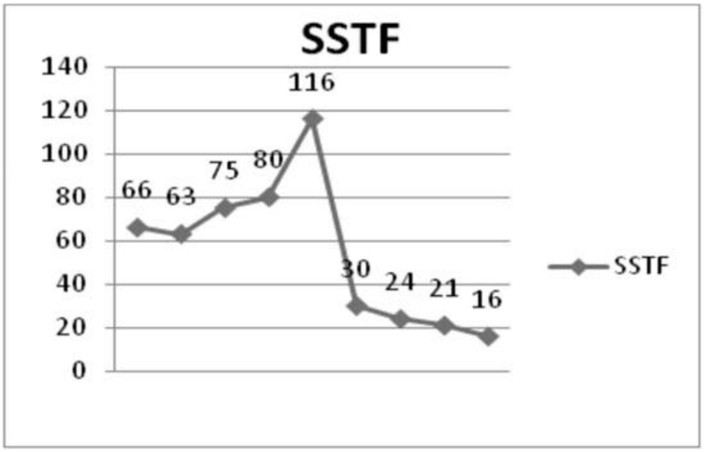
3. DIFFERENT DISK SCHEDULING ALGORITHMS

1. FCFS: - The simplest form of Disk scheduling is First come First served algorithm. But usually it does not provide the faster service as compared to others. In this case the track request which comes first will served first. Considering an example-we have taken the following track request for accessing the tracks as-(16, 75, 24, 21, 30, 80, 116, 63) & initial head position is 66.



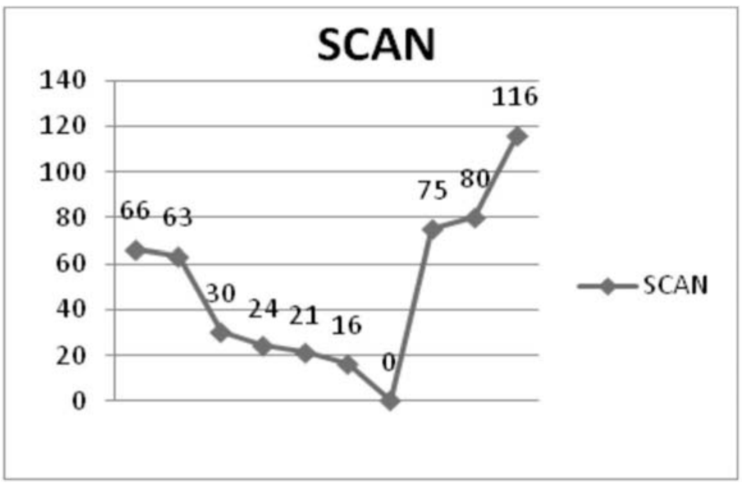
The Total head movement is 50+59+51+3+9+50+36+53=311

2. SSTF: - Shortest Seek Time First selects the request with the minimum seek time from the current head position. Since seek time increases with the number of cylinders traversed by the head. This scheduling process is given to those processes which need the shortest seek time even if these requests are not the first ones in the queue.



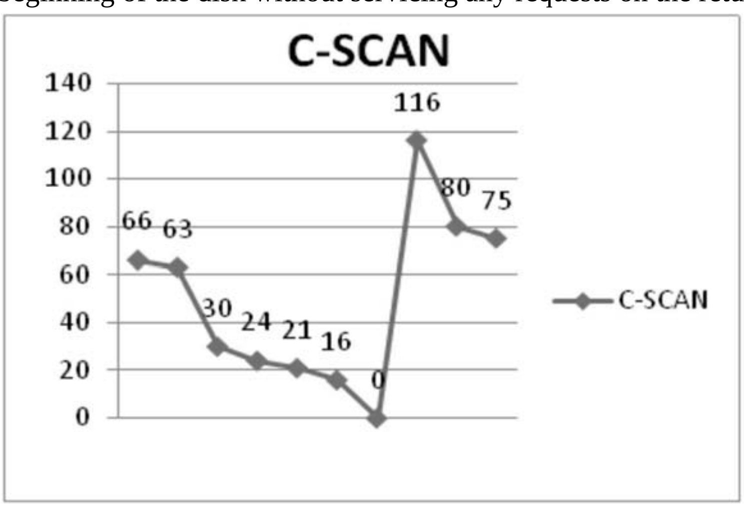
The Total head movement is
3+12+5+36+86+6+3+5=156

3. SCAN: - This algorithm is also called Elevator algorithm. In this the disk arm starts at one end of the disk & moves toward the other end, servicing requests until the head will move to the other end of the disk, where the head movement is reversed & the servicing continues. In this case, head moves to the starting point i.e (0).



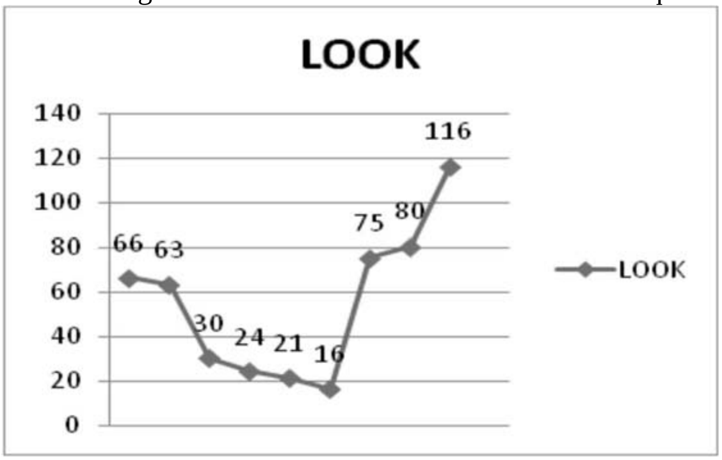
The Total head movement is
3+33+6+3+5+16+75+5+36=182

4. C-SCAN: -It refers to the circular scan. In this case the head moves from one end of the disk to the other servicing the requests in between. However, when it reaches to the other end, it immediately returns to the beginning of the disk without servicing any requests on the return trip.



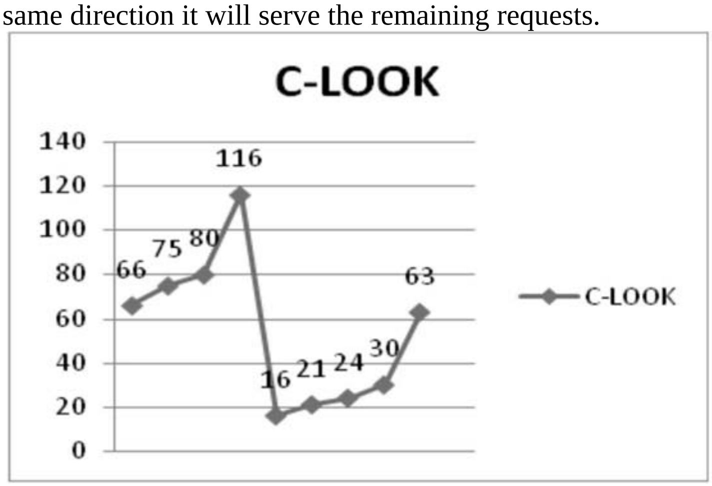
The Total head movement is
3+33+6+3+5+16+116+36+5=223

5. LOOK : -Look is similar to Scan. Look will change directions when it reaches the last request in the current direction. Again it moves to the other end till the last request in same direction.



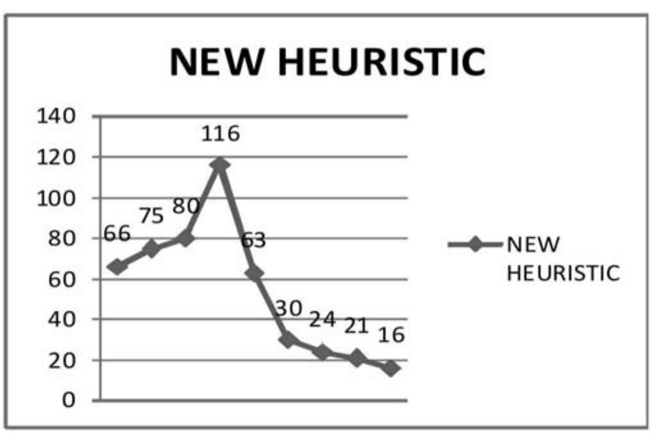
The Total head movement is
3+33+6+3+5+59+5+36=150

6. C-LOOK : -It refers to Circular Look. Head moves from one end of the disk & reaches at the last request again it moves to the other end without servicing any request on the return trip & after reaching the last request in same direction it will serve the remaining requests.



The Total head movement is
9+5+36+100+5+3+6+33=197

7. NEW HEURISTIC : - At first sort (in ascending order) of all cylinders as input Blocks by using any sorting method. Find the distance between the smallest block number and current head position and the distance between the largest block number and the current head position and then move to the side which has minimum distance.

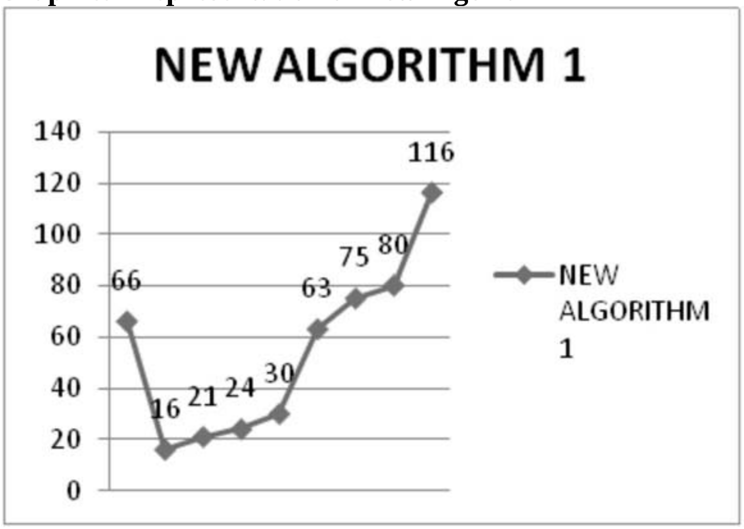


The Total head movement is
9+5+36+53+33+6+3+5=150

4. PROPOSED ALGORITHMS: -

8. NEW ALGORITHM 1: -It is similar to the Look policy. In this algorithm the head position moves to the starting position or first request which is in ascending order then moves to the other request one by one & provide services to all the requests.

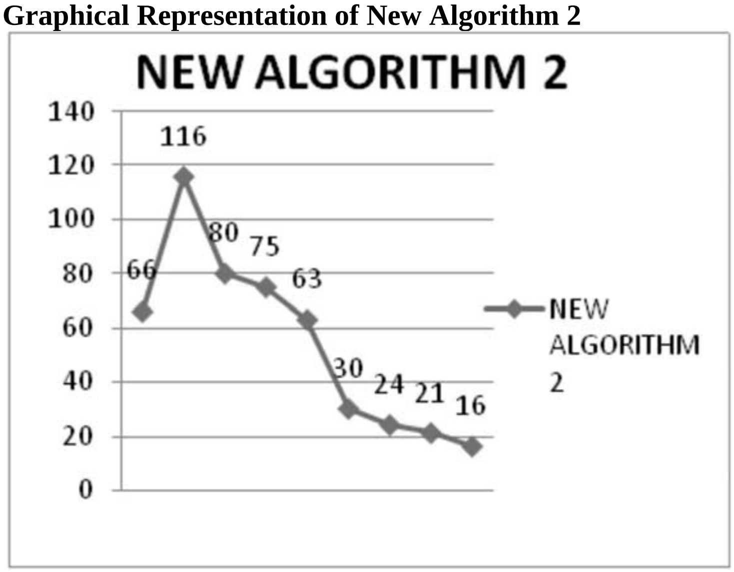
Graphical Representation of New Algorithm 1



The Total head movement is
50+5+3+6+33+12+5+36=150

9. NEW ALGORITHM 2: - In this algorithm, the head position moves to the ending position or ending request first which is in ascending order then moves to the other end and serve the request one by one.

Graphical Representation of New Algorithm 2



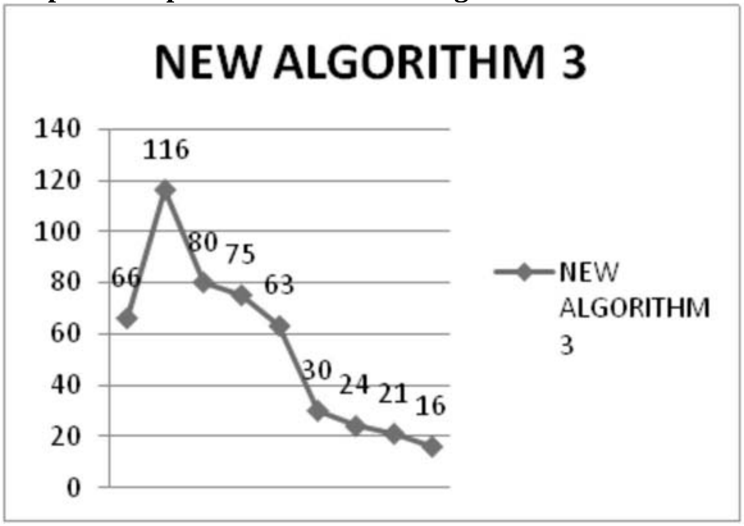
The Total head movement is
50+36+5+12+33+6+3+5=150

10. NEW ALGORITHM 3: -In this algorithm, we first sort all the request in ascending order then **1)** Find out the distance between head position and smallest track request & no. of request in same side. If both values are minimum comparative to other side then head moves to the starting position first then it moves to the other end.

2) If not, then find out the distance between head position and largest track request & no. of requests in same side. If both values are minimum in comparison to other side then head moves to the ending position first, then it moves to the other end.

3) In case both the conditions may false then head moves to the minimum distance first then to other side.

Graphical Representation of New Algorithm 3



The Total head movement is
50+36+5+12+33+6+3+5=150

ALGORITHM: 1) check whether Ready Queue is empty or not.

If empty

Then

Halt

Else

Go to step 2

2) Input the initial Head Position (IHP).

3) Sort all the Track Requests (TR) in ascending order using any sorting method.

4) **If (IHP-STR<IHP-ETR) &&(no. of request on L.H.S of IHP<no. of request on R.H.S of IHP)** Then Calculate Initial seek time (IST) & scan from the IHP to starting track request in descending order.

5) Else if (IHP-STR>IHP-ETR) && (no. of request on L.H.S of IHP>no. of request on R.H.S of IHP)

6) GOTO step 7

7) Calculate Initial seek time (IST) & scan to the IHP to Ending Track Request(ETR) in ascending order.

8) Else

9) Calculate the minimum distance from IHP & moves to that side.

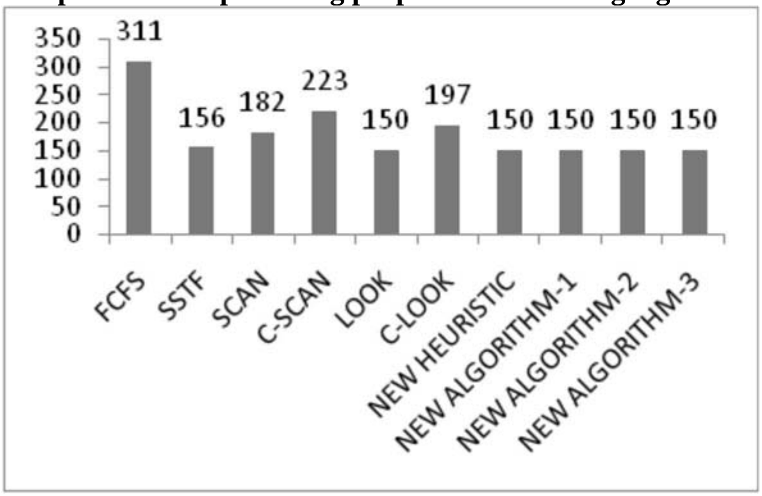
10) Total Seek time=

11) End

Comparative table among proposed and existing algorithms:

S.NO	NAME OF ALGORITHM	NUMBER OF HEAD MOVEMENT
1.	FCFS	311
2.	SSTF	156
3.	SCAN	182
4.	C-SCAN	223
5.	LOOK	150
6.	C-LOOK	197
7.	NEW HEURISTIC	150
8.	NEW ALGORITHM-1	150
9.	NEW ALGORITHM-2	150
10.	NEW ALGORITHM-3	150

Comparative Graph among proposed and existing algorithms:



5. Comparative Study of Disk Scheduling Technique by changing in Head position: we have considered many track request & after analysis we get different scheduling technique which gives the minimum seek time or rotational delay according to change in Head position. There are basically 3 situations where we get different respond from these algorithms.

1) Head at the starting position

2) Head at the middle position

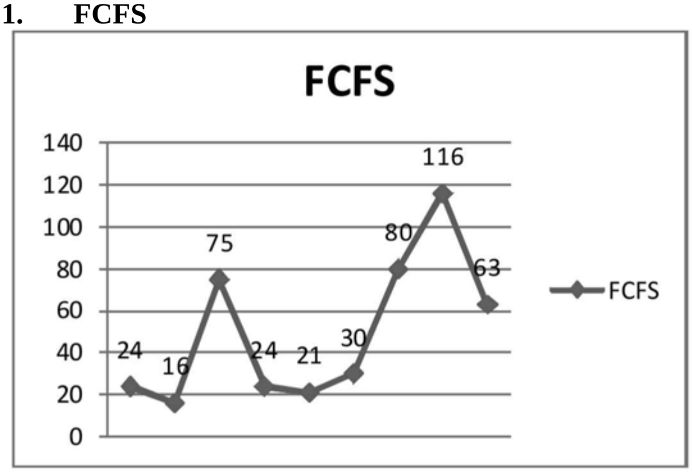
3) Head at the ending position

It has been observed that these scheduling policies depend on Head Position & the distance between the track requests.

1) Head at the starting position: In this situation the head position is at the starting of track request. We have taken the following track request for accessing the track as: -

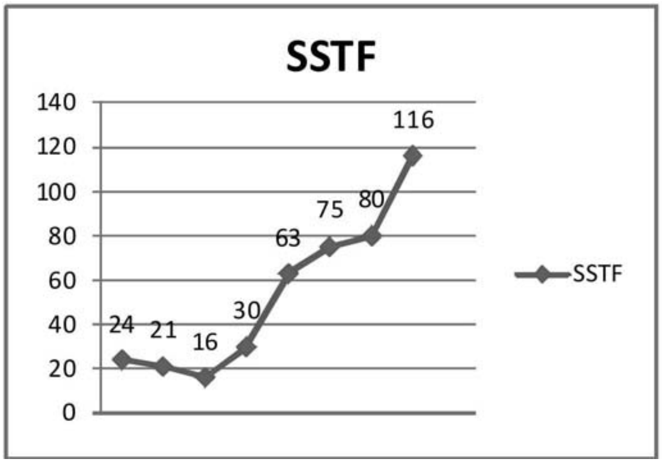
(16, 75, 24, 21, 30, 80, 116, 63) & head position is 24. Following results have been analyzed through simulation.

1. FCFS



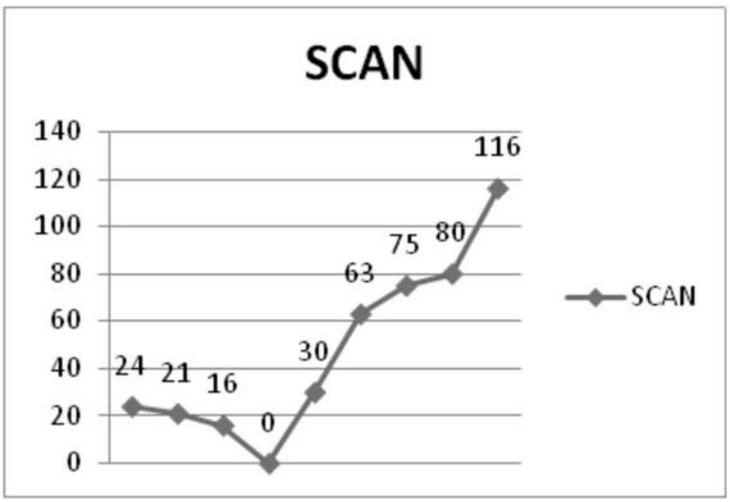
The Total head movement is
8+59+51+3+9+50+36+53=269

2. SSTF



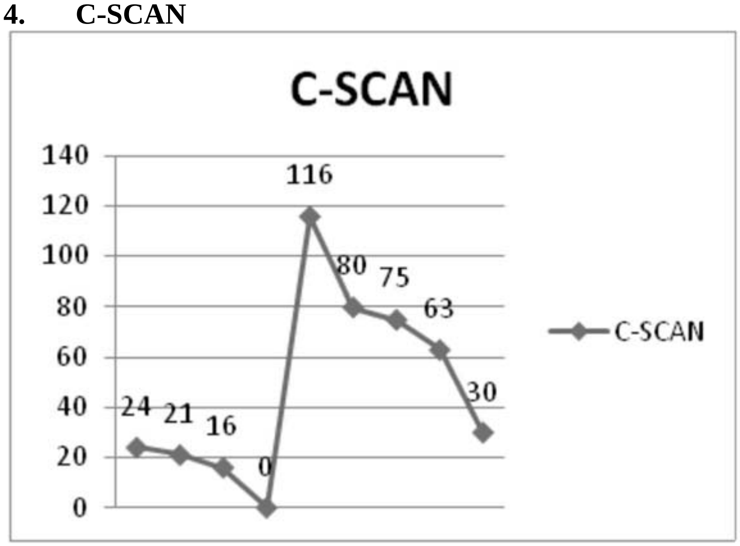
The Total head movement is
3+5+14+33+12+5+36=108

3. SCAN



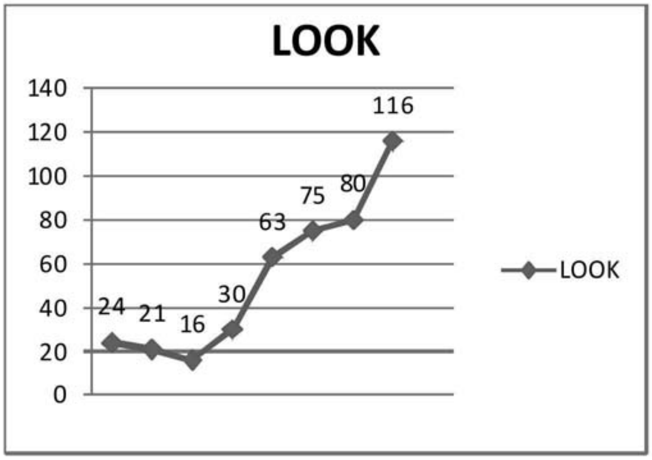
The Total head movement is
3+5+16+30+33+12+5+36=140

4. C-SCAN



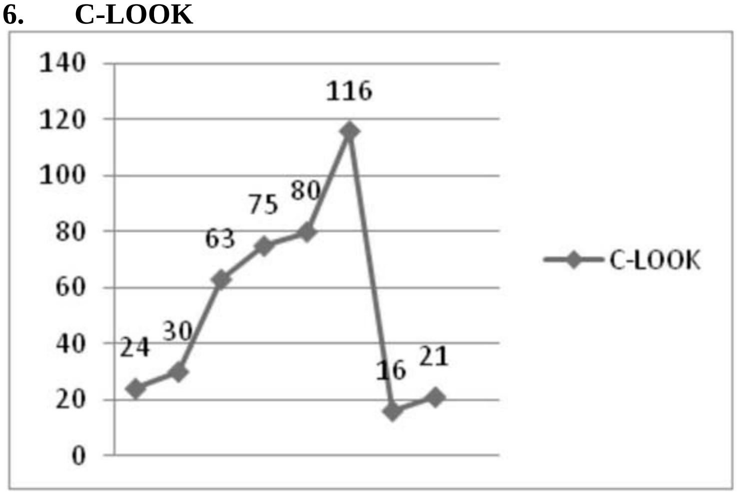
The Total head movement is
3+5+16+116+36+5+12+33=226

5. LOOK



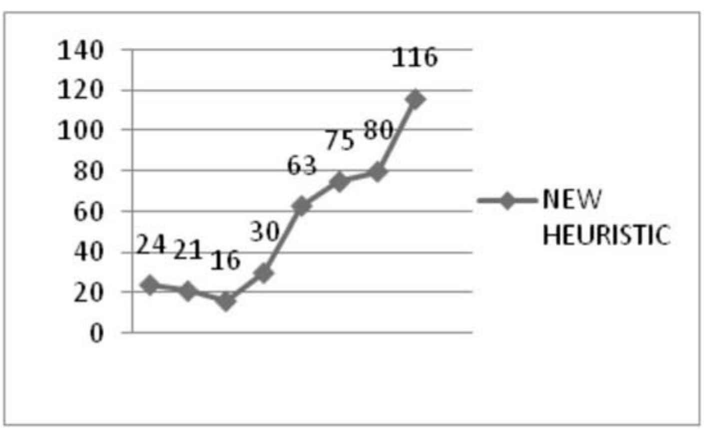
The Total head movement is
3+55+14+33+12+5+36=108

6. C-LOOK



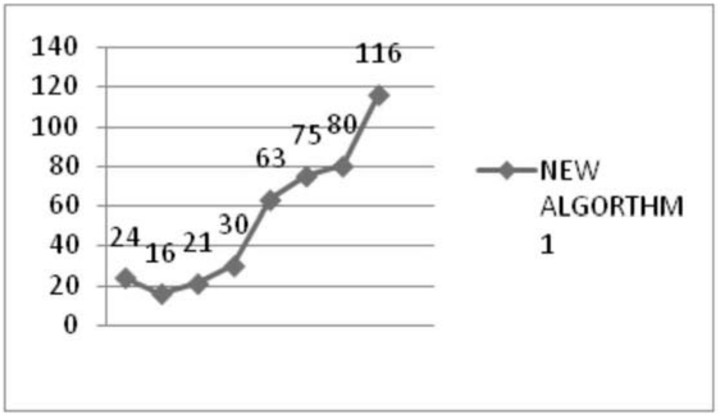
The Total head movement is
6+33+12+5+36+100+5=197

7. NEW HEURISTIC



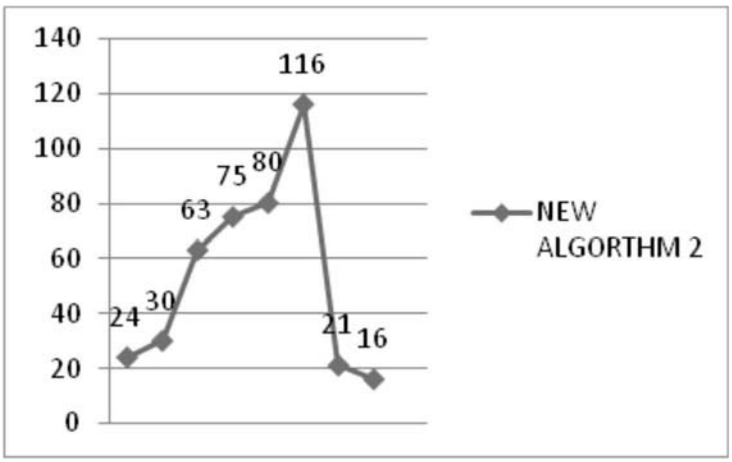
The Total head movement is
3+5+14+33+12+5+36=108

8. NEW ALGORITHM 1



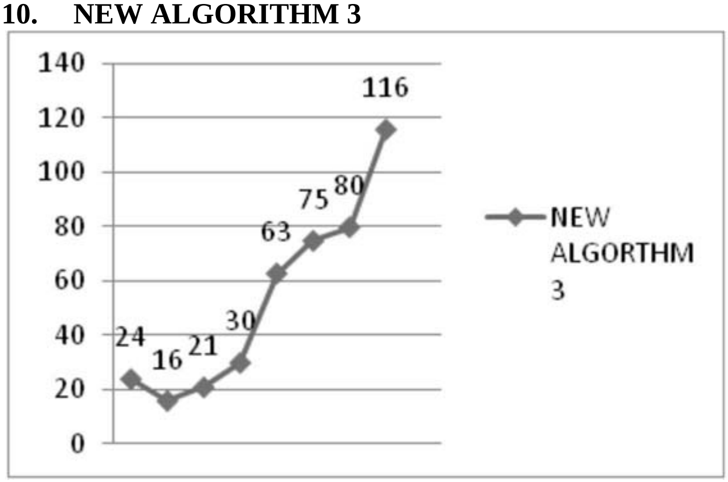
The Total head movement is
8+5+9+33+12+5+36=108

9. NEW ALGORITHM 2



The Total head movement is
6+33+12+5+36+95+5=192

10. NEW ALGORITHM 3

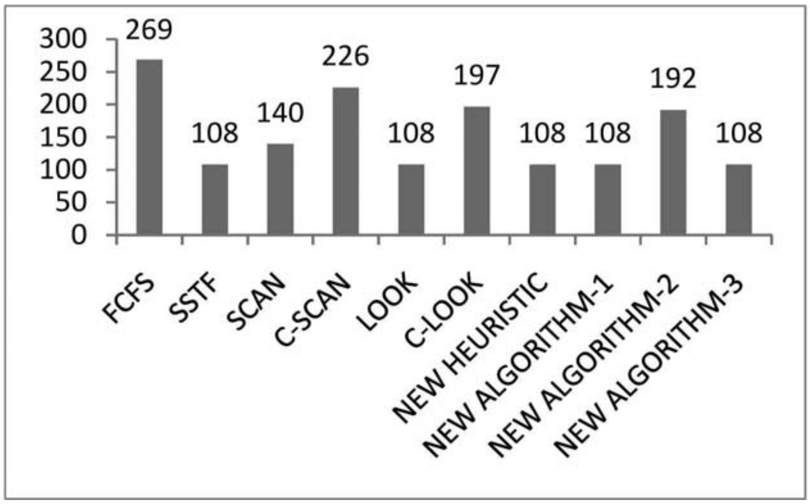


The Total head movement is
8+5+9+33+12+5+36=108

Comparative table among proposed and existing algorithms by changing its Head Position:-

S.NO	NAME OF ALGORITHM	NUMBER OF HEAD MOVEMENT
1.	FCFS	269
2.	SSTF	108
3.	SCAN	140
4.	C-SCAN	226
5.	LOOK	108
6.	C-LOOK	197
7.	NEW HEURISTIC	108
8.	NEW ALGORITHM-1	108
9.	NEW ALGORITHM-2	192
10.	NEW ALGORITHM-3	108

Comparative Graph among proposed and existing algorithms by changing its Head Position:-

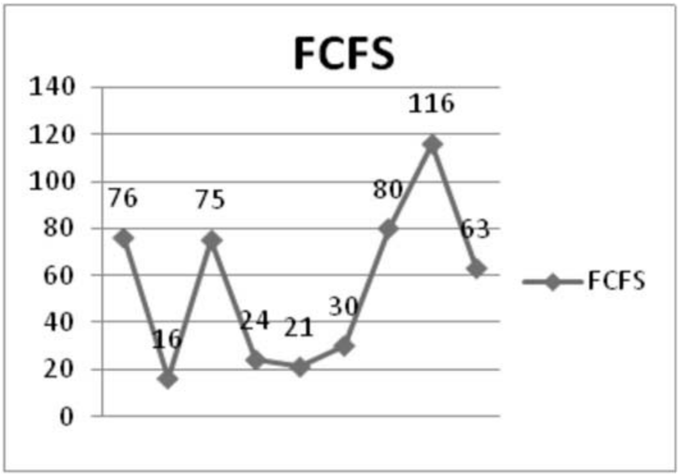


We observe that by changing head position in the same track request we get different results So different algorithm produces different results in several cases. So if the head is in starting position then **SSTF, LOOK, New Heuristic, Newalgorithm1, Algorithm-3** produce better result than other algorithms.

2) **Head at the middle position: -**In this situation the head position is in the middle of track request. As we discussed earlier in starting. In this case which algorithm gives better performance is: **-LOOK, New Heuristic, New Algorithm-1, New Algorithm-2, New Algorithm-3.**

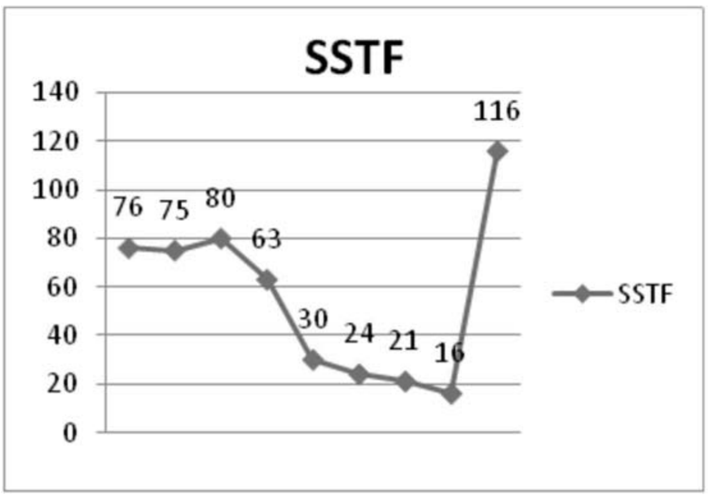
3) **Head at the ending position: -**In this situation the head position is in the ending of track request. We have taken the following track request for accessing the track as: **-** (16, 75, 24, 21, 30, 80, 116, 63) & head is 76.

1. **FCFS**



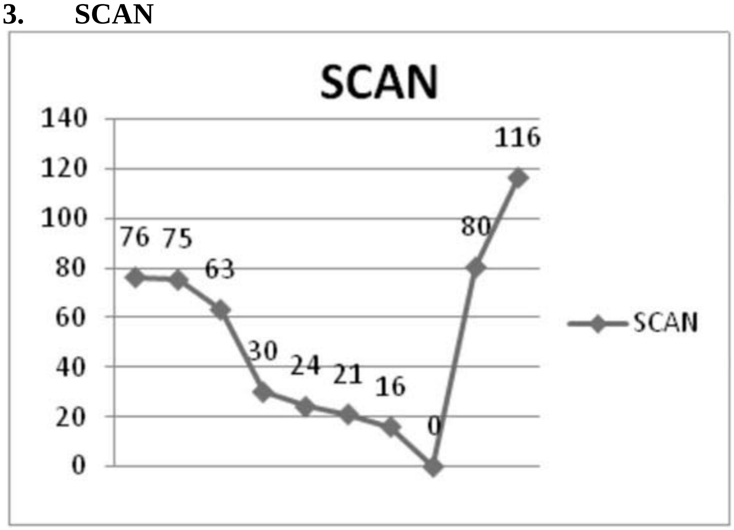
The Total head movement is
60+59+51+3+9+50+36+53=321

2. **SSTF**



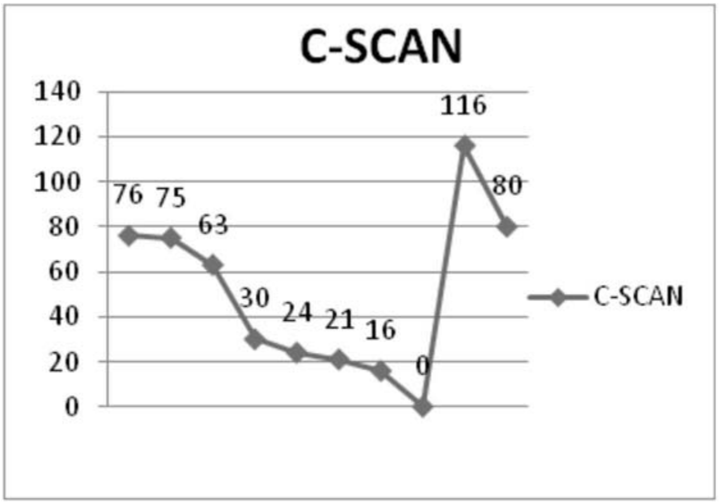
The Total head movement is
1+5+17+33+6+3+5+100=170

3. **SCAN**



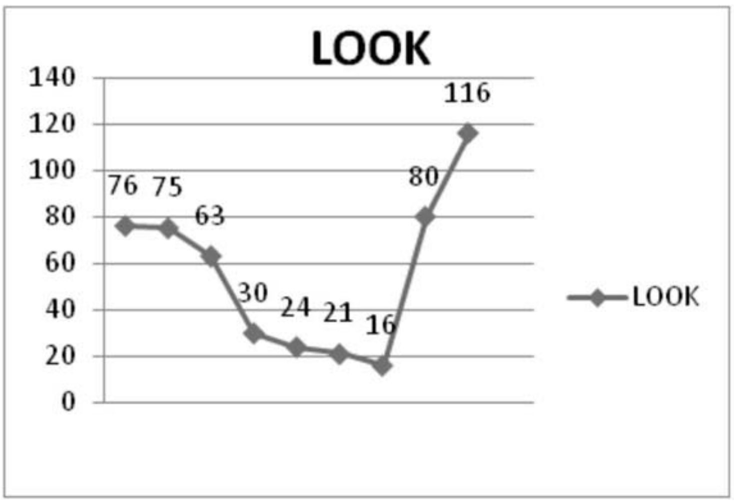
The Total head movement is
1+12+33+6+3+5+16+80+36=192

4. C-SCAN



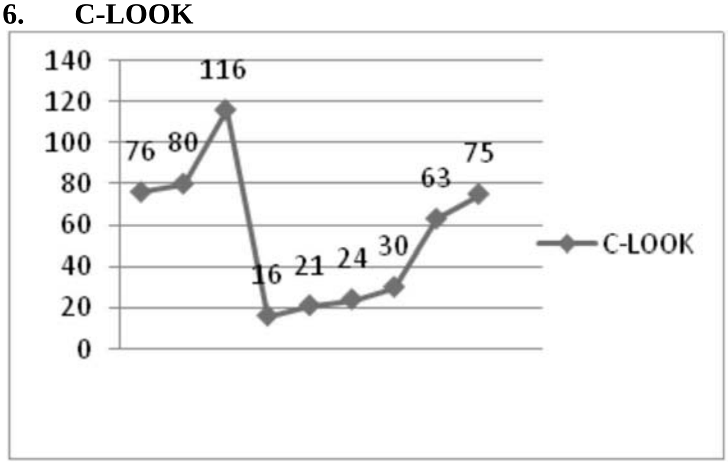
The Total head movement is
1+12 +33+6+3+5+16+116+36=228

5. LOOK



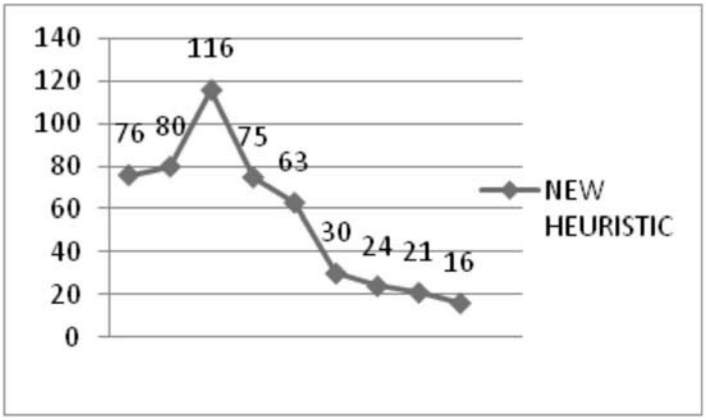
The Total head movement is
1+12+33+6+3+5+64+36=160

6. C-LOOK



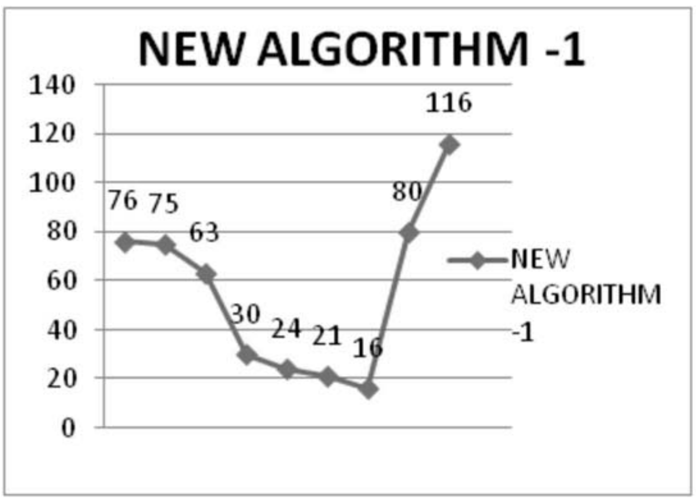
The Total head movement is
4+36+100+5+3+6+33+12=199

7. NEW HEURISTIC



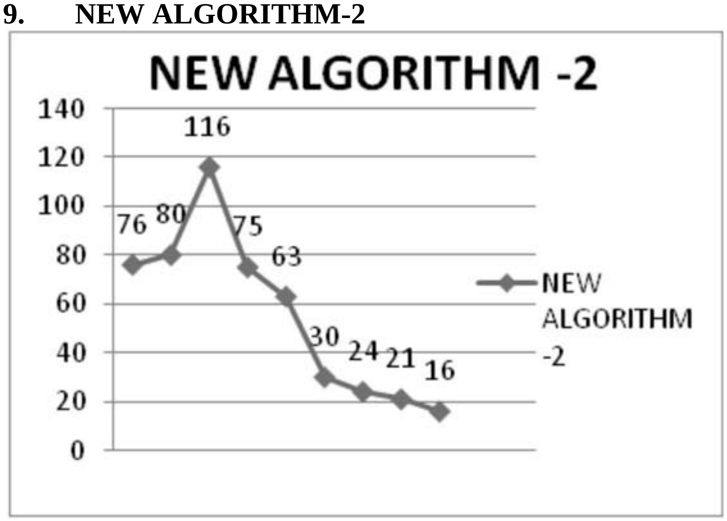
The Total head movement is
4+36+41+12+33+6+3+5=140

8. NEW ALGORITHM-1



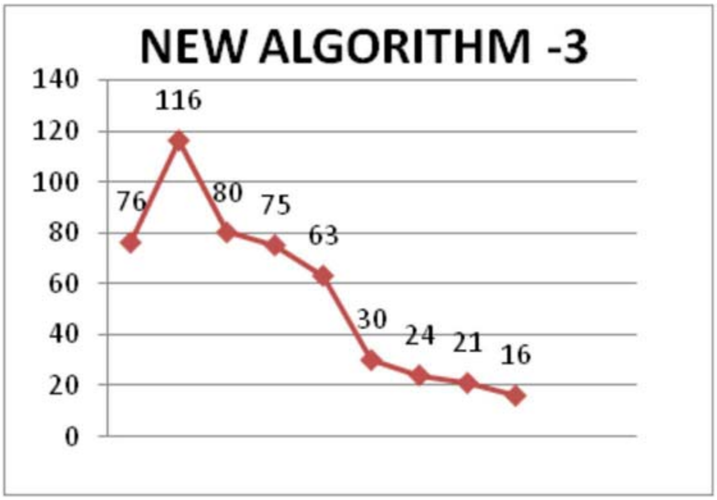
The Total head movement is
1+12+33+6+3+5+64+36=160

9. NEW ALGORITHM-2



The Total head movement is
4+36+41+12+33+6+3+5=140

10. NEW ALGORITHM-3

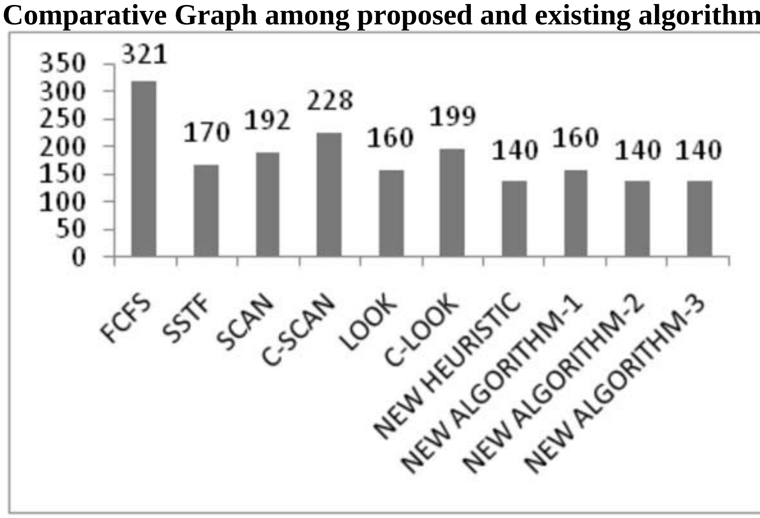


The Total head movement is
40+36+5+12+33+6+3+5=140

Comparative table among proposed and existing algorithms by changing its Head Position:-

S.NO	NAME OF ALGORITHM	NUMBER OF HEAD MOVEMENT
1.	FCFS	321
2.	SSTF	170
3.	SCAN	192
4.	C-SCAN	228
5.	LOOK	160
6.	C-LOOK	199
7.	NEW HEURISTIC	140
8.	NEW ALGORITHM-1	160
9.	NEW ALGORITHM-2	140
10.	NEW ALGORITHM-3	140

Comparative Graph among proposed and existing algorithms by changing its Head Position:-



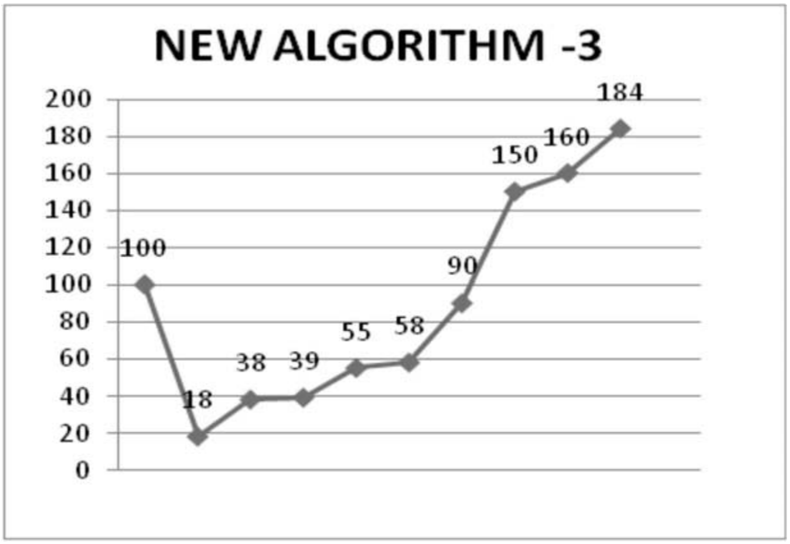
In this situation the head position is in the ending of track request. So far by changing the position of head which algorithm gives better performance is: **- New Heuristic, New Algorithm-2, New Algorithm-3.**

At last we can say that sometimes number of head movement is equal to SSTF, LOOK scheduling. But NEW ALGORITHM-3 is applicable to all situations. So we can easily apply this algorithm to real time system.

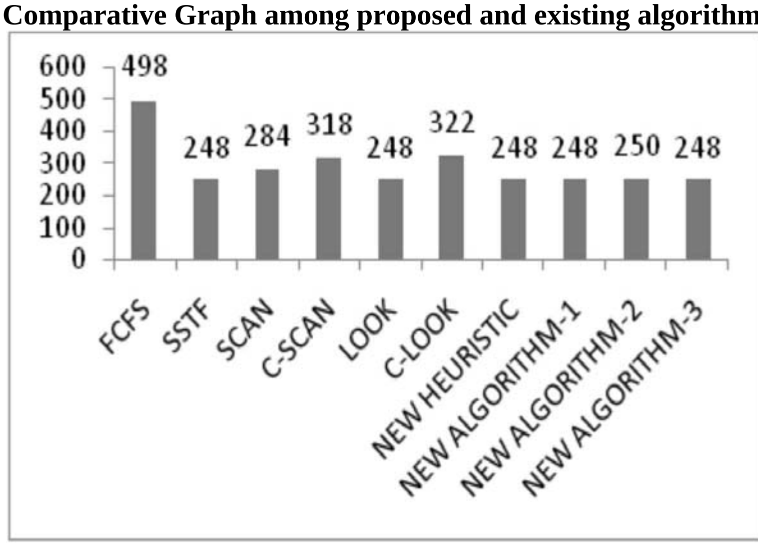
Comparative Study of Disk Scheduling Technique with the distance between Track request : - In Disk scheduling techniques we observed that by changing in head position we get different responses but one another factor in this is Distance between the Track Request. sometimes the track requests are near to head position but we are not able to get minimum seek time due to the distance between them. In some cases the minimum no. of requests are in before the head position but there distance is too large so head will have to move on another side. so here we apply **New Algorithm-3** to overcome this problem. Let us consider it with an example as below: -

Example 1: - (55, 58, 39, 18, 90, 160, 150, 38, 184) & head is at 100 position.

1) First we have to arrange this in ascending order so we get :- (18,38,39,55,58,90,150,160,184)
2) So here we find that head position is 100. There are 6 Requests before the 100 & the distance is 82. And 3 Requests are after the 100 & the distance is 84.
3) So to solve this kind of problem we apply **New Algorithm-3.**After applying this we get minimum seek time in any situation.



Comparative Graph among proposed and existing algorithms with the Distance between the track request: -

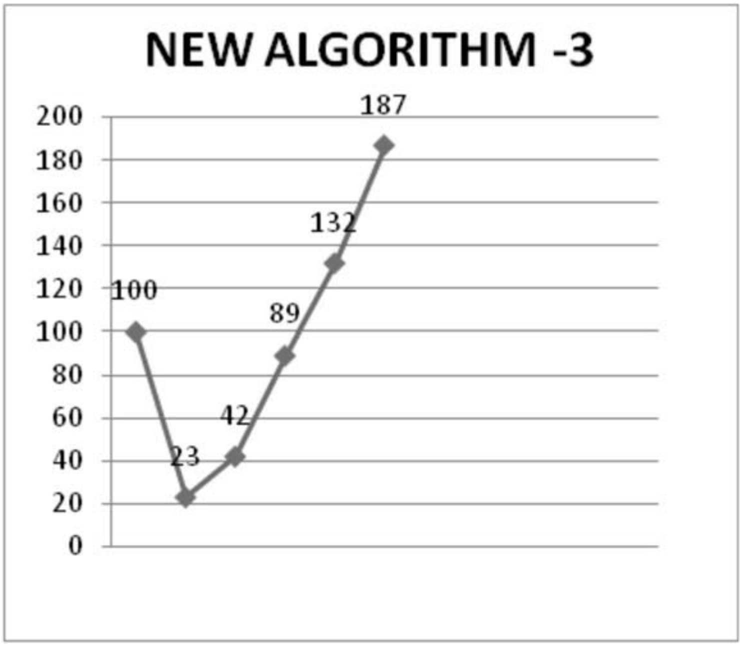


Example 2: - (23, 89, 132, 42, 187) & the head position is 100.

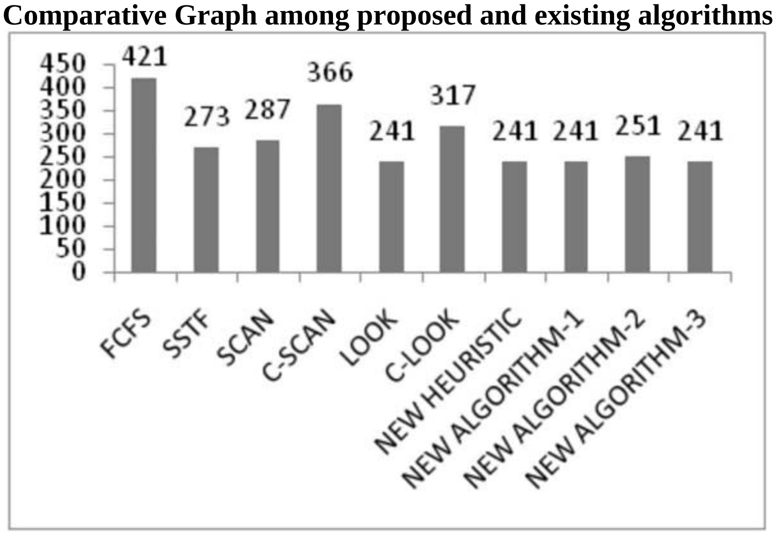
1) First we have to arrange this in ascending order so we get :- (23,42,89,132,187)

2) We found that head position is 100. There are 3 Requests before the 100 & the distance is 77. And 2 Requests are after the 100 & the distance is 87.

3) Again we applied **NEW ALGORITHM-3.**After applying this we get minimum seek time in any situation.



Comparative Graph among proposed and existing algorithms with the Distance between the track request:-



6. Conclusion

Our proposed **Algorithm-3** shows better performance than other existing disk scheduling algorithms (FIFO,SSTF,SCAN,C-SCAN and LOOK,C-LOOK).The seek time has been improved by this algorithm which increases the efficiency of the disk performance. In future we can implement **New Algorithm-3** in real time systems. In this paper we have compared all the algorithms by changing their head position. It shows the effective responses with manipulation of distance in track requests. So we can easily prove that algorithm 3 is applicable in any case either the position of head or the distance between track requests.

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UPPER BOUNDS ON THE PROBABILITY OF ERROR AND WEIGHTED MEANS DIVERGENCE MEASURES

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ABSTRACT :
In the present communication, we have considered some weighted means such as weighted arithmetic, weighted harmonic, weighted geometric, weighted root- square means and mixed weighted means. The difference of these means has been considered as the divergence and the resulting divergence is convex function. Some upper bounds for the probability of error in terms of weighted means divergence measures have been proved exploring weighted f-divergence.
Keywords: ***probability of error, weighted Arithmetic mean, weighted Geometric mean, weighted Harmonic mean, weighted root-square mean and weighted mixed mean, weighted f – Divergence AMS classification : 26D15.***

Introduction:-

It is a well known fact that means-Arithmetic, Geometric, Harmonic, Root-Square, Heron's mean and Logarithmic means have wide application in Mathematical sciences, Statistical analysis, Medical Sciences, Budget Analysis, Planning, Environmental sciences, and many others. Kapur and Sharma [7] have applied - Arithmetic, Geometric, and Harmonic means in information measures for increasing and decreasing probability distributions and characterized the newly monotonic measures of information M-entropy, M-inaccuracy ,and M-divergence measures and their concavity/convexity.

Generalizations of these means have created some interest among Liu, H. Meng, X-J [10], who introduced contra harmonic mean as Seifferts mean established different inequalities. Logarithmic mean which can be expressed in terms of Gauss's hyper geometric function 2 $F_{1,}$ has many applications. For example, a variant of Johnson's functional equation, involving logarithmic mean appears in heat conduction problems.

Herorian and Seiffert means have applications in geometry, topology, ordinary differential equations and fuzzy sets. For example, Runga-Kutta methods are based on Herorian mean. A lot of work is being done by Zhi-hua Zhang and Yu-Dong Wu [14], H.N. Shi, J. Zhang and Da Mao [4] and Huan Nan Shi and Zua Zhang [5] Ladishlav mate Jicka [9] have studied many types of means and characterized then and established bounds for them and concavity and convexity of those means.

Taneja [11] considered the difference of means due to the fact that divergence measures viz. Kullback-Leibler [8] divergence being the difference of inaccuracy measure and Shannon entropy, this divergence has been applied to many areas. R.P. Singh etal [15,16] established inequalities & concavity of the divergence. Further, R.P. Singh & S. Nagar [17] classified classical divergence measures into logarithmic & non symmetric categories.

Recently Javier E. Contravas-Reyes and Rainaldo B.Arellano-Valle [6] have applied Kullback-Leibler's [8] divergence measure for multivariate skew normal distributions to study seismic catalogue of Service seismologic national of Chile containing 6,714 after shocks on a map [32 – 40^{o} S] x [69 – 75.5^{o} E] for a period between 27 February 2010 to July 2011.

Also Zhi – Hua - Zhang and Yu - Dong Wu [14] have established some new bounds for Logarithmic means viz.

$$L(a,b)=\begin{cases}\dfrac{a-b}{\log a-\log b}, & a\neq b\\ a, & a=b\end{cases} \tag{1.1}$$

And the bounds are

$$G\leq L\leq M_{1/3}\leq M_{1/2}\leq H_1\leq M_{2/3}\leq A \tag{1.2}$$

Stated by J.Ch. Kuang [12]

$$\left.\begin{aligned}A&=A(a,b)=\frac{a+b}{2}\\ G&=G(a,b)=\sqrt{ab}\\ H_1&=H_1(a,b)=\frac{a+\sqrt{ab}+b}{3}\end{aligned}\right\} \tag{1.3}$$

and the generalized power type Herorian mean studied by G.Jia and J.D.Cao [3]

$$\left.\begin{aligned}H_r(a,b)&=\left\{\frac{a^r+(ab)^{r/2}+b^r}{3}\right\}^{1/r}, \quad r\neq 0\\ &=\sqrt{ab} \quad ,r=0\end{aligned}\right\} \tag{1.4}$$

and t-order power mean defined by

$$\left.\begin{aligned}M_t(a,b)&=\left(\frac{a^t+b^t}{3}\right)^{1/t}, \quad t\neq 0\\ &=\sqrt{ab} \quad , \quad t=0\end{aligned}\right\} \tag{1.5}$$

the inequality studied by T.P. Lin & J.Ch.Kuang [12] is given by

$G\leq L\leq M_{1/2}$. (1.6)

Another generalization of Heronian Mean is given by Zh. G. Xiao and Zh. H. Zhang [14] as follows:

$$H(a,b,k)=\frac{1}{k+1}\sum_{i=1}^{k}a^{\frac{k-i}{k}}b^{\frac{1}{k}}\text{................}(1.7)$$

and

$$h(a,b)=\frac{1}{k}\sum_{i=1}^{k}a^{\frac{k+1-i}{k+1}}b^{\frac{1}{k+1}}\text{.....................}(1.8)$$

As mentioned above, there are many generalization of means due to their applications.

Recently Taneja [11] who generalized means as mean of order t, t ≠ 0 as follows:

$$M_t(a,b) = \begin{cases} \left(\dfrac{a^t + b^t}{2}\right)^{\frac{1}{t}}, & t \neq 0 \\ \sqrt{ab}, & t = 0 \\ \max(ab), & t = \infty \\ \min(ab), & t = -\infty \end{cases} \tag{1.9}$$

$\forall a,b, \in R, a,b > 0$ and some particular cases

$$\left.\begin{array}{llll} t = -1, & M_{-1}(a,b) = & H(a,b) = & \dfrac{2ab}{a+b} \\ t = 0, & M_0(a,b) = & G(a,b) = & \sqrt{ab} \\ t = \dfrac{1}{2}, & M_{\frac{1}{2}}(a,b) = & N_1(a,b) = & \left(\dfrac{\sqrt{a}+\sqrt{b}}{2}\right)^2 \\ t = 1, & M_1(a,b) = & A(a,b) = & \dfrac{a+b}{2} \\ t = 2, & M_2(a,b) = & G(a,b) = & \left(\dfrac{a^2+b^2}{2}\right)^{\frac{1}{2}} \end{array}\right\} \tag{1.10}$$

also define some mixed means, such as

$$N_1(a,b) = \left(\frac{\sqrt{a}+\sqrt{b}}{2}\right)^2 \tag{1.11}$$

$$N_2(a,b) = \left(\frac{\sqrt{a}+\sqrt{b}}{2}\right)\left(\sqrt{\frac{a+b}{2}}\right) = \quad N_1(a,b) \quad A(a,b) \tag{1.12}$$

$$N_3(a,b) = \left(\frac{a+\sqrt{ab}+b}{3}\right)\left(\sqrt{\frac{a+b}{2}}\right) = \frac{2A(a,b)+G(a,b)}{3} \tag{1.13}$$

and $$S(a,b) = \sqrt{\frac{a^2+b^2}{2}} \tag{1.14}$$

Taneja [11] considered the difference of means as a divergence measure and exploited Csiszar I [1] f-divergence for probability distribution and has studied their convexities.

2. WEIGHTED f-DIVERGENCE AND PROBABILITY OF ERROR

The divergence between two probability density functions defined by Csiszar [1] is given by

$$C_f(p,q) = \int_x f\left(\frac{p(x)}{q(x)}\right) q(x)dx \tag{2.1}$$

Here the function f(x), for $x \in (0,\infty)$ is convex function satisfying the condition

$$f(0)=\lim_{x\to o} f(x); 0f\begin{pmatrix}0\\0\end{pmatrix}=0$$

$$0f\begin{pmatrix}a\\0\end{pmatrix}=\lim_{t\to\infty} f\begin{pmatrix}a\\t\end{pmatrix} \tag{2.2}$$

$$=a\lim_{x\to\infty}\begin{pmatrix}f(x)\\x\end{pmatrix}$$

It is obvious that $C_f(p,q)\geq f(1) and C_f(p,q)=f(1)$, where p(x)=q(x) . Thus $C_f(p,q)-f(1)$ is a divergence measure is the sense $C_f(p,q)-f(1)\geq 0$. Boekee and Van der Lubbe [2] have introduced the average f-divergence between two hypothesis $C_1$and C_2 in terms of their "a posterior'' probabilities. This average f–divergenceis defined as

$$C_f(c_1,c_2)=\int_x f\begin{pmatrix}p(c_1/x)\\q(c_2/x)\end{pmatrix}p(c_1/x)\,p(y)dx$$

$$=E_x\left\{f\begin{pmatrix}p(c_1/x)\\p(c_2/x)\end{pmatrix}p(c_1/x)\right\} \tag{2.3}$$

2.1 EXTENSION TO WEIGHTED f – DIVERGENCE:

In the recent years a lot of work has been done by various researchers towards the weighted entropy, weighted inaccuracy and weighted divergence measures. The weighted divergence was defined as follows:

$$D(P\,||\,Q;W)=\sum_{i=1}^{n} w_i p_i \log\frac{p_i}{q_i} \tag{2.4}$$

by considering the following information scheme:

$$I.S.=\begin{pmatrix}p_1,\ p_2,.....................p_n\\ q_1,\ q_2,.......................q_n\\ w_1,w_2,.......................w_n\end{pmatrix} \tag{2.5}$$

In the direction of weighted divergence a lot of work has been already done by many authors. But here our main objective is to introduce weight f- divergence corresponding to Csiszar's [1] –f divergence in the following way:

$$C_f(P,Q:W)=\int_x w\begin{pmatrix}p(x)\\q(x)\end{pmatrix}q(x)dx \tag{2.6}$$

Let us introduce the function

$$f^*(u,w)=wuf\left(\frac{1-u}{u}\right)\quad . \tag{2.7}$$

Setting u = u(x) = P (C_2/x), it is easy to see from P (C_1/x) =1-P (C_2 /x) that

$$C_f(C_1,C_2;W)=\int_X f^*\begin{pmatrix}P(C_1/x;w)\\P(C_2/x;w)\end{pmatrix}P(C_1/x)\,p(x)dx$$

$$C_f(C_1,C_2;W)=E_X\left\{f\begin{pmatrix}p(c_1/x\\p(c_2/x\end{pmatrix}p(C_1/x)\right\} \tag{2.8}$$

From Vajda [13], it follows that **f*** is convex in [0, 1] and is strictly convex iff f(u) is strictly convex. Taneja [11] has proved that

$$Pe\le\frac{1}{2f\infty-f_1}\left[f\infty-\overline{C_f}(C_1,C_2)\right] \tag{2.9}$$

provided f_∞ is finite,

where P_e is the probability of error. and f_1= f(1) =0, is finite, then (2.9) becomes

$$P_e\le\frac{1}{2}\left[1-\frac{1}{f_\infty}C_f(C_1,C_2)\right]. \tag{2.10}$$

3. WEIGHTED MEAN DIVERGENCES

We consider here the following weighted mean divergences:

$$M_{SA}(a,b;w)=S(a,b;w)-A(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.1)$$

$$M_{SG}(a,b;w)=S(a,b;w)-G(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.2)$$

$$M_{SH}(a,b;w)=S(a,b;w)-H(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.3)$$

$$M_{AG}(a,b;w)=A(a,b;w)-G(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.4)$$

$$M_{AN}(a,b;w)=A(a,b;w)-N(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.5)$$

$$M_{AN_2}(a,b;w)=A(a,b;w)-N_2(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.6)$$

$$M_{SN_1}(a,b;w)=S(a,b;w)-N_1(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.7)$$

$$M_{SN_2}(a,b;w)=S(a,b;w)-N_2(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.8)$$

$$M_{SN_3}(a,b;w)=S(a,b;w)-N_3(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

$$M_{N_2N_1}(a,b;w)=N_2(a,b;w)-N_1(a,b;w) \quad \ldots\ldots\ldots\ldots \quad (3.10)$$

- **Functional Forms of means without weight are given as follows:**

$$M_{SA}(a,b)=S(a,b)-A(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\frac{x+1}{2}\;=f_{SA}(x)\qquad\ldots\ldots\ldots\ldots\quad(3.11)$$

$$M_{SG}(a,b)=S(a,b)-G(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\sqrt{x}\;=f_{SG}(x)\qquad\ldots\ldots\ldots\ldots\quad(3.12)$$

$$M_{SH}(a,b)=S(a,b)-H(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\frac{2x}{x+1}\;=f_{SH}(x)\qquad\ldots\ldots\ldots\quad(3.13)$$

$$M_{AG}(a,b)=A(a,b)-G(a,b)\quad=\frac{x+1}{2}-\sqrt{x}\;=f_{AG}(x)\qquad\ldots\ldots\ldots\ldots\ldots\quad(3.14)$$

$$M_{AN}(a,b)=A(a,b)-N(a,b)\quad=\frac{x+1}{2}-\frac{2x}{x+1}\;=f_{AN}(x)\qquad\ldots\ldots\ldots\ldots\quad(3.15)$$

$$M_{AN_2}(a,b)=A(a,b)-N_2(a,b)\quad=\frac{x+1}{2}-\left(\frac{\sqrt{x}+1}{2}\right)\left(\sqrt{\frac{x+1}{2}}\right)\;=f_{AN_2}(x)\qquad(3.16)$$

$$M_{SN_1}(a,b)=S(a,b)-N_1(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\left(\frac{\sqrt{x}+1}{2}\right)^2\;=f_{SN_1}(x)\qquad\ldots\ldots\quad(3.17)$$

$$M_{SN_2}(a,b)=S(a,b)-N_2(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\frac{\sqrt{x+1}}{2}\;=f_{SN_2}(x)\qquad\ldots\ldots\quad(3.18)$$

$$M_{SN_3}(a,b)=S(a,b)-N_3(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\frac{x+1}{2}\;=f_{SN_3}(x)\qquad\ldots\ldots\ldots\quad(3.19)$$

$$M_{N_2N_1}(a,b)=N_2(a,b)-N_1(a,b)\quad=\sqrt{\frac{x^2+1}{2}}-\frac{x+1}{2}\;=f_{N_2N_1}\qquad\ldots\ldots\ldots\quad(3.20)$$

4. BOUNDS ON PROBABILITY OF ERROR IN TERMSOF WEIGHTED MEAN DIVERGENCE

In this section, we shall give bounds on the probability of error in terms of weighted means divergences based on the results (2.9) and (2.10).

Theorem 1: Let us consider the measure

$$M_{SA}(C_1,C_2;W)=E_X\left\{f_{SA}^*\left(\frac{p(C_1/x;W)}{p(C_2/x;W)}\right)p(C_1/x)\right\}\tag{4.1}$$

where $\qquad f_{SA}^*(x;w)=wx\quad f_{SA}\begin{pmatrix}1-x\\x\end{pmatrix}$

we have $\qquad f_{SA}^*(x)=\sqrt{\frac{x^2+1}{2}}-\frac{x+1}{2},\qquad\forall x\in(x,\infty)\qquad(4.2)$

$$f_{SA}^{*}(x;w)=\frac{w}{2}\left[\sqrt{2\{x^2+(1-x^2)\}}-1\right]= f_{SA}^{*}(1-x;w)$$

now (4.4)

$$P_e \le \frac{1}{2}\left[1-\left(\frac{2}{\sqrt{2}-1}\right)M_{SG}(C_1,C_2;W)\right]$$

$$f_{SA_\infty}^{*}=\lim_{x\to\infty}\frac{f_{SA}(x)}{x}=\frac{1}{2}(\sqrt{2}-1) \tag{4.5}$$

$$f_{SA}(1)=0 \tag{4.6}$$

From (2.10) together with (4.4), (4.5) and (4.6) give the following upper bound on the probability of error

$$P_e \le \frac{1}{2}\left[1-\left(\frac{2}{\sqrt{2}-1}\right)M_{SG}(C_1,C_2;W)\right] \tag{4.7}$$

Theorem 2: Show that $P_e \le \frac{1}{2}\left[1-\left(\frac{2}{\sqrt{2}-1}\right)M_{SG}(C_1,C_2;W)\right]$

Proof: let us consider the measure

$$M_{SG}(C_1,C_2;W)=E_X\left\{f_{SG}^{*}\begin{pmatrix}P(C_1/x);W\\ P(C_2/x);W\end{pmatrix}P(C_1/x)\right. \tag{4.8}$$

$$f_{SG}^{*}(x;w)=wxf_{SG}\begin{pmatrix}1-x\\ x\end{pmatrix} \tag{4.9}$$

where $f_{SG}(x)=\sqrt{\frac{x^2+1}{2}}-\sqrt{x}, \qquad \forall x\in(0,\infty)$

hence $$f_{SG}^{*}(x;w)=w\left[\frac{\sqrt{2}}{2}\sqrt{x^2+(1-x)^2}-\sqrt{x}(1-x)\right]= f_{SG}^{*}(1-x;w) \tag{4.10}$$

also $$f_{(SG)_\infty}=\lim_{x\to\infty}\frac{f_{SG}(x)}{x}=\frac{\sqrt{2}}{2} \tag{4.11}$$

and $$f_{SG}(1)=0 \tag{4.12}$$

from (2.10) together with (4.10),(4.11) and(4.12),the upper bound on the probability of error is

$$P_e \le \frac{1}{2}\left[1-\frac{2}{\sqrt{2}}M_{SG}(C_{1,}C_2;W)\right] . \tag{4.13}$$

Theorem 3: To show that $$P_e \le \frac{1}{2}\left[1-\frac{2}{\sqrt{2}}M_{SH}(C_{1,}C_2;W)\right] \tag{4.14}$$

Proof: Let us consider $$M_{SH}(C_1,C_2;W)=E_X\left\{f_{SH}^{*}\begin{pmatrix}P(C_1/x);W\\ P(C_2/x);W\end{pmatrix}P(C_1/x)\right. \tag{4.15}$$

where $$f_{SH}^{*}(x;w)=wxf_{SH}\begin{pmatrix}1-x\\ x\end{pmatrix} \tag{4.16}$$

and $f_{SH}(x)=\sqrt{\frac{x^2+1}{2}}-\frac{2x}{x+1}, \qquad \forall x\in(0,\infty)$

now $$f_{SH}^{*}(x;w)=w\left[\frac{\sqrt{2}}{2}\sqrt{x^2+(1-x)^2}-2x(1-x)\right]=f_{SH}^{*}(1-x;w) \tag{4.17}$$

$$f_{SH}(1)=0 \tag{4.18}$$

$$f_{(SH)_\infty}=\lim_{x\to\infty}\frac{f_{SH}(x)}{x}=\frac{\sqrt{2}}{2} \tag{4.19}$$

using (2.10) together with (4.17),(4.18) and(4.19),the upper bound on the probability of error is $P_e\le\frac{1}{2}\left[1-\frac{2}{\sqrt{2}}M_{SH}(C_{1,}C_2;W)\right]$

Theorem 4: To show that $$P_e\le\frac{1}{2}\left[1-2M_{AG}(C_{1,}C_2;W)\right] \tag{4.20}$$

Proof: $$M_{AG}(C_1,C_2;W)=E_X\left\{f_{AG}^{*}\begin{pmatrix}P(C_1/x):W\\P(C_2/x):W\end{pmatrix}P(C_1/x)\right\} \tag{4.21}$$

where $$f_{AG}^{*}(x;w)=wxf_{AG}\begin{pmatrix}1-x\\x\end{pmatrix} \tag{4.22}$$

and $$f_{AG}(x)=\frac{x+1}{2}-\sqrt{x}\,,\qquad \forall x\in(0,\infty)$$

In this case, we have

$$f_{AG}^{*}(x;w)=w\left[\frac{1}{2}-\sqrt{x}(1-x)\right]=f_{AG}^{*}(1-x;w) \tag{4.23}$$

and $$f_{(AG)_\infty}=\lim_{x\to\infty}\frac{f_{AG}(x)}{x}=\frac{1}{2} \tag{4.24}$$

$$f_{AG}(1)=0$$

using (2.10) together with (4.23),(4.24) and(4.25),the upper bound on the probability of error is $P_e\le\frac{1}{2}\left[1-2M_{AG}(C_{1,}C_2;W)\right]$.

Theorem 5 To show that $$P_e\le\frac{1}{2}\left[1-2M_{AG}(C_{1,}C_2;W)\right] \tag{4.26}$$

Proof: let us consider $$M_{AH}(C_1,C_2;W)=E_X\left\{f_{AH}^{*}\begin{pmatrix}P(C_1/x):W\\P(C_2/x):W\end{pmatrix}P(C_1/x)\right\} \tag{4.27}$$

Where $$f_{AH}^{*}(x;w)=wxf_{AH}\begin{pmatrix}1-x\\x\end{pmatrix} \tag{4.28}$$

and $$f_{AH}(x)=\left(\frac{x+1}{2}-\frac{2x}{1+x}\right),\qquad \forall x\in(0,\infty)$$

also $$f_{AH}^{*}(x;w)=\frac{w}{2}\left[(2x-1)^2\right]=f_{AH}^{*}(1-x;w) \tag{4.29}$$

$$f_{(AH)_\infty}=\lim_{x\to\infty}\frac{f_{AH}(x)}{x}=\frac{1}{2} \tag{4.30}$$

and $$f_{AH}(1)=0 \tag{4.31}$$

now using (2.10) together with (4.29),(4.30) and(4.31),the upper bound on the probability of error is $P_e \leq \frac{1}{2}\left[1-2M_{AG}(C_{1,}C_2;W)\right]$.

Theorem 6: To show that $$P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{2-\sqrt{2}}\right)M_{AN_2}(C_{1,}C_2;W)\right] \quad (4.32)$$

Proof: let us consider $$M_{AN_2}(C_1,C_2;W)=E_X\left\{f^*_{AN_2}\left(\frac{P(C_1/x);W}{P(C_2/x);W}\right)P(C_1/x)\right. \quad (4.33)$$

where $$f^*_{AN_2}(x;w)=wxf_{AN_2}\left(\frac{1-x}{x}\right) \quad (4.34)$$

and $$f_{AN_2}(x)=\frac{x+1}{2}-\left(\frac{\sqrt{x}+1}{2}\right)\left(\sqrt{\frac{x+1}{2}}\right),\ \forall x\in(0,\infty)$$

$$f^*_{AN_2}(x;w)=w\left[\frac{1}{2}-\frac{\sqrt{2}}{4}(\sqrt{x}+\sqrt{(1-x)}\right]=f^*_{AN_2}(1-x;w) \quad (4.35)$$

and $$f_{(AN_2)_\infty}=\lim_{x\to\infty}\frac{f_{AN_2}(x)}{x}=\frac{1}{2}-\frac{\sqrt{2}}{4}=\left(\frac{2-\sqrt{2}}{4}\right) \quad (4.36)$$

$$f_{AN_2}(1)=0 \quad (4.37)$$

now using (2.10) together with (4.29),(4.30) and(4.31),the upper bound on the probability of error is $P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{2-\sqrt{2}}\right)2M_{AN_2}(C_{1,}C_2;W)\right]$.

Theorm7: To show that

$$P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{2-\sqrt{2}}\right)2M_{AN_2}(C_{1,}C_2;W)\right] \quad (4.38)$$

Proof: Let us consider

$$M_{SN_1}(C_1,C_2;W)=E_X\left\{f^*_{SN_1}\left(\frac{P(C_1/x);W}{P(C_2/x);W}\right)P(C_1/x)\right. \quad (4.39)$$

where $$f^*_{SN_1}(x;w)=wxf_{SN_1}\left(\frac{1-x}{x}\right) \quad (4.40)$$

and $$f_{AN_2}(x)=\sqrt{\frac{x^2+1}{2}}-\left(\frac{\sqrt{x}+1}{2}\right)^2,\qquad \forall x\in(0,\infty)$$

$$f^*_{SN_1}(x;w)=w\left[\frac{\sqrt{2}}{2}(\sqrt{x^2+(1-x)^2}-\frac{1}{4}\{1+2\sqrt{x(1-x)}\right]=f^*_{SN_1}(1-x;w) \quad (4.41)$$

and $$f_{(SN_1)_\infty}=\lim_{x\to\infty}\frac{f_{SN_1}(x)}{x}=\frac{\sqrt{2}}{2}-\frac{1}{4}=\left(\frac{2\sqrt{2}-1}{4}\right) \quad (4.42)$$

$$f_{SN_1}(1)=0 \quad (4.43)$$

now using (2.10) together with (4.41),(4.42) and(4.43),the upper bound on the probability of error is $P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{2\sqrt{2}-1}\right)M_{SN_1}(C_{1,}C_2;W)\right]$.

Theorem 8: To show that

$$P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{2-\sqrt{2}}\right)2M_{AN_2}(C_{1,}C_2;W)\right] \tag{4.44}$$

Proof: Let us consider

$$M_{SN_2}(C_1,C_2;W) = E_X\left\{f^*_{SN_2}\begin{pmatrix}P(C_1/x);W\\P(C_2/x);W\end{pmatrix}P(C_1/x)\right\} \tag{4.45}$$

where $$f^*_{SN_2}(x;w) = wxf_{SN_2}\begin{pmatrix}1-x\\x\end{pmatrix} \tag{4.46}$$

and $$f_{SN_2}(x) = \sqrt{\frac{x^2+1}{2}} - \left(\frac{\sqrt{x}+1}{2}\right)\left(\sqrt{\frac{x+1}{2}}\right),\ \forall x \in (0,\infty)$$

$$f^*_{SN_2}(x;w) = w\left[\frac{\sqrt{2}}{4}\left\{2\sqrt{x^2+(1-x)^2}-\sqrt{x}-\sqrt{(1-x)}\right\}\right] = f^*_{SN_2}(1-x;w) \tag{4.47}$$

and $$f_{(SN_2)_\infty} = \lim_{x\to\infty}\frac{f_{SN_2}(x)}{x} = \frac{\sqrt{2}}{4}. \tag{4.48}$$

$$f_{SN_2}(1) = 0 \tag{4.49}$$

now using (2.10) together with (4.47),(4.48) and(4.49),the upper bound on the probability of error is $P_e \leq \frac{1}{2}\left[1-\left(\frac{4}{\sqrt{2}}\right)M_{SN_2}(C_{1,}C_2;W)\right]$.

Theorm 9: To show that

$$P_e \leq \frac{1}{2}\left[1-\left(\frac{6}{3\sqrt{2}-2}\right)M_{AN_3}(C_{1,}C_2;W)\right] \tag{4.50}$$

Proof: Let us consider

$$M_{SN_3}(C_1,C_2;W) = E_X\left\{f^*_{SN_3}\begin{pmatrix}P(C_1/x);W\\P(C_2/x);W\end{pmatrix}P(C_1/x)\right\} \tag{4.51}$$

where $$f^*_{SN_3}(x;w) = wxf_{SN_3}\begin{pmatrix}1-x\\x\end{pmatrix} \tag{4.52}$$

and $$f_{SN_3}(x) = \sqrt{\frac{x^2+1}{2}} - \left(\frac{x+\sqrt{x}+1}{3}\right), \qquad \forall x \in (0,\infty)$$

$$f^*_{SN_3}(x;w) = w\left[\frac{\sqrt{2}}{2}\left\{\sqrt{x^2+(1-x)^2}-\frac{1}{3}(1+\sqrt{x(1-x)}\right\}\right] = f^*_{SN_3}(1-x;w) \tag{4.53}$$

and $$f_{(SN_3)_\infty} = \lim_{x\to\infty}\frac{f_{SN_3}(x)}{x} = \frac{\sqrt{2}}{2}-\frac{1}{3} = \frac{3\sqrt{2}-2}{6}. \tag{4.54}$$

$$f_{SN_3}(1)=0 \tag{4.55}$$

now using (2.10) together with (4.52),(4.53) and(4.54),the upper bound on the probability of error is $P_e \le \frac{1}{2}\left[1-\left(\frac{6}{3\sqrt{2}-2}\right)M_{SN_3}(C_{1,}C_2;W)\right]$.

Theorm 10: To show that

$$P_e \le \frac{1}{2}\left[1-\left(\frac{4}{2\sqrt{2}-1}\right)M_{N_2N_1}(C_{1,}C_2;W)\right] \tag{4.55}$$

Proof: Let us consider

$$M_{N_1N_2}(C_1,C_2;W)=E_X\left\{f^*_{N_2N_1}\begin{pmatrix}P(C_1/x);W\\P(C_2/x);W\end{pmatrix}P(C_1/x)\right\} \tag{4.56}$$

where $$f^*_{N_2N_1}(x;w)=wxf_{N_2N_1}\begin{pmatrix}1-x\\x\end{pmatrix} \tag{4.57}$$

and $$f_{N_2N_1}(x)=\left(\frac{\sqrt{x}+1}{2}\right)\left(\sqrt{\frac{x+1}{2}}\right)-\left(\frac{\sqrt{x}+1}{2}\right),\qquad \forall x\in(0,\infty)$$

$$f^*_{N_2N_1}(x;w)=w\left[\frac{\sqrt{2}}{4}\left\{\sqrt{x+(1-x)}-\frac{1}{4}(1+2\sqrt{x(1-x)}\right\}\right]=f^*_{N_2N_1}(1-x;w) \tag{4.58}$$

and

$$f_{(N_2N_1)\infty}=\lim_{x\to\infty}\frac{f_{N_2N_1}(x)}{x}=\frac{\sqrt{2}}{2}-\frac{1}{4}=\frac{2\sqrt{2}-1}{4}. \tag{4.59}$$

$$f_{N_2N_1}(1)=0 \tag{4.60}$$

Now using (2.10) together with (4.58), (4.59) and (4.60), the upper bound on the probability of error is $P_e \le \frac{1}{2}\left[1-\left(\frac{4}{2\sqrt{2}-1}\right)M_{N_2N_1}(C_{1,}C_2;W)\right]$.

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STOCHASTIC MODEL FOR THE YULE PROCESS AND ESTIMATION OF THE CONVEX FUNCTION OF GIP SECRETION IN RESPONSE TO ORAL GLUCOSE TOLERANCE TEST

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ABSTRACT :
To evaluate the Glucose dependent insulinotropic polypeptide (GIP) gastrointestinal peptide hormone regulating postprandial insulin release from pancreatic Beta-cells. The standardized oral glucose tolerance test was performed between n=52 patients and n=28 healthy controls. GIP levels were measured sequentially for 120 minutes, after glucose administration. GIP secretion was unchanged. There is no difference in GIP secretion between patients and healthy controls. In this paper, the problem is investigated considering Yule process and estimation of convex function with focus on exponential or Markovian case.
Key Words: ***Incretin effect, β-cells, Yule process, CMJ process.***
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1. Introduction

The incretins, glucose dependent insulinotropic polypeptide(GIP) are gastrointestinal peptide hormones regulating postprandial insulin release from pancreatic β-cells, the so-called "incretin effect"[14]. Healthy controls were matched by sex and screened for the metabolic syndrome and liver disease by medical history and a blood sample including liver function tests(LFT), fasting glucose and lipids. Exclusion criteria for controls were alcohol consumption, smoking, any liver disease, elevated LFTs, elevated lipids, IR or intake of any drug with a known influence on glucose homeostasis. Oral glucose tolerance test (OGTT) was performed in all subjects after an overnight fast according to a standardized protocol using 75g of glucose in 300 ml tap water (300 kcal). Baseline vital parameters, height, weight and BMI were taken. Blood samples were drawn Mean fasting GIP plasma levels were not different between the groups. After OGTT neither peak nor total GIP concentrations differed significantly between patients and controls.

There was complete loss of insulin secretion in response to GIP [6], [3]. From clinical studies there is substantial evidence that incretin-based therapies exhibit various beneficial effects in patients like preventions of weight gain apart from their glucose-lowering properties [13]. GIP is unlikely to explain glucose-induced hyperinsulinemia. Possible explanations are involvement of other incretins or non-hormonal regulators of insulin secretion, or the compensatory hyperplasia of β-cells and hypersecretion of insulin in the insulin-resistant state [8], [4].

To estimate the general branching model, where particles have not necessarily exponential life lengths and give birth at a constant rate b. The process that counts the number of the alive through time is a Crump-Mode-Jagers process [4] which is binary and homogenous. In the case of exponential lifetimes, population dynamics are given by linear birth and death processes. The problem is tackled by considering Yule process and estimation of convex function with focus on exponential or Markovian case.

Notations:

$\wedge$	finite positive measure on $(0, +\infty)$
ρ_x	birth point process
μ_x	non-negative random variable
Z	Scale function
ω	Convex function

2. Crump Mode Jagers process

Let Y= (Y(t), t$\geq$ 0) be the process counting the number of extant particles through time. We denote the life span distribution by $\wedge(.)/a$, where $\wedge$ is the finite measure on $(0, +\infty]$ with total mass 'a' and is called a lifespan measure [2].

The total population process Y belongs to a large class of branching processes called Crump-Mode-Jagers or CMJ processes. In this processes, also called general branching processes [12], [11], one associates with each particle x in the population a non-negative random variable μ_x and a point process ρ_x called birth point process. One assumes that the sequence $(\mu_x, \rho_x)_x$ not necessarily exponential but μ_x and ρ_x are not necessarily independent. Then, the CMJ process is defined as

$$Y(t) = \sum_x 1\{\gamma_x \leq t < \gamma_x + \mu_x\}, t \geq 0, \qquad \text{---------- (1)}$$

where for any particle x in the population, γ_x is birth time.

In our particular case, the common distribution of life spans is $\wedge(.)/a$ and conditional on lifespan, the birth point process of a practical is distributed as a Poisson point process during life. We can say that the CMJ process Y is homogenous and binary. We will say that Y is subcritical, critical, or supercritical according to the mean number of patients.

$$s := \int_0^\infty v \wedge (dv), \qquad \text{--------- (2)}$$

is less than, equal to, or greater than 1.

The advantage of homogenous binary CMJ processes is that they allow for explicit computations, for example about one-dimensional marginal of Y (viewing the forthcoming proposition 2.a). More precisely, for $\mu \geq 0$, define

$$\omega(\mu) := \mu - \int_{(0,\infty)} (1 - e^{-\mu v}) \wedge (dv), \qquad \text{---------- (3)}$$

And let q be the greatest root of ω. Notice that ω is convex, $\omega(0)$ =0, and $\omega'(0)$=1-s. As a consequence,

q = 0 if Y is subcritical or critical,

q > 0 if Y is supercritical. ---------- (4)

Let Z be the so-called scale function [7, page 194] associated with ω, that is, the unique increasing continuous function $(0,\infty)\to(0,\infty)$ satisfying

$$\int_0^\infty Z(x)e^{-\mu x}dx=\frac{1}{\omega(\mu)},\ \mu>q. \qquad \text{---------- (5)}$$

Propostion 2.a. [2], [1]. The one-dimensional marginals of Y are given by

$$P(Y(t)=0)=1-\frac{Z^{'}(t)}{aZ(t)} \qquad \text{---------- (6)}$$

and for $m\geq 1$,

$$P(Y(t)=m)=\left(1-\frac{1}{Z(t)}\right)^{m-1}\frac{Z^{'}(t)}{aZ(t)^2} \qquad \text{---------- (7)}$$

In other words, conditional on being nonzero, Y(t) is distributed as a geometric random variable with success probability $1/Z(t)$.

If Ext: = $\{Y(t)\underset{t\to\infty}{\to} 0\}$ denotes the extinction event of Y, according to [2], as a consequence of the last Proposition,

$$P(Ext)=1-\frac{q}{a}. \qquad \text{---------- (8)}$$

Thus, by (4) extinction occurs when Y is (sub) critical and $P(Ext^c)>0$ when it is supercritical.

The proposition 2. b. justifies the fact that q is called the Malthusian parameter of the population in the supercritical case.

Proposition 2.b. [2]. If s>1, conditional on the survival event Ext^c,

$$e^{-qt}Y(t)\underset{t\to\infty}{\to}\rho \qquad \text{--------- (9)}$$

Where ρ is exponential with parameter $\omega^{'}(q)$. In fact, convergence in distribution is proved in [2] and convergence holds according to [10], (see [9], page 285]).

3. Exponential Case

An interesting case that we will focus on is the exponential (or Markovian) case, when the common distribution of life lengths is exponential with parameter c, that is, $\wedge(dv)=ace^{-cv}dv$ or $\wedge(dv)=a\beta_\infty(dv)$. In that case, Y is, respectively, a linear birth and death process with birth rate and death rate c or a pure birth process (or Yule process) with parameter a.

In this case, Y and Y* are Markov processes and the quantities defined in section 2 are computable. Indeed, we have

$$\omega(\mu) = \frac{\mu(\mu - a + c)}{\mu + c}, \; q = a - c, \qquad \text{--------- (10)}$$

$$s = 1 - \omega'(0) = \frac{a}{c}, \quad \omega'(q) = 1 - \frac{c}{a}.$$

It is also possible to compute the function Z, defined by [6], while it is generally unknown. From [2, page 393], we have

$$Z(x) = \begin{cases} \dfrac{ae^{qx} - c}{q}, & if\ a \neq c, \quad x \geq 0, \\ 1 + ax, & if\ a = c, \end{cases} \qquad \text{--------- (11)}$$

and in all cases

$$Z'(x) = ae^{qx}, \quad x \geq 0. \qquad \text{--------- (12)}$$

4. Example

The incretin glucose-dependent insulinotropic polypeptide (GIP) gastrointestinal peptide hormones regulating postprandial insulin release from pancreatic β-cells. A standardized oral glucose tolerance test was performed between n = 52 patients and n = 28 healthy controls as defined in [5]. The GIP secretion is shown in Fig(1) after glucose administration. GIP secretion in response to OGTT is not different in patients vs. controls.

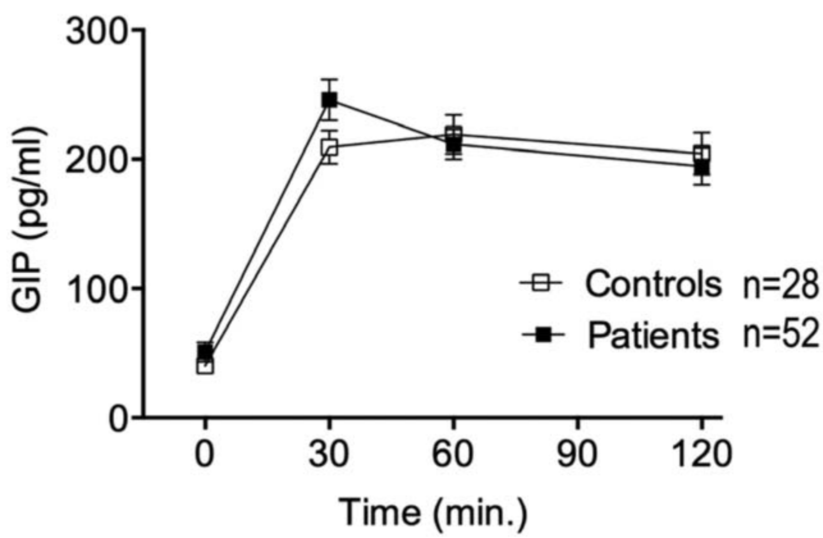

Fig. (1)

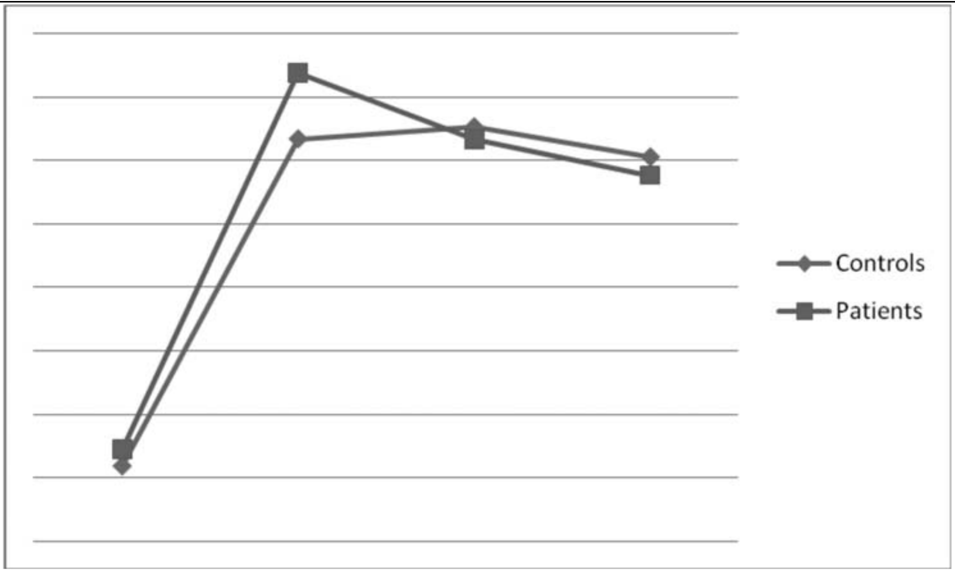

Fig. (2)

5. Conclusion

Evaluation of GIP secretion levels were measured sequentially for 120 minutes in patients and healthy control is fitted with the convex function with focus on exponential or Markovian case by considering Yule process. The results of GIP levels remain unchanged that is, there is no difference in the GIP secretion between patients and healthy controls. The results coincide with the mathematical and medical report.

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TESTING OF RANDOMNESS OF THE NUMBERS GENERATED BY FISHER YATES

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ABSTRACT :
Proper randomness of the numbers generated by Fisher and Yates has been examined by D. Chakrabarty in 2010 by computing the probability of occurrence of each digit in the table of generated numbers and then applying the theoretical probabilistic concept of randomness.
The same has been tested by B.K. Sarmah and D. Chakrabarty (Nov. 2014 IJESRT) by applying the Chi-square test for testing the significance of difference between observed frequency of each of the digit in the table and the corresponding theoretical (expected) frequency.
In this paper, the randomness of the digits have been tested by applying t-test for amount of deviation of the observed number of occurrence and the theoretical(expected) number of occurrences of the respective digits and hence the numbers. The test shows that the numbers generated by Fisher and Yates deviated significantly from proper randomness.
Keywords: ***Random number generated by Fisher and Yates, testingof randomness, t-test.***

Introduction

Drawing of random sample has been found to be vital or basic necessity in most of the researches/investigations especially of applied sciences. The convenient practical method of selecting a random sample consists of the use of Table of Random Numbers. Existing tables of random numbers, used commonly, are the ones due to Fisher and Yates (Constructed in 1938), L.H.C. Tippett (Constructed in 1927), Kendall and Babington Smith (Constructed in 1955) and Rand Corporation (constructed in 1955) and recently D.Chakarbarty (2013) generate one table of two digit & Three digits random number.

The random number table have been subjected to various statistical tests of randomness. These tests have limitations to decide on proper randomness of the numbers occurring in the corresponding tables. As a Consequence it is not guaranteed that the numbers in each of these tables are properly random. This leads to think of testing the proper randomness of the numbers in this tables. In the present study, an attempt has been made to test this. The study, here, has been made on the testing of randomness of the table of numbers constructed by Fisher and Yates only.

By the existing statistical methods, it is only possible to know whether the randomness of the numbers of a table is proper. It is only possible to know whether the deviation of the degree of its randomness is significant.

In order to test the proper randomness of the random numbers table constructed by Fisher and Yates t-test has been applied.

Materials and Methods:

Fisher and Yates random number table consists of a total of 7500 two digited numbers giving in all 15000 digits.

To know whether the number in random number table of Fisher and Yates are proper or not student's t-test for amount of deviation is applied.

Let 'd' be the variable denoting the measure of the deviation (amount of deviation) of the observed number of occurrences of the respective digit.

Suppose, di(i=0,1….9) are independent observed values of the deviatioin variables.

If the table of number is random then di=0, for all i, in the ideal situation . However, due to chance error, di may assume non zero value.

Thus the values of di's are due to chance error but not due to any assignable error if the table is random.

The chance variables are i.i.d. N.(o,σ) variables.

Thus testing of randomness is equivalent to testing of the hypothesis H_o

That E(d_i) = 0, for all i,

Let us consider the statistic t for testing H_o

i.e. $t = \frac{\overline{d} - E(\overline{d})}{S.E.(\overline{d})} \sim t_{n-1}$

where,

$\overline{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$

We have,

E $(\overline{d}) = \frac{1}{n} \sum E(d_i) = 0$

When H_o is true

Also,

$var\,\overline{(d)} = \frac{\sigma^2}{n}$, σ^2 is unknown

However unbiased estimate of σ^2 is

$s^2 = \frac{1}{n-1} \sum (d_i - \overline{d})^2$

$= \frac{1}{n-1}[\Sigma\, {d_i}^2 - \frac{(\Sigma d_i)^2}{n}]$

Which implies unbiased estimate of

$var\,\overline{(d)} = \frac{s^2}{n}$

$and\ S.D.\overline{(d)} = \frac{s}{\sqrt{n}}$

Therefore statistic t for testing H_o becomes

$t = \frac{\overline{d}}{s/\sqrt{n}}$ when H_o is true and this t follows student's t distribution with (n-1) d.f.

Steps in the Method:

In order to test the proper randomness of the numbers of Fisher and Yates table one is required to proceed with the following steps:

Step1: In the first step, observe the occurrences of the digits 0 to 9 for first 1000 trails, second 1000 trials ……….. … up to 15^{th} 1000 trails as shown in the table.

Step2: In the second step, compute the theoretical expected frequencies. This is done by dividing trails i.e. 1^{st} 1000 ………… and 15^{th} 1000 by 10 assuming that the digit 0 to 9 occurs equal number of times.

Step 3: In the third step, compute the amount of deviation of observed occurrences of digits and expected occurrences of digits.

Step 4: In the fourth step, compute the value of student's t for each of the trails.

Step 5: Compare the t value with corresponding theoretical values.

Step 6: Draw conclusion as per the result obtained in step 5.

Results and discussion:

The results obtained on operating the steps (Nos. 1 to 5) on the random numbers table constructed by Fisher and Yates have been observed. It is observed from the table that occurrences of digit 0 to 9 are not equal.

Conclusion:

From the table prepared for observed frequency of occurrence of digits along with respective expected frequency (shown in bracket) it is observed that the calculated value of t is significant. That is calculated value of 't' have been found to be significant on comparing them with the corresponding theoretical values.

Hence it may be concluded that the table of numbers constructed by Fisher and Yates deviates significantly from proper randomness.

Therefore Fisher and Yates Random Numbers Table Cannot be treated as properly random.

Table: Observed frequency of occurrence of digits along with the respective expected frequency (Shown in bracket), amount of deviation (di) and the values of students 't' statistic from Fisher and Yates.

Digits	0	1	2	3	4	5	6	7	8	9	t^2-value
1st 1000	9 91 (100)	7 93 (100)	4 104 (100)	2 102 (100)	2 102(100)	3 103 (100)	9 109 (100)	8 108 (100)	12 88 (100)	8 108 (100)	6.17
2ND 1000	19 119 (100)	9 91 (100)	7 107 (100)	18 118 (100)	8 1 08 (100)	7 93 (100)	7 93 (100)	4 104 (100)	18 (100) 82	15 (100) 85	5.56
3rd 1000	2 98 (100)	4 96 (100)	5 95 (100)	8 108 (100)	1 99 (100)	1 101 (100)	7 93 (100)	2 98 (100)	5 (100) 105	7 107 (100)	4.54
4th 1000	3 97 (100)	1 99 (100)	13 113 (100)	8 92 (100)	3 103 (100)	7 93 (100)	6 106 (100)	4 96 (100)	1 (100) 101	0 100 (100)	3.99
5th 1000	7 93 (100)	1 101 (100)	11 89 (100)	4 104 (100)	5 105 (100)	3 103 (100)	1 101 (100)	16 84 (100)	10 (100) 110	10 110 (100)	3.83
6TH 1000	15 85 (100)	11 89 (100)	9 109 (100)	6 106 (100)	9 91 (100)	17 83 (100)	4 104 (100)	16 116 (100)	0 (100) 100	17 117 (100)	5.06
7TH 1000	7 107 (100)	3 103 (100)	12 88 (100)	1 99 (100)	2 98 (100)	9 109 (100)	8 92 (100)	10 110 (100)	2 98 (100)	4 96 (100)	4.44
`8TH 1000	3 103 (100)	6 106 (100)	3 97 (100)	3 103 (100)	1 99 (100)	13 87 (100)	13 113 (100)	2 102 (100)	12 88 (100)	2 102 (100)	3.72
9TH 1000	1 101 (100)	4 96 (100)	10 90 (100)	1 99 (100)	13 87 (100)	10 110 (100)	6 106 (100)	1 99 (100)	14 (100) 114	(100) 98	3.81
10TH 1000	9 91 (100)	2 102 (100)	15 85 (100)	5 95 (100)	15 115 (100)	4 96 (100)	5 105 (100)	16 116 (100)	8 92 (100)	3 103 (100)	4.93
11TH 1000	13 113 (100)	5 95 (100)	2 102 (100)	10 110 (100)	8 92 (100)	0 100 (100)	4 104 (100)	19 81 (100)	13 113 (100)	0 100 (100)	3.22
12 TH 1000	2 102 (100)	0 100 (100)	8 92 (100)	9 109 (100)	8 92 (100)	8 92 (100)	2 102 (100)	12 112 (100)	4 (100) 96	3 103 (100)	4.39
13TH 1000	1 101 (100)	11 89 (100)	7 93 (100)	0 100 (100)	0 100 (100)	1 101 (100)	31 131 (100)	5 105 (100)	23 77 (100)	3 (100) 103	2.37
14TH 1000	7 93 (100)	3 97 (100)	16 116 (100)	9 109 (100)	10 110 (100)	4 96 (100)	3 103 (100)	26 74(100)	3 (100) 103	1 (100) 99	3.49
15 TH 1000	4 104 (100)	6 94 (100)	15 85 (100)	14 114 (100)	5 105 (100)	9 91 (100)	10 110 (100)	7 93 (100)	21 121 (100)	7 93 (100)	5.44

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NEW METHOD OF 1'S INTERVAL LINEAR ASSIGNMENT PROBLEMS

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ABSTRACT :
This paper deals with the Assignment problems arise in different situations where we have to find an optimal way to assign n jobs to m other objects as per their efficiency using maximizes and Alternate optimal assignment models. Using ones interval linear assignment methods and the existing Hungarian method a numerical example has been solved and compared. We consider interval analysis concept for solving interval linear assignment problems.
Keywords: ***Assignment Problems, Interval Linear Assignment Problems, Maxima and Minima, Alternative optimal solution***
MSC Code: 90B80

1. Introduction

The assignment problem is one of the earliest application of linear integer programming problem. Different methods have been presented in literature for assignment problem and various articles have been published on the subject [1], [2] and [3] .The Hungarian method is more convenient method among them. This iterative method is based on adding or subtracting a constant to every element of a row or column of the cost matrix, in a minimization model and create some zeros in the given cost matrix and then try to find a complete assignment in terms of zeros. By a complete assignment for a cost matrix $n\times n$, we mean an assignment plan containing exactly n assigned independent zeros, one in each row and one in each column. The main concept of assignment problem is to find the optimum allocation of a number of resources to an equal number of demand points. An assignment plan is optimal if it optimizes the total cost or effectiveness of doing all the jobs. This paper attempts to propose a method for solving assignment problem which is different from the preceding one.

The standard assignment problem can be seen as a relaxation of more complex combinatorial optimization problems such as traveling salesman problem, quadratic assignment problem etc. It can also be considered as a particular transportation problem with all supplies and demands equal to 1. The assignment problem has also several variations such as the semi-assignment problem and the k-cardinality assignment problem. The reader interested in more details about these two problems or other variations should study for a comprehensive survey of the assignment problem variations.

2. Definition

Arithmetic Operations in interval

The interval form of the parameters may be written as where is the left value [$\underline{x}$] and is the right value [$\overline{x}$]of the interval respectively. We define the centre is m = $\frac{\overline{x}+\underline{x}}{2}$ and w = $\overline{x}-\underline{x}$ is the width of the interval [$\overline{x},\underline{x}$]

Let [$\overline{x}, \underline{x}$] and [$\overline{y}, \underline{y}$] be two elements then the following arithmetic are well known

(i) [$\overline{x}, \underline{x}$] +[$\overline{y}, \underline{y}$] = [$\overline{x} + \overline{y}$, $\underline{x} + \underline{y}$]

(ii) [$\overline{x}, \underline{x}$] × [$\overline{y}, \underline{y}$] =[min{$\overline{x}\underline{y}$ $\underline{x}\overline{y}$, $\overline{x}\overline{y}$, $\underline{x}\underline{y}$} ,max{$\overline{x}\underline{y}$ $\underline{x}\overline{y}$, $\overline{x}\overline{y}$, $\underline{x}\underline{y}$}]

(iii) [$\overline{x}, \underline{x}$] ÷ [$\overline{y}, \underline{y}$] =[min{$\overline{x} \div \underline{y}$ $\underline{x} \div \overline{y}$, $\overline{x} \div \overline{y}$, $\underline{x} \div \underline{y}$} , max{$\overline{x} \div \underline{y}$ $\underline{x} \div \overline{y}$, $\overline{x} \div \overline{y}$, $\underline{x} \div \underline{y}$}] provide if [$\overline{y}$, $\underline{y}$] ≠ [0, 0],

(iv) [$\overline{x}, \underline{x}$] - [$\overline{y}, \underline{y}$] = [$\overline{x}$-$\underline{y}$, $\underline{x}$-$\overline{y}$] Provide if [$\overline{y}$, $\underline{y}$] ≠ [0, 0],

3. Mathematical Model of Assignment Problem

Let X_{ij} denote the assignment of facility I to job j such that

$$Xij = \begin{cases} 1 & if\ facility\ is\ assigned\ to\ job\ j \\ 0 & otherwise \end{cases}$$

Then,the mathematical formulation of the standard assignment problem (SAP) is as follows:

$$\text{Min } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{1} x_{ij} = 1 \qquad i=1,2\text{----------}n$$

$$\sum_{i=1}^{1} x_{ij} = 1 \qquad j= 1,2 \text{--------}n$$

$$x_{ij} = \{0 \text{ or } 1\} \qquad i ,j =1, 2\text{-------}n$$

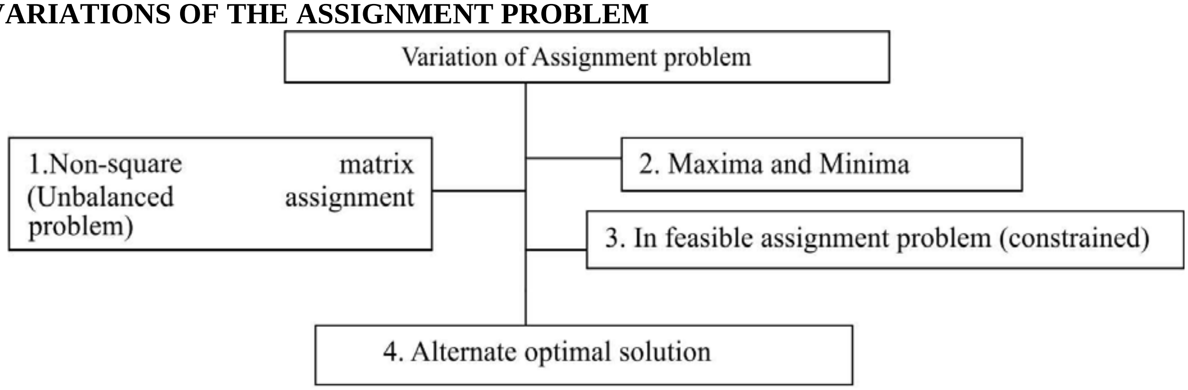
where for all i, j = 1, …, n, cij is the cost of assigning agent I to task j, Xij = 1 means that agent i is assigned to task j and Xij = 0 means that agent i is not assigned to task j.The first set of constraints implies that each agent is assigned to one and only one task and the second set of constraints implies that to each task is assigned one and only one agent.

In addition to the minimization of assignment cost, an assignment problem may consider other objective functions such as the minimization of completion time. When the assignment problem is considered with the minimization of assignment cost as the objective function, it is called the cost minimizing assignment problem.

Note:-

It may be noted that assignment problem is a variation of transportation problem with two characteristics 1.The cost matrix is a square matrix 2.The optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix

4. VARIATIONS OF THE ASSIGNMENT PROBLEM



1. Non-square matrix (Unbalanced assignment problem):

Such a problem is found to exist when the number of facilities is not equal to the number of jobs. Since the Hungarian method of solution requires a square matrix, fictitious facilities or jobs may be added and zero costs be assigned to the corresponding cells of the matrix . These cells are then treated the same way as the real cost cells during the solution procedure.

2. Maxima and Minima method:

Sometimes the assignment problem may deal with maximization of the objective function. The maximization problem has to be changed to minimization before the Hungarian method may be applied. This transformation may be done in either of the following two ways:

a. by subtracting all the elements from the largest element of the matrix.

b. multiplying the matrix elements by-1

3. In feasible assignment problem (constrained):

A constrained assignment occurs in the cell (i ,j) of the assignment cost matrix if i^{th} person is unable to perform j^{th} job. Such problems can be solved by assigning a very heavy cost (infinite cost) to the corresponding cell. Such a job will then be automatically excluded from further consideration.

In such cases, the cost of performing that particular activity by a particular resource is considered to be very large (written as M or ∞) so as to prohibit the entry of this pair of resources- activity into the final solution.

4. Alternate optimal solution:

Sometimes, it is possible to have two or more ways to strike off all zero elements in the reduce matrix for a given problem. In such cases there will be alternate optimal solutions with the same cost. Alternate optimal solutions offer a great flexibility to the management sine it can select the one which is most suitable to its requirement.

5. Algorithm for ones Assignment problem

Step1. In a minimization (maximization) case, find the minimum (maximum) element of each row in the assignment matrix (say *ai*) and write it on the right handside of the matrix.Then divide each element of i^{th} row of the matrix by a_i. These operations create at least one ones in each rows. In term of ones for each row and column do assign, otherwise go to step 2.

Step 2. Find the minimum (maximum) element of each column in assignment matrix (say *bj*), and write it below jth column. Then divide each element of j^{th} column of the matrix by *bj* .These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.

Step 3. Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than *n*, then the complete assignment is not possible, while if the number of lines is exactly equal to *n*, then the complete assignment is obtained.

Step 4. If a complete assignment program is not possible in step 3, then select the smallest (largest) element (say *dij*) out of those which do not lie on any of the lines in the above matrix. Then divide by *dij* each element of the uncovered rows or columns. This operation create some new ones to this row or column. If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained

Example 1(Maximization problem):

Beta Corporation has four plants each of which can manufacture any one of four productss . Given the revenue and cost data below, obtain which product each plant should produce to maximize profit.

Cost matrix with crisp entries

	I	II	III	IV
A	32	48	22	36
B	48	40	38	24
C	42	40	48	30
D	30	32	28	48

Step 1: We take a linear assignment problem as an example problem and solved this problem by traditional Hungarian method .The assignment cost of assigning any operator to any one machine is given in the following table

Cost matrix with crisp entries

	I	II	III	IV
A	[31,33]	[47,49]	[21,23]	[35,37]
B	[47,49]	[39,41]	[37,39]	[23,25]
C	[41,43]	[39,41]	[47,49]	[29,31]
D	[29,31]	[31,33]	[27,29]	[47,49]

Step 2: To convert the maximization problem one has to be changed to minimization before the Hungarian method may be applied. By divided all the elements from the largest element [47,49] of the matrix.

Next, Then divide each element of ith row of the matrix by a_i. These operations create ones to each rows, and now find the minimum element of each column in assignment matrix (say *bj*), and write it below that column. Then divide each element of jth column of the matrix by *bj*. Thus resulting matrix becomes

	I	II	III	IV
A	$[\frac{31}{47},\frac{33}{49}]$	[1,1]	$[\frac{21}{47},\frac{23}{49}]$	$[\frac{35}{47},\frac{37}{49}]$
B	[1,1]	$[\frac{39}{47},\frac{41}{49}]$	$[\frac{37}{47},\frac{39}{49}]$	$[\frac{23}{47},\frac{25}{49}]$
C	$[\frac{41}{47},\frac{43}{49}]$	$[\frac{39}{47},\frac{41}{49}]$	[1,1]	$[\frac{29}{47},\frac{31}{49}]$
D	$[\frac{29}{47},\frac{31}{49}]$	$[\frac{31}{47},\frac{33}{49}]$	$[\frac{27}{47},\frac{29}{49}]$	[1,1]

Step 3. Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

	I	II	III	IV
A	$[\frac{31}{47},\frac{33}{49}]$	$[\mathbf{1},\mathbf{1}]$	$[\frac{21}{47},\frac{23}{49}]$	$[\frac{35}{47},\frac{37}{49}]$
B	$[\mathbf{1},\mathbf{1}]$	$[\frac{39}{47},\frac{41}{49}]$	$[\frac{37}{47},\frac{39}{49}]$	$[\frac{23}{47},\frac{25}{49}]$
C	$[\frac{41}{47},\frac{43}{49}]$	$[\frac{39}{47},\frac{41}{49}]$	$[\mathbf{1},\mathbf{1}]$	$[\frac{29}{47},\frac{31}{49}]$
D	$[\frac{29}{47},\frac{31}{49}]$	$[\frac{31}{47},\frac{33}{49}]$	$[\frac{27}{47},\frac{29}{49}]$	$[\mathbf{1},\mathbf{1}]$

We are applying the proposed interval Hungarian method and solve this problem. We get an maximum assignment cost is [188,196] and optimal assignment as A ,B,C,D machines are assigned to II ,I, III ,IV operators respectively .

Example 4(Alternate optimal solution):

An automobile workshop wishes to put four mechanics to four different jobs. The mechanics have somewhat different kinds of skills and they exhibit different levels of efficiency from one job to another. The manager of the workshop has estimate the number of man-hours that would be required for each job-man combination. This is given in the matrix form in adjacent table

Mechanic \ Job	A	B	C	D
1	5	2	2	8
2	7	9	2	2
3	7	4	6	4
4	5	7	8	5

Find the optimum assignment that will result in minimum man –hours needed.

Step 1: We take a linear assignment problem as an example problem and solve this problem by traditional Hungarian method .The assignment cost of assigning any operator to any one machine is given in the following table

Cost matrix with crisp entries

	A	B	C	D
1	[4,6]	[1,3]	[1,3]	[7,9]
2	[6,8]	[8,10]	[1,3]	[1,3]
3	[6,8]	[3,5]	[5,7]	[3,5]
4	[4,6]	[6,8]	[7,9]	[4,6]

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a minimum problem.

Step 2: Then divide each element of ith row of the matrix by a_i. These operations create ones to each rows and now find the minimum element of each column in assignment matrix (say bj), and write it below that column. Then divide each element of jth column of the matrix by bj .

We make the 'one's-assignments' as shown in the table .It may be note that an assignment problem can have more than one optimal solution the other solution is shown in table

Mechanic \ Job	A	B	C	D
1	$[\frac{4}{1},\frac{6}{3}]$	**[1,1]**	~~[1,1]~~	$[\frac{7}{1},\frac{9}{3}]$
2	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{8}{1},\frac{10}{3}]$	**[1,1]**	~~[1,1]~~
3	$[\frac{6}{3},\frac{8}{5}]$	~~[1,1]~~	$[\frac{5}{3},\frac{7}{5}]$	**[1,1]**
4	**[1,1]**	$[\frac{6}{4},\frac{8}{6}]$	$[\frac{7}{4},\frac{9}{6}]$	~~[1,1]~~

Optimal solution I		
Mechanic	Job	Man -Hours
1	B	[1,3]
2	C	[1,3]
3	D	[3,5]
4	A	[4,6]

Mechanic \ Job	A	B	C	D
1	$[\frac{4}{1},\frac{6}{3}]$	~~[1,1]~~	**[1,1]**	$[\frac{7}{1},\frac{9}{3}]$
2	$[\frac{6}{1},\frac{8}{3}]$	$[\frac{8}{1},\frac{10}{3}]$	~~[1,1]~~	**[1,1]**
3	$[\frac{6}{3},\frac{8}{5}]$	**[1,1]**	$[\frac{5}{3},\frac{7}{5}]$	~~[1,1]~~
4	**[1,1]**	$[\frac{6}{4},\frac{8}{6}]$	$[\frac{7}{4},\frac{9}{6}]$	~~[1,1]~~

Optimal solution II		
Mechanic	Job	Man -Hours
1	C	[1,3]
2	D	[1,3]
3	B	[3,5]
4	A	[4,6]

Conclusions:

This paper presents a two different models for solving 1's assignment problems . The proposed interval Hungarian method is effective and useful in this interval context. Using this method we can solve real world linear assignment problems where entries of the cost matrix are in interval form.

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FUZZY REGULAR GRAPH PROPERTIES WITH IF-THEN RULES

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ABSTRACT :
In this paper derivation and approaches are worked on the basis of IF-THEN rules in regular fuzzy graphs and totally regular fuzzy graphs. Some amazing facts with peculiar results for new parameters, by analyzing IF-THEN rule in fuzzy graphs are received. Fuzzy strongest path for the parameters are derived by presenting the fuzzy graph concept.
Keywords: ***Fuzzy strength path, Fuzzy strongest path, Fuzzy IF-THEN rule.***

Introduction

Graph theory is proved to be tremendously useful in modeling the essential features of systems with finite components Graphical models are used to represent telephone network, railway network, communication problems, traffic network etc.

Graph theoretic models provide a useful structure upon which analytic techniques can be used. A graph is also used to model a relationship between a given set of objects. Each object is represented by a vertex and the relationship between them is represented by an edge if the relationship is unordered and by means of a directed edge if the objects have an ordered relation between them. Relationship among the objects need not always be precisely defined criteria; when we think of an imprecise concept, the fuzziness arises.

In 1965, L.A. Zadeh introduced a mathematical frame work to explain the concept of uncertainty in real life through the paper presentation in a seminar. Recently G. Nirmala etal (2013) discussed the various concepts related with fuzzy graph along with their properties in this paper some peculiar result for new parameters, are obtained by analyzing IF-THEN rule in fuzzy graphs

2. Basic Concepts

Definition 2.1

A fuzzy graph is a pair of functions $\sigma : V \to [0,1]$, $\mu : V \times V \to [0,1]$ for all u, v in V, we have $\mu(u,v) \le \min[\sigma(u), \sigma(v)]$

Example 2.1

The following figure represents a fuzzy graph $G:(\sigma,\mu)$, where $\sigma = \{u/0.5, v/0.4, w/0.3, x/0.2, y/0.7\}$ and $\mu = \{(u,v)/0.4, (v,w)/0.3, (w,x)/0.5, (x,y)/0.5, (y,u)/0.2\}$

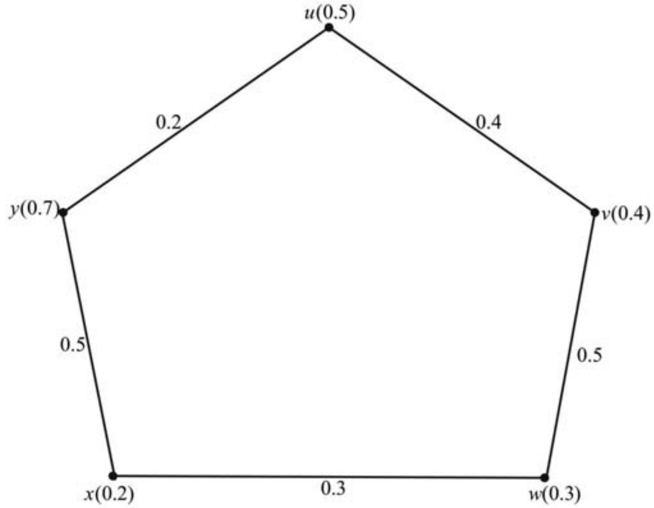


Fig.1. Fuzzy Graph

Definition 2.2

Let $G=(\sigma,\mu)$ be a fuzzy graph on $G^*=(V,E)$. If $d_G(v)=K$ for all $v\in V$ that is if each vertex has same degree K, then G is said to be a regular fuzzy graph of degree K or a $K-$degree regular fuzzy graph.

Example 2.2

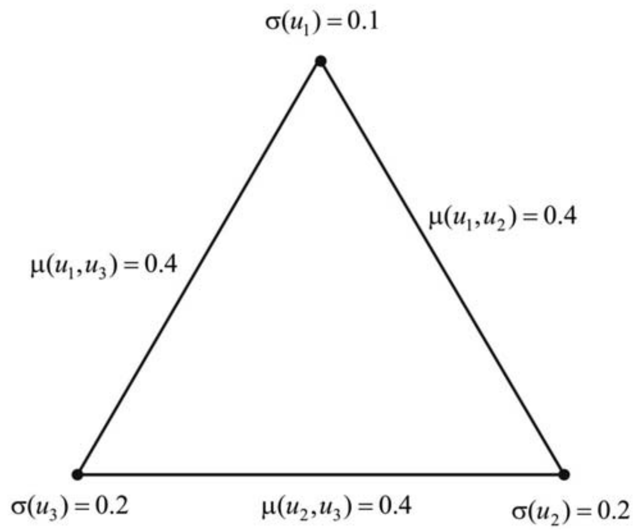


Fig.2. $K-$**degree regular fuzzy graph.**

$$d[\sigma(u_1)]=\mu(u_1,u_3)+\mu(u_1,u_2)=0.4+0.4=0.8$$
$$d[\sigma(u_2)]=\mu(u_1,u_2)+\mu(u_2,u_3)=0.4+0.4=0.8$$
$$d[\sigma(u_3)]=\mu(u_1,u_3)+\mu(u_2,u_3)=0.4+0.4=0.8$$

$\therefore$ Degree of the fuzzy regular graph is 0.8.

Definition 2.3

Let $G(\sigma,\mu)$ be a fuzzy graph with $G(V,E)$. The total degree of a vertex $u\in V$ is defined by,

$$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u)$$
$$= d_G(u) + \sigma(v)$$
$$\therefore td_G(u) = d_G(u) + \sigma(u)$$

If each vertex of G has the same total degree $K,$ then G is said to be a totally regular fuzzy graph of total degree K or a K-totally regular fuzzy graph.

Example 2.3

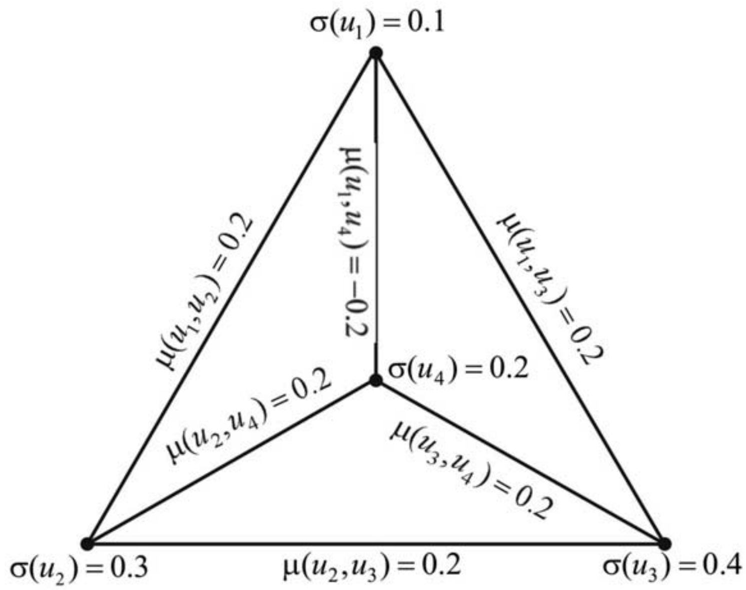


Fig.3. K-totally regular fuzzy graph.

Here,

$K-$degree of a regular fuzzy graph is,

$$d[\sigma(u_1)] = \mu(u_1,u_2) + \mu(u_1,u_4) + \mu(u_1,u_3) = 0.2 + 0.2 + 0.2 = 0.6$$
$$d[\sigma(u_2)] = \mu(u_1,u_2) + \mu(u_2,u_4) + \mu(u_2,u_3) = 0.2 + 0.2 + 0.2 = 0.6$$
$$d[\sigma(u_3)] = \mu(u_1,u_3) + \mu(u_3,u_4) + \mu(u_2,u_3) = 0.2 + 0.2 + 0.2 = 0.6$$
$$d[\sigma(u_4)] = \mu(u_1,u_4) + \mu(u_2,u_4) + \mu(u_3,u_4) = 0.2 + 0.2 + 0.2 = 0.6$$

$\therefore$ Totally regular fuzzy graph value is,

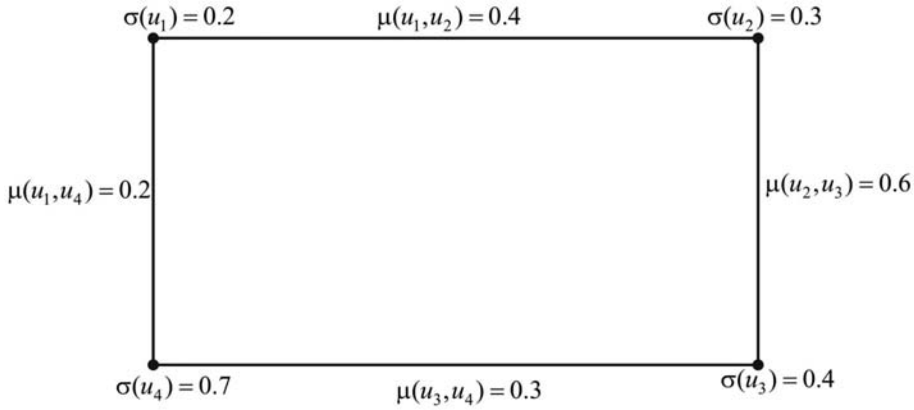
$$td(u_1) = d[\sigma(u_1)] + \sigma(u_1) = 0.6 + 0.1 = 0.7$$
$$td(u_2) = d[\sigma(u_2)] + \sigma(u_2) = 0.6 + 0.3 = 0.9$$
$$td(u_3) = d[\sigma(u_3)] + \sigma(u_3) = 0.6 + 0.4 = 1.0$$
$$td(u_4) = d[\sigma(u_4)] + \sigma(u_4) = 0.6 + 0.2 = 0.8$$

Definition 2.4

The order of a fuzzy graph G is, $O(G) = \sum_{u \in V} \sigma(u)$

The size of a fuzzy graph G is, $S(G) = \sum_{uv \in E} \mu(uv)$

Example 2.4



Here,

The order of a fuzzy graph G is,

$$O(G) = \sum_{u \in V} \sigma(u)$$
$$= \sigma(u_1) + \sigma(u_2) + \sigma(u_3) + \sigma(u_4)$$
$$= 0.2 + 0.3 + 0.4 + 0.7$$
$$\therefore O(G) = 1.6$$

The size of a fuzzy graph G is,

$$S(G) = \sum_{uv \in E} \mu(uv)$$
$$= \mu(u_1,u_2) + \mu(u_2,u_3) + \mu(u_3,u_4) + \mu(u_1,u_4)$$
$$= 0.4 + 0.6 + 0.3 + 0.2$$
$$\therefore S(G) = 1.5$$

Definition 2.5

A single fuzzy IF-THEN rule assumes the form, IF x is A, THEN y is B. where A and B are linguistic values defined by fuzzy sets on the ranges (universe of discourse) X and Y, respectively.

The IF-part of the rule "x is A" is called the antecedent or premise, while the THEN-part of the rule "y is B" is called the consequent or conclusion.

Example 2.5

IF there is heavy rain and strong winds, THEN there must be severe flood warning.

The vague terms heavy, strong and severe are fuzzy sets, qualifying the variables rain, wind and flood warning respectively.

3. Fuzzy Strongest Path with Fuzzy rule

Result 3.1

Let $G:(\sigma,\mu)$ is a fuzzy graph. In this fuzzy graph, in each and every path, if the membership value is minimum, then the paths are called fuzzy strength paths.

IF the fuzzy strength paths are equal, THEN it is called fuzzy strongest path.

Example 3.1

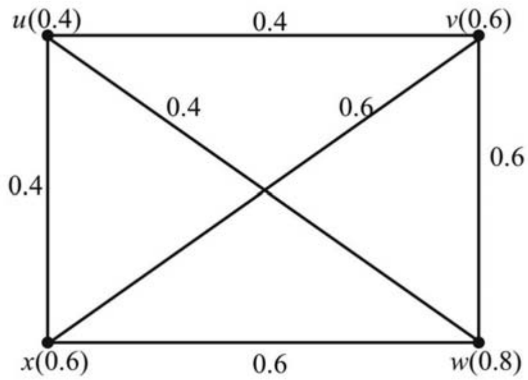


Fig.4. Fuzzy strongest path

The above graph shows five path between $u-x$.

S. No.	Path	$u-x$ paths	Membership values	Strength value
1.	P_1	$u-x$	0.4	0.4
2.	P_2	$u-v-x$	0.4, 0.6	0.4
3.	P_3	$u-w-x$	0.4, 0.6	0.4
4.	P_4	$u-v-w-x$	0.4, 0.6, 0.6	0.4
5.	P_5	$u-w-v-x$	0.4, 0.6, 0.4	0.4

From the table strength value of all the paths from $u-x$ is 0.4.

Therefore, the strength of each of these path is 0.4 and hence all paths are Fuzzy strongest paths.

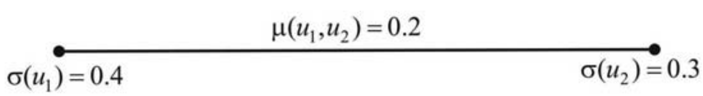
4. Fuzzy regular graph with fuzzy rule

Result 4.1

Any connected fuzzy graph with two vertices is a regular fuzzy graph.

Proof 4.1

Let $G:(\sigma,\mu)$ be a fuzzy graph with $G:(v,E)$.



$$d[\sigma(u_1)] = 0.2$$
$$d[\sigma(u_2)] = 0.2$$

Therefore, degree of the fuzzy regular graph is 0.2.

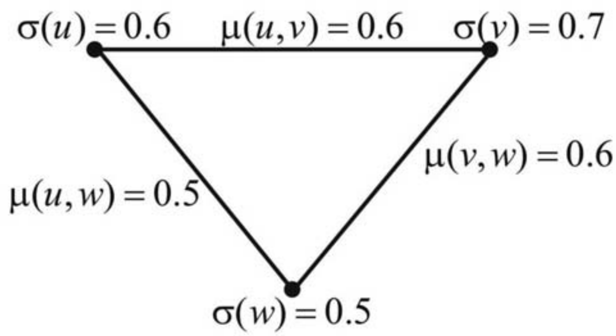
Hence it is proved.

Result 4.2

IF fuzzy graph $G(\sigma,\mu)$ is a complete fuzzy graph, THEN G is not a regular fuzzy graph.

Proof 4.2

Consider a fuzzy graph $G(\sigma,\mu)$ with $G(V,E)$.



In a fuzzy graph $G(\sigma,\mu)$,

$\sigma(u)=0.6,\ \sigma(v)=0.7,\ \ \sigma(w)=0.5$ and

$\mu(u,v)=0.6,\ \ \mu(v,w)=0.6,\ \ \mu(u,w)=0.5$

IF membership grades are assigned to edges, THEN the values will be:

$$d[\sigma(u)]=\mu(u,v)+\mu(u,w)=0.6+0.5=1.1$$
$$d[\sigma(v)]=\mu(u,v)+\mu(v,w)=0.6+0.6=1.2$$
$$d[\sigma(w)]=\mu(u,w)+\mu(u,v)=0.6+0.5=1.1$$
$$d[\sigma(u)]=d[\sigma(w)]=1.1\ \neq d[\sigma(v)]=1.2$$

$\therefore G$ is not a regular fuzzy graph.

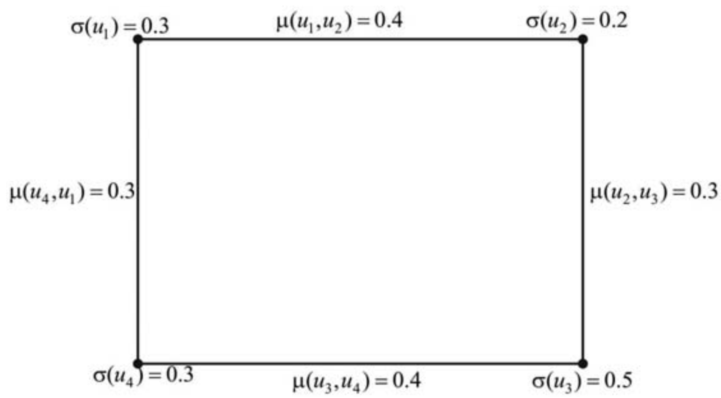
Hence it is proved.

Result 4.3

IF G is a regular fuzzy graph, THEN G is not a totally regular fuzzy graph.

Proof 4.3

Consider a fuzzy graph $G:(\sigma,\mu)$ with $G:(V,E)$.



Here,

$\sigma(u_1)=0.3,\ \ \sigma(u_2)=0.2,\ \ \sigma(u_3)=0.5,\ \ \sigma(u_4)=0.3$

and $\mu(u_1,u_2)=0.4,\ \mu(u_2,u_3)=0.3,\ \ \mu(u_3,u_4)=0.4,\ \mu(u_4,u_1)=0.3.$

IF the membership grades of edges which are incident on any degree of a vertex $\sigma(v_i)$ is added, THEN the sum of corresponding membership values of vertices vary.

Then,

$$d[\sigma(u_1)] = \mu(u_1,u_2) + \mu(u_1,u_4) = 0.4 + 0.3 = 0.7$$
$$d[\sigma(u_2)] = \mu(u_1,u_2) + \mu(u_2,u_3) = 0.4 + 0.3 = 0.7$$
$$d[\sigma(u_3)] = \mu(u_3,u_4) + \mu(u_2,u_3) = 0.4 + 0.3 = 0.7$$
$$d[\sigma(u_4)] = \mu(u_1,u_4) + \mu(u_3,u_4) = 0.3 + 0.4 = 0.7$$

$\therefore G$ is a regular fuzzy graph.

IF membership grades are assigned to edges, THEN the total degree of a vertex is,

$$td_G(u) = d_G(u) + \sigma(u)$$
$$\therefore td[\sigma(u_1)] = 0.7 + 0.3 = 1.0$$
$$td[\sigma(u_2)] = 0.7 + 0.2 = 0.9$$
$$td[\sigma(u_3)] = 0.7 + 0.5 = 1.2$$
$$td[\sigma(u_4)] = 0.7 + 0.3 = 1.0$$
$$td[\sigma(u_1)] \neq td[\sigma(u_2)]$$

$\therefore G$ is not a totally regular fuzzy graph.

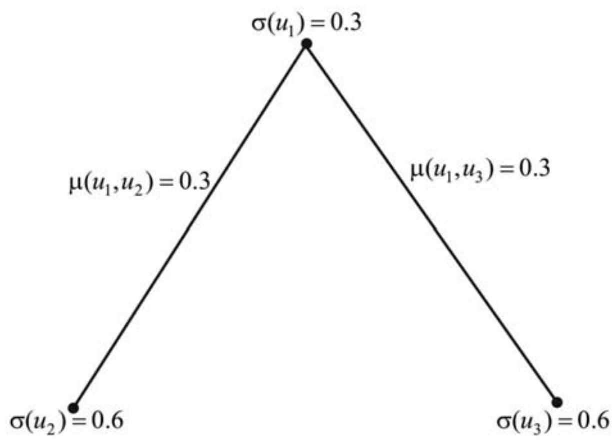
Hence the theorem is proved.

Result 4.4

IF G is a totally regular fuzzy graph, THEN G is a regular fuzzy graph.

Proof 4.4

Consider a fuzzy graph $G(\sigma,\mu)$ with $G(V,E)$.



In this fuzzy graph,

$\sigma(u_1) = 0.3,\ \sigma(u_2) = 0.6,\ \sigma(u_3) = 0.6$ and $\mu(u_1,u_2) = 0.4,\ \mu(u_1,u_3) = 0.3.$

IF membership grades are assigned to edges, THEN the totally regular fuzzy graph value is,

$$td(u_1) = 0.6 + 0.3 = 0.9$$
$$td(u_2) = 0.3 + 0.6 = 0.9$$
$$td(u_3) = 0.3 + 0.6 = 0.9$$

$\therefore G$ is a totally regular fuzzy graph.

$$d(u_1) = 0.6, d(u_2) = 0.3, d(u_3) = 0.3$$

$$d(u_1) \neq d(u_2) = d(u_3).$$

$\therefore G$ is not a regular fuzzy graph.

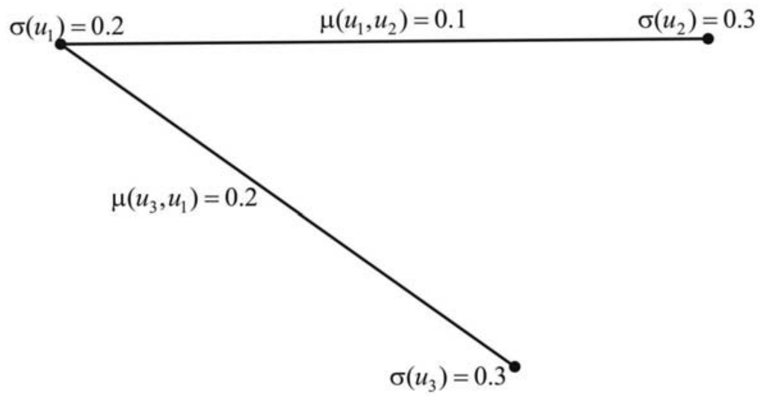
Hence the theorem.

Result 4.5

IF G is not a regular fuzzy graph, THEN G is not a totally regular fuzzy graph.

Proof 4.5

Consider a fuzzy graph $G(\sigma, \mu)$ with $G(V, E)$.



In this fuzzy graph,

$$\sigma(u_1) = 0.2, \quad \sigma(u_2) = 0.3, \quad \sigma(u_3) = 0.3 \text{ and } \mu(u_1, u_2) = 0.1, \quad \mu(u_3, u_1) = 0.2.$$

Here,

$$d(u_1) = 0.3, \quad d(u_2) = 0.1, \quad d(u_3) = 0.2$$

$$d(u_1) \neq d(u_2) \neq d(u_3)$$

$\therefore G$ is not a regular fuzzy graph.

$$td(u_1) = 0.5, \quad td(u_2) = 0.4, \quad td(u_3) = 0.5$$

$$td(u_1) = td(u_3) = 0.5 \neq td(u_2) = 0.4$$

$\therefore G$ is not a totally regular fuzzy graph.

Hence the theorem.

Theorem 4.1

The size of a $K-$regular fuzzy graph $G : (\sigma, \mu)$ on $G^* : (V, E)$ is $\frac{PK}{2}$, where $P = |V|$.

Proof 4.1

The size of $G(\sigma, \mu), S(G) = \sum_{uv \in E} \mu(uv)$

since, G is a $K-$regular fuzzy graph. Then $d_G(v) = K$ for all $v \in V$.

$$\sum_{v \in V} d_G(v) = 2 \sum_{uv \in V} \mu(uv)$$

$$= 2S(G)$$

$$\therefore 2S(G) = \sum_{v \in V} d_G(v)$$

$$= \sum_{v \in V} K$$

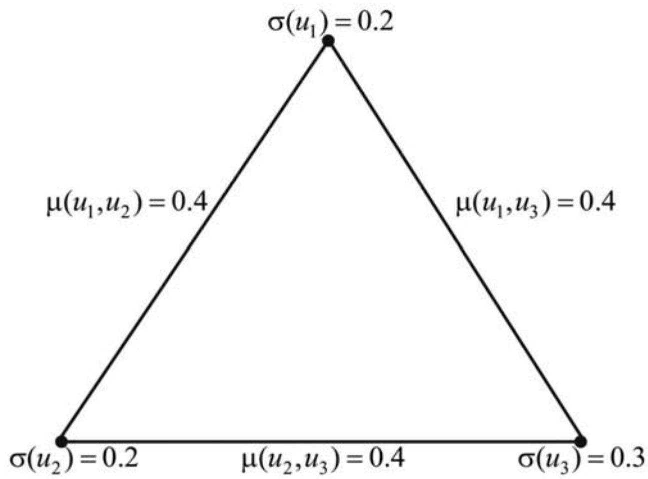
$$= PK$$

$$\Rightarrow 2S(G) = PK$$

$$\therefore S(G) = \frac{PK}{2}$$

(i.e.) The size of a K -regular fuzzy graph $= \dfrac{PK}{2}$.

Example



In this fuzzy graph,

$d(v_i) = 0.8$ for $i = 1, 2, 3.$

i.e., $d(v_1) = 0.8,\quad d(v_2) = 0.8,\quad d(v_3) = 0.8,$ and $P = |V| = 3.$

$G(\sigma, \mu) = 0.8$ is the value of a regular fuzzy graph.

i.e., $K = 0.8$

$$\Rightarrow S(G) = \frac{PK}{2}$$

$$= \frac{3(0.8)}{2}$$

$$= 1.2$$

$\therefore$ Size of a k-regular fuzzy graph = 1.2.

Theorem 4.2

IF $G : (\sigma, \mu)$ is a k-totally regular fuzzy graph, THEN $2S(G) + O(G) = pq,$ where $P = |V|$.

Proof 4.2

Since, G is a k-totally regular fuzzy graph.

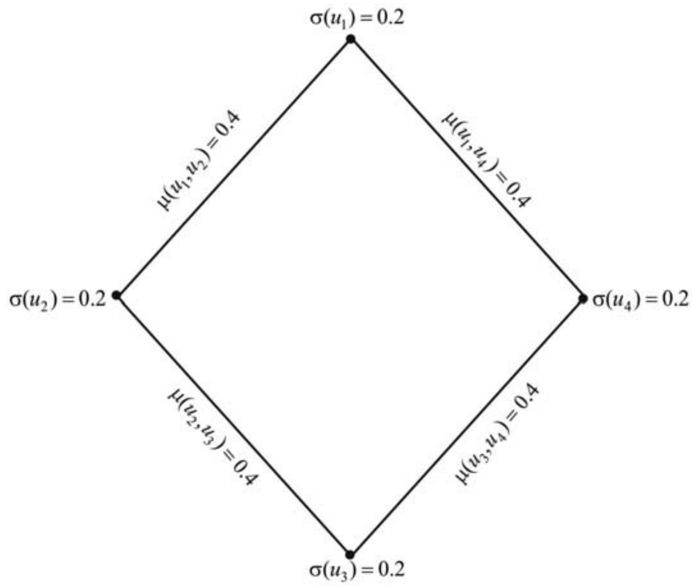
$$q = td(v) = d_G(v) + \sigma(u) \text{ for all } v \in V$$

$$\sum_{v \in V} q = \sum_{v \in V} d_G(v) + \sum_{u \in V} \sigma(u)$$

$$pq = 2S(G) + O(G)$$

$$\therefore 2S(G) + O(G) = pq.$$

Example



In this fuzzy graph,

$$d(v_i) = 0.8 \text{ for } i = 1, 2, 3, 4.$$

i.e., $d(v_1) = 0.8, \quad d(v_2) = 0.8, \quad d(v_3) = 0.8, \quad d(v_4) = 0.8$

and $P = |V| = 4, q = 1.0, k = 0.8, O(G) = 0.8$

$$\Rightarrow S(G) = \frac{PK}{2}$$

$$= \frac{4(0.8)}{2} = 1.6$$

$$\therefore S(G) = 1.6.$$

$$2S(G) + O(G) = pq$$

$$2(1.6) + 0.8 = 4(1)$$

$$3.2 + 0.8 = 4$$

$$4 = 4$$

Hence the theorem is proved.

Theorem 4.3

IF G is a k-regular fuzzy graph and *r*-totally regular fuzzy graph, THEN $O(G) = P(r - k)$.

Proof 4.3

Since, the size of a k-regular fuzzy graph is $\dfrac{PK}{2}$.

i.e., $S(G) = \dfrac{PK}{2}$

$\Rightarrow 2S(G) = PK$

and $2S(G) + O(G) = \mathrm{Pr}$

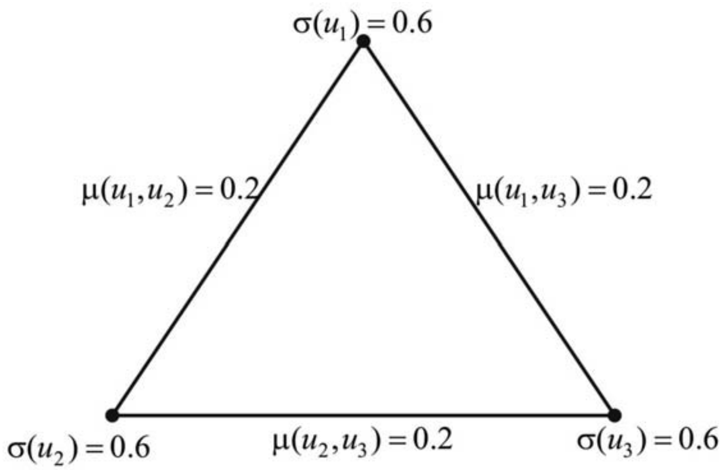
So, $O(G) = \mathrm{Pr} - 2S(G)$

$= \mathrm{Pr} - Pk$

$= P(r-k)$

$\therefore O(G) = P(r-k).$

Example



In this fuzzy graph,

$d(u_1) = 0.4, \quad d(u_2) = 0.4, \quad d(u_3) = 0.4$

$P = |V| = 3, k = 0.4, r = 1$

$S(G) = \dfrac{3(0.4)}{2} = 0.6$

$\therefore S(G) = 0.6, O(G) = 1.8$

$O(G) = \mathrm{Pr} - 2S(G)$

$O(G) = P(r-k)$

$1.8 = 3(1-0.4)$

$1.8 = 3(0.6)$

$1.8 = 1.8$

Hence the theorem.

Conclusion

In this formulated work, IF-THEN rule evolves interesting results in new parameters of regular fuzzy graph and totally regular fuzzy graph. The fuzzy strongest path by applying different approaches is arrived.

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THRESHOLD EFFECT ON A FUZZY QUEUE MODEL WITH BATCH ARRIVAL

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ABSTRACT :
The objective of the paper is to model the performance measures of a simple queue system with batch arrival. The arrival and service rates both are considered to be in fuzzy parameters. The mathematical description of the model has been presented for two and three parameters The various queue characteristics are extracted from the fuzzy state probability function governing the behaviour of the system. α – cut and Zadeh extension principle has been applied to transfer the trapezoidal queue system into crisp queue measures. The batch size follows geometrical distribution. The evaluation of thresholding is applied to eliminate small membership values to avoid unnecessary computations.

Key Words: ***Fuzzy variable, fuzzy numbers, threshold effect, queue characteristics.***

1. INTRODUCTION:

Queueing theory already has a long history and has been used to solve many practical problems arising in manufacturing, communication, transportation and other fields. The theory uses mathematical tools to predict the behaviour of a system. Prediction deals with the behaviour to have n customers in the system, stochastic processes describing the arrival and service of customers.

In the literature, customers inter-arrival times and customers service times are required to follow certain probability distributions with fixed parameters. However, in many real world applications, the parameters distribution may only be characterized subjectively i.e. the arrivals & services are typically described in any language. In other words, these systems of parameters are both possibilistic & probabilistic. Fuzzy queues are potentially much more useful and realistic than the commonly used crisp queues.

Different people judge and evaluate reality differently. Fuzzy logic, like probability theory, deals with uncertainty while unlike probability theory it deals with degree of occurrence of an event, whereas the later deals only with occurrence of event. There are queuing systems whose arrival and service rates can be described by fuzzy linguistic variables. Fuzzy control of queueing system is an application of fuzzy logic theory to control of queues. It is a combination of artificial intelligence, Operation Research and optimal control. Various Researchers applied fuzzy logic concepts in queueing system under different situations. Li & Lee (1989) investigated the analytical results for two special queues $M/F/1/\infty$ and $FM/FM/1/\infty$ where F denotes fuzzy time & FM denotes fuzzified exponential distribution. Negi and Li (1992) proposed a procedure using α-cut. T.P. Singh et.al.(2011) studied fuzzy network queue model in blocking situation. Further T.P. Singh, Kusum & Deepak Gupta (2012) investigated the fuzzy queue model with bulk arrival for interdependent communication system and later on this work was extended by Arti & T. P. Singh (2013) for voice packetized system. Pourdarvish & Shokry (2009) developed a method that is able to provide fuzzy performance measures for service queues in which exponential arrival rate and service rate both are considered in fuzzy. In this paper we extend the study made by Pourdarwish & Shokry (2009) taken in consideration the parameters in trapezoidal fuzzy variables with α-cut. We have analysed the model with threshold effect. The evaluation of threshold eliminates small membership values to avoid unnecessary computations. The fuzzified exponential distribution has been used in this study.

2.1 TRAPEZOIDAL FUZZY NUMBER:

$$\widetilde{A} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1 & a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise} \end{cases}$$

2.2 TRAPEZOIDAL FUZZY NUMBER OPERATION:

Let $\widetilde{A} = (a_1,a_2,a_3, a_4)$ and $\tilde{B} = (b_1,b_2,b_3, b_4)$ be two trapezoidal fuzzy number, then the arithmetic operation on $\widetilde{A}$ and $\tilde{B}$ are given as follows :

Addition $\widetilde{A} + \tilde{B} = [a_1+b_1, a_2+b_2, a_3+b_3, a_4+ b_4]$

Subtraction $\widetilde{A} - \tilde{B} = [a_1-b_4, a_2-b_3, a_3-b_2, a_4- b_1]$

Multiplication $\widetilde{A} * \tilde{B} = [a_1b_1, a_2b_2, a_3b_3, a_4b_4]$

Division $\widetilde{A} / \tilde{B} = [a_1/b_4, a_2/b_3, a_3/b_2, a_4/ b_1]$

Provided $\widetilde{A}$ *and* $\tilde{B}$ are all non-zero positive numbers.

2.3 DEFUZZIFICATION OF TRAPEZOIDAL FUZZY NUMBER:

If $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then its associated crisp number is given by Chen's etal. graded mean formula as follows:

$$A = \frac{a_1+2a_2+2a_3+a_4}{6}$$

3. MODEL DESCRIPTION:

Assume a single service channel with Poisson process of batch size X and fuzzified exponential inter-arrival service model of a queue system with infinite capacity and service discipline FIFO basis i.e. in notation form the model is of $M^{[x]}/M/1/\infty$/FIFO type. Let λ_x be arrival rate of Poisson Process of batch size X and c_x be assigned probability. The number of customers in any arrival is a random variable X where $c_x = \frac{\lambda_x}{\lambda}$, where λ is composite arrival rate of all the batches of size X.

i.e. $\lambda = \sum_{i=1}^{\infty} \lambda_i$

The objective is to model the queueing system first with crisp parameters, extend it to fuzzified parameters and further to evaluate threshold effect .

4. BALANCE EQUATIONS AND SOLUTION OF THE MODEL:

Define $P_n(t)$ probability that there are n units in the system at any instant t. The differential difference equations governing the model can be derived easily using general Birth Death arguments.

$$P_n(t + \Delta t) = (1 - \lambda\Delta t)(1 - \mu\Delta t)P_n(t) + \mu\Delta t P_{n+1}(t) + \lambda\Delta t \sum_{k=1}^{n} P_{n-k}(t)c_k \qquad n\geq 1$$

$$P_0(t + \Delta t) = (1 - \lambda\Delta t)P_0(t) + \mu\Delta t P_{n+1}(t) \qquad n=0$$

on simplification, we get

$P_n'(t) = -(\lambda+\mu)P_n(t) + \mu P_{n+1}(t) + \lambda \sum_{k=1}^{n} P_{n-k}(t)c_k$, n≥1

$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$ n=0

IN STEADY STATE:

The steady state condition is reached when the behaviour of the system becomes independent of the time. When t→∞ the steady state equations are

$0 = -(\lambda+\mu) P_n + \mu P_{n+1} + \lambda \sum_{k=1}^{n} P_{n-k} c_k$ n≥1 (1)

$0 = -\lambda P_0 + \mu P_1$ n=0 (2)

in order to solve the system of equations (1) & (2) we apply generating function technique (g. f. t.), defining generating function as

$$\left.\begin{aligned} P(z) &= \sum_{n=0}^{\infty} P_n z^n && (\| z \| \leq 1) \\ C(z) &= \sum_{n=0}^{\infty} c_n z^n = \sum_{n=1}^{\infty} c_n z^n && (\| z \| \leq 1) \end{aligned}\right\} \quad (3)$$

Where P(z) & C(z) are generating functions of steady state probability P_n and batch size c_n respectively.

Multiplying (1) with z^n & summing over n=0 to ∞, after simplification, we get

$0 = -\lambda \sum_{n=0}^{\infty} P_n z^n - \mu \sum_{n=1}^{\infty} P_n z^n + \frac{\mu}{z} \sum_{n=1}^{\infty} P_n z^n + \lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} P_{n-k} c_k z^n$ (4)

Here $\sum_{k=1}^{n} P_{n-k} c_k z^n$ is the probability function for the sum of steady state system size and batch size is simply a convolution formula for discrete random variables

Hence, $\sum_{n=1}^{\infty} \sum_{k=1}^{n} P_{n-k} c_k z^n = \sum_{k=1}^{\infty} c_k z^k \sum_{n=k}^{\infty} P_{n-k} z^{n-k} = C(z)P(z)$ (5)

using (3) & (5) in (4),we have

$-\lambda P(z) - \mu [P(z) - P_0] + \frac{\mu}{z} [P(z) - P0] + \lambda C(z) P(z) = 0$ (6)

solving (6),we get

$P(z) = \frac{\mu(1-z) P_0}{\mu(1-z) - \lambda z [1 - C(z)]}$ $(\| z \| \leq 1)$ (7)

Using condition P(1) =1,we can find P_0

$P(1) = \lim_{z\to 1} \frac{\mu (1-z) P_0}{\mu(1-z) - \lambda z[1 - C(z)]}$ $\frac{0}{0}$ form

Applying L' Hospital rule, we get

$$1 = \lim_{z\to 1} \frac{-\mu P_0}{-\mu - \lambda[1 - C(z)] + \lambda z \frac{d}{dz} C(z)}$$

$1 = \frac{\mu P_0}{-\mu + \lambda E(x)}$

$P_0 = 1 - \frac{\lambda}{\mu} E(x)$

$P_0 = 1 - \rho$, where $\rho = \frac{\lambda}{\mu} E(x)$ with $\rho < 1$

Assume that the batch size c_x is geometrically distributed:

i.e. c_x= (1-α)α^{x-1} ; 0 < $\alpha < 1$ (α is a parameter)

C (z)= $(1-\alpha)\sum_{n=0}^{\infty}\alpha^{n-1} z^n$, ($\| z \| \leq 1$) from (3)

C (z) =$\frac{(1-\alpha)z}{1-\alpha z}$ (8)

From (7), we get

P (z) =$\frac{\mu(1-z) P_0}{\mu(1-z)-\lambda z[1-\frac{z(1-\alpha)}{(1-\alpha z)}]}$

(9)

Using P (1) = 1 in (9), we get

$$P_0 = 1-\frac{\lambda}{\mu(1-\alpha)}=1-\frac{\lambda}{\mu}E(x)=1-\rho$$

Using (8) in (7) we have

P (z) =$\frac{\mu(1-z)P_0}{\mu(1-z)-\lambda z\,[1-\frac{z(1-\alpha)}{(1-\alpha z)}]}$

P (z) = $\frac{\mu\,(1-\alpha z)P_0}{\mu(1-\alpha z)-\lambda z}$

P (z) = $\frac{\mu(1-\alpha z)(1-\rho)}{\mu(1-\alpha z)-\lambda z}$

using $\rho = \frac{\lambda}{\mu(1-\alpha)}$,we get

P (z) = $\frac{(1-\alpha z)(1-\rho)}{(1-\alpha z)-\rho(1-\alpha)z}$

P (z) = $\frac{(1-\alpha z)(1-\rho)}{1-\{\alpha+(1-\alpha)\rho\}z}$

P (z) =$(1-\rho)\,[\frac{1}{1-\{\alpha+(1-\alpha)\rho\}z}-\frac{\alpha z}{1-\{\alpha+(1-\alpha)\rho\}z}]$

Now we apply the formula for the sum of a geometric series,

i.e. $\frac{1}{1-r}=\sum_{n=0}^{\infty} r^n$

P (z) =$(1-\rho)\,[\sum_{n=0}^{\infty}[\{\alpha+(1-\alpha)\rho\}z]^n-\sum_{n=0}^{\infty}\alpha\{\alpha+(1-\alpha)\rho\}^n z^{n+1}]$

$since P_n$ is the coefficient of z^n in P(z)

Hence

$P_n = (1-\rho)\,[\{\alpha+(1-\alpha)\rho\}^n-\alpha\{\alpha+(1-\alpha)\rho\}^{n-1}]$

$P_n = (1-\rho)\{\alpha+(1-\alpha)\rho\}^{n-1}(1-\alpha)\rho$ $n \geq 0$

4.1 MODEL formulation FOR TWO PARAMETERS λ_1 & λ_2

From (3)

$C(z) = c_1 z + c_2 z^2$

C (z)= $\frac{\lambda_1 z+\lambda_2 z^2}{\lambda}$ $\quad (c_x = \frac{\lambda_x}{\lambda})$

Using in (7), we get

P (z) =$\frac{\mu(1-z)(1-\rho)}{\mu(1-z)-\lambda z\,[1-C\,(z)]}$

P (z) =$\frac{\mu(1-z)(1-\rho)}{\mu(1-z)-\lambda z\,[1-\{\frac{\lambda_1 z+\lambda_2 z^2}{\lambda}\}]}$

Differentiating with respect to z, we get

$$P'(z) = \mu(1-\rho)\left[\frac{-\mu(1-z)+z(\lambda-\lambda_1 z-\lambda_2 z^2)-(1-z)(-\mu-\lambda+2\lambda_1 z+3\lambda_2 z^2}{\{\mu(1-z)-z(\lambda-\lambda_1 z-\lambda_2 z^2)\}^2}\right] \tag{10}$$

Using L' Hospital's rule twice, we get

$$L = P'(1) = \lim_{z\to 1} P'(z) = \frac{\mu(1-\rho)(\lambda_1+3\lambda_2)}{\{\mu-\lambda_1-2\lambda_2\}^2} \tag{11}$$

With $\rho = \frac{(\lambda_1+2\lambda_2)}{\mu}$

If we define $\frac{\lambda_1}{\mu}$ as ρ_1 and $\frac{\lambda_2}{\mu}$ as ρ_2, then Expected batch size E(x), is $\frac{(\lambda_1+2\lambda_2)}{\lambda}$

and ρ may be written as $\rho = \rho_1 + 2\rho_2$

Divide N^r and D^r by $\mu^2(1-\rho)$ in equation (11), we get

$$L = \left[\frac{\rho_1+3\rho_2}{1-\rho_1-2\rho_2}\right] \tag{12}$$

$\frac{\partial L}{\partial \rho_1} = \frac{1+\rho_2}{\{1-\rho_1-2\rho_2\}^2} > 0$ and $\frac{\partial L}{\partial \rho_2} = \frac{3-\rho_1}{\{1-\rho_1-\rho_2\}^2} > 0$

This shows system queue length is minimum at this stage.

4.2 MODEL Formulation FOR THREE PARAMETERS λ_1, λ_2 & λ_3.

From (3)

$C(z) = c_1 z + c_2 z^2 + c_3 z^3$

C (z)= $\frac{\lambda_1 z+\lambda_2 z^2+\lambda_3 z^3}{\lambda}$ $\quad (c_x = \frac{\lambda_x}{\lambda})$

Using in (7), we get

P (z) =$\frac{\mu(1-z)(1-\rho)}{\mu(1-z)-\lambda z\,[1-C\,(z)]}$

P (z) =$\frac{\mu(1-z)(1-\rho)}{\mu(1-z)-\lambda z\,[1-\{\frac{\lambda_1 z+\lambda_2 z^2+\lambda_3 z^3}{\lambda}\}]}$

Differentiating with respect to z, we get

$$P'(z) = \mu(1-\rho)\left[\frac{-\mu(1-z)+z(\lambda-\lambda_1 z-\lambda_2 z^2-\lambda_3 z^3)-(1-z)(-\mu-\lambda+2\lambda_1 z+3\lambda_2 z^2+4\lambda_3 z^3}{\{\mu(1-z)-z(\lambda-\lambda_1 z-\lambda_2 z^2-\lambda_3 z^3)\}^2}\right] \tag{13}$$

Using L' Hospital's rule twice, we get

$$L = P'(1) = \lim_{z\to 1} P'(z) = \frac{\mu(1-\rho)(\lambda_1+3\lambda_2+6\lambda_3)}{\{\mu-\lambda_1-2\lambda_2-3\lambda_3\}^2} \tag{14}$$

With $\rho = \frac{(\lambda_1+2\lambda_2+3\lambda_3)}{\mu}$

If we define $\frac{\lambda_1}{\mu}$ as ρ_1 and $\frac{\lambda_2}{\mu}$ as $\rho_2$and $\frac{\lambda_3}{\mu}$ as $\rho_3$then Expected batch size E(x), is $\frac{(\lambda_1+2\lambda_2+3\lambda_3)}{\lambda}$

and ρ may be written as $\rho = \rho_1 + 2\rho_2 + 3\rho_3$

Divide N[r] and D[r] by $\mu^2(1-\rho)$ in equation (14), we get

$$L = \left[\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right] \quad (15)$$

$\frac{\partial L}{\partial \rho_1} = \frac{1+\rho_2+3\rho_3}{\{1-\rho_1-2\rho_2-3\rho_3\}^2} > 0$, $\frac{\partial L}{\partial \rho_2} = \frac{3-\rho_1-3\rho_3}{\{1-\rho_1-2\rho_2-3\rho_3\}^2} > 0$ and $\frac{\partial L}{\partial \rho_3} = \frac{6-3\rho_1-3\rho_2}{\{1-\rho_1-2\rho_2-3\rho_3\}^2} > 0$

This shows system queue length is minimum at this stage.

5. EVALUATION WITH THRESHOLDING:

Threshold is a mechanism to control the level of uncertainty in a fuzzy system and to discard low possibilities in the antecedent domain and to eliminate the corresponding implication results in the consequent domain. It is easier to estimate the effect of threshold on antecedents than on consequents because consequent membership functions are subject to aggregation and defuzzification before the effects can be observed.

The simplest form of threshold is the α-cut that limits the spread of membership function on the universe of discourse. If the level of threshold is above the intersection points then the membership function do not overlap. The effect of threshold creates dead zone. The α-cut threshold creates a sudden drop in the evaluation of membership function that is not desired.

$$\mu(x) = 0 \ if \ \mu(x) \leq \alpha$$

Now we suppose that $\lambda \ and \ \mu$ are approximately known and can be represented by fuzzy numbers $\tilde{\lambda}$ and $\tilde{\mu}$ can be represented respectively

$$\tilde{\lambda}(\beta) = [\tilde{\lambda}_1(\beta), \tilde{\lambda}_2(\beta)]$$

$$\tilde{\mu}(\beta) = [\tilde{\mu}_1(\beta), \tilde{\mu}_2(\beta)]$$

Considering batch size distributed geometrically

$\tilde{P}_n = (1-\rho)\{\alpha + (1-\alpha)\rho\}^{n-1}(1-\alpha)\rho$ //S

Where S= $\{\rho \in \tilde{\rho}(\beta)\}$

$$\tilde{L}(\beta) = [\tilde{L}_1(\beta), \tilde{L}_2(\beta)]$$

By using Little's formulae, we may find other effective measures in fuzzy parameters.

Then queue length in system

$$\tilde{L}(\alpha) = \left[\frac{\tilde{\rho}_{(1,L)}+3\tilde{\rho}_{(2,L)}}{1-\tilde{\rho}_{(1,L)}-2\tilde{\rho}_{(2,L)}}; \frac{\tilde{\rho}_{(1,U)}+3\tilde{\rho}_{(2,U)}}{1-\tilde{\rho}_{(1,U)}-2\tilde{\rho}_{(2,U)}}\right] \quad (13)$$

Using Little's Formulae

$$\widetilde{W}_q(\alpha) = \left[\frac{1}{\tilde{\mu}_U}\left(\frac{\tilde{\rho}_{(1,L)}+3\tilde{\rho}_{(2,L)}}{1-\tilde{\rho}_{(1,L)}-2\tilde{\rho}_{(2,L)}}\right); \frac{1}{\tilde{\mu}_L}\left(\frac{\tilde{\rho}_{(1,U)}+3\tilde{\rho}_{(2,U)}}{1-\tilde{\rho}_{(1,U)}-2\tilde{\rho}_{(2,U)}}\right)\right]$$

$$also\ \widetilde{W}_s(\alpha) = \left[\frac{1}{\tilde{\mu}_U}+\left(\frac{\tilde{\rho}_{(1,L)}+3\tilde{\rho}_{(2,L)}}{1-\tilde{\rho}_{(1,L)}-2\tilde{\rho}_{(2,L)}}\right); \frac{1}{\tilde{\mu}_L}+\left(\frac{\tilde{\rho}_{(1,U)}+3\tilde{\rho}_{(2,U)}}{1-\tilde{\rho}_{(1,U)}-2\tilde{\rho}_{(2,U)}}\right)\right]$$

NUMERICAL ILLUSTRATION:

Consider$\tilde{\lambda}_1 = (1,2,3,4), \tilde{\lambda}_2 = (2,3,4,5), \tilde{\mu} = (15,16,17,18)$ such that

$$\tilde{\rho} = \frac{(\tilde{\lambda}_1 + 2\tilde{\lambda}_2)}{\tilde{\mu}} < 1$$

Applying α-cut using 2.1

$$\tilde{\lambda}_1 = (\alpha + 1, 4 - \alpha)$$
$$\tilde{\lambda}_2 = (\alpha + 2, 5 - \alpha)$$
$$\tilde{\mu} = (\alpha + 15, 18 - \alpha)$$
$$\tilde{\rho}_1 = \left(\frac{\alpha + 1}{18 - \alpha}, \frac{4 - \alpha}{15 + \alpha}\right)$$
$$\tilde{\rho}_2 = \left(\frac{\alpha + 2}{18 - \alpha}, \frac{5 - \alpha}{15 + \alpha}\right)$$

Substituting these values in (12), we get

$$\tilde{L}(\alpha) = \left(\frac{7 + 4\alpha}{13 - 4\alpha}, \frac{19 - 4\alpha}{1 + 4\alpha}\right)$$
$$\widetilde{W}_q(\alpha) = \left[\frac{1}{(18 - \alpha)}\left(\frac{7 + 4\alpha}{13 - 4\alpha}\right); \frac{1}{(15 + \alpha)}\left(\frac{19 - 4\alpha}{1 + 4\alpha}\right)\right]$$
$$\widetilde{W}_s(\alpha) = \left[\frac{1}{(18 - \alpha)} + \left(\frac{7 + 4\alpha}{13 - 4\alpha}\right); \frac{1}{(15 + \alpha)} + \left(\frac{19 - 4\alpha}{1 + 4\alpha}\right)\right]$$

For different values of α, the fuzzy queue length of the system and waiting time in system can be depicted in the following table as:

Table:1

α	$\tilde{L}(\alpha)$	$\widetilde{W}_S(\alpha)$
0	(.538;19)	(.593;19.06)
.2	(.639;10.11)	(.695;10.175)
.4	(.754;6.69)	(.81;6.754)
.5	(.81;5.66)	(.86;5.72)
.6	(.88;4.88)	(.937;4.94)
.8	(1.04;3.76)	(1.09;3.82)
.9	(1.12;3.34)	(1.18;3.40)
1	(1.22;3)	(1.27;3.06)

From the table 1, we have

$\tilde{L} = (.538,1.22,3,19) and\ \widetilde{W}_s = (.593,1.27,3.06,19.06)$

Fuzzy queue length and waiting time in system can be converted into **linguistic variables such as**

very short, short, average, large. It is because in many real world applications, the parameters may not be estimated precisely e.g. arrival and service pattern of the customers are more suitably described in linguistic terms such as frequently arrival or fast, slow or moderate services rather than probability distribution. Both inter arrival times and service time are more possibilistic than probabilistic in many situations.

Effect of an α-cut threshold on the membership function shown in figures:

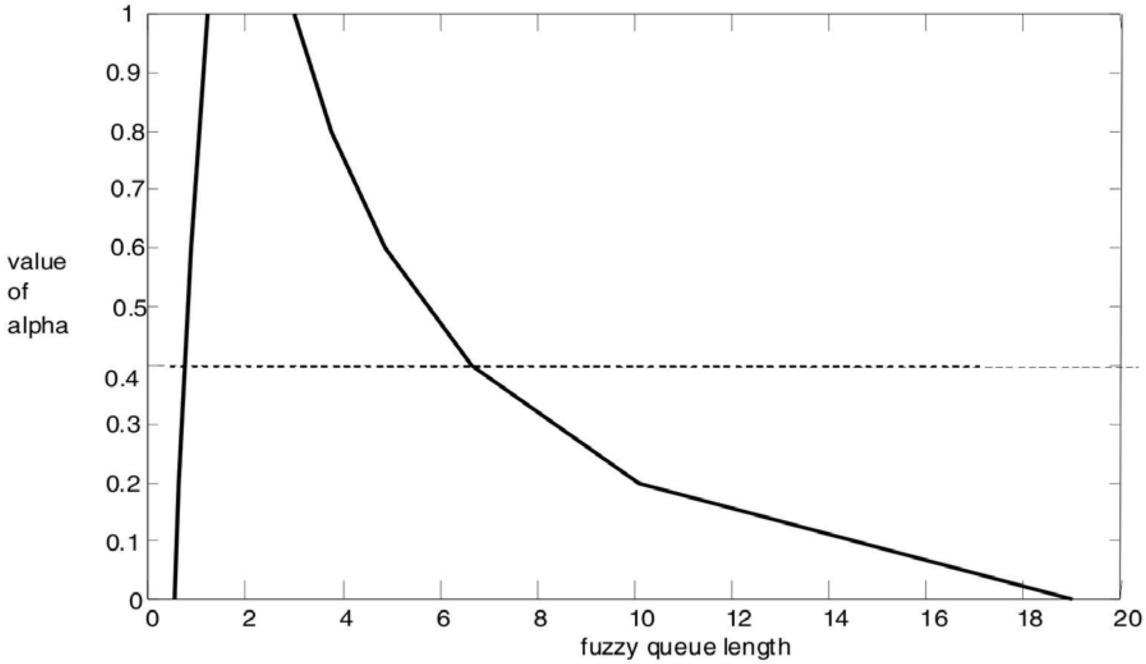

Figure 1 : Alpha cut on fuzzy queue length

Change in the membership functions due to threshold

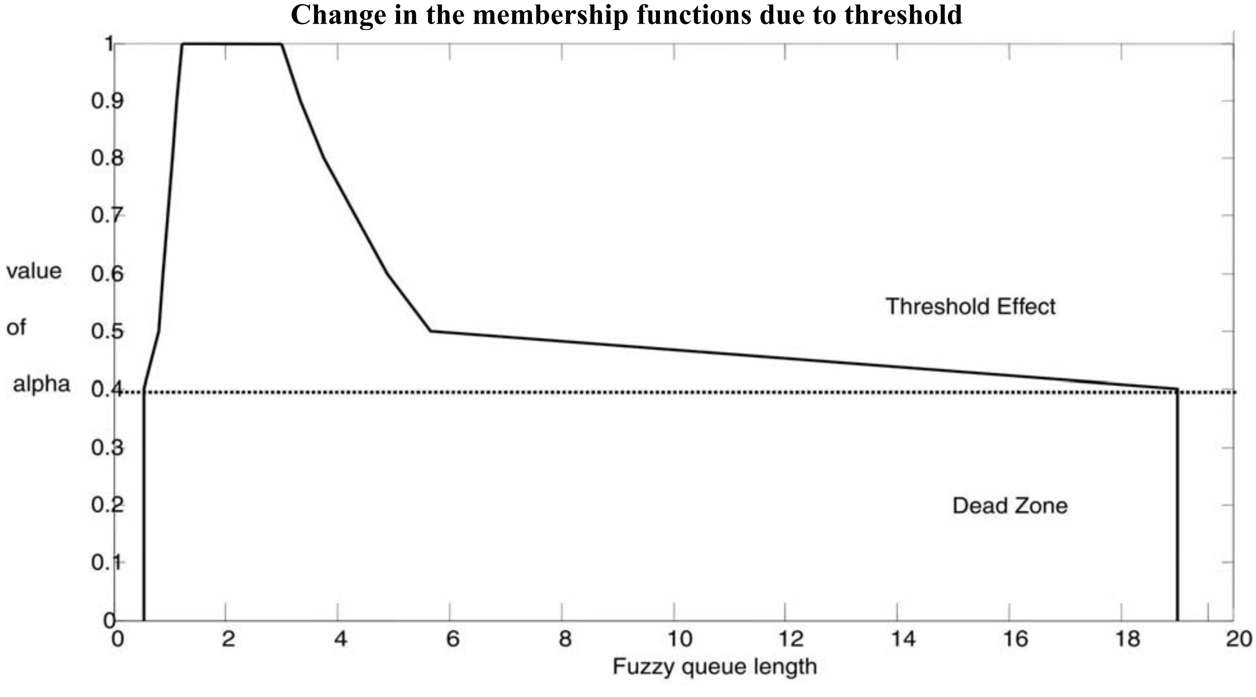

Figure 2: Change in membership function due to threshold

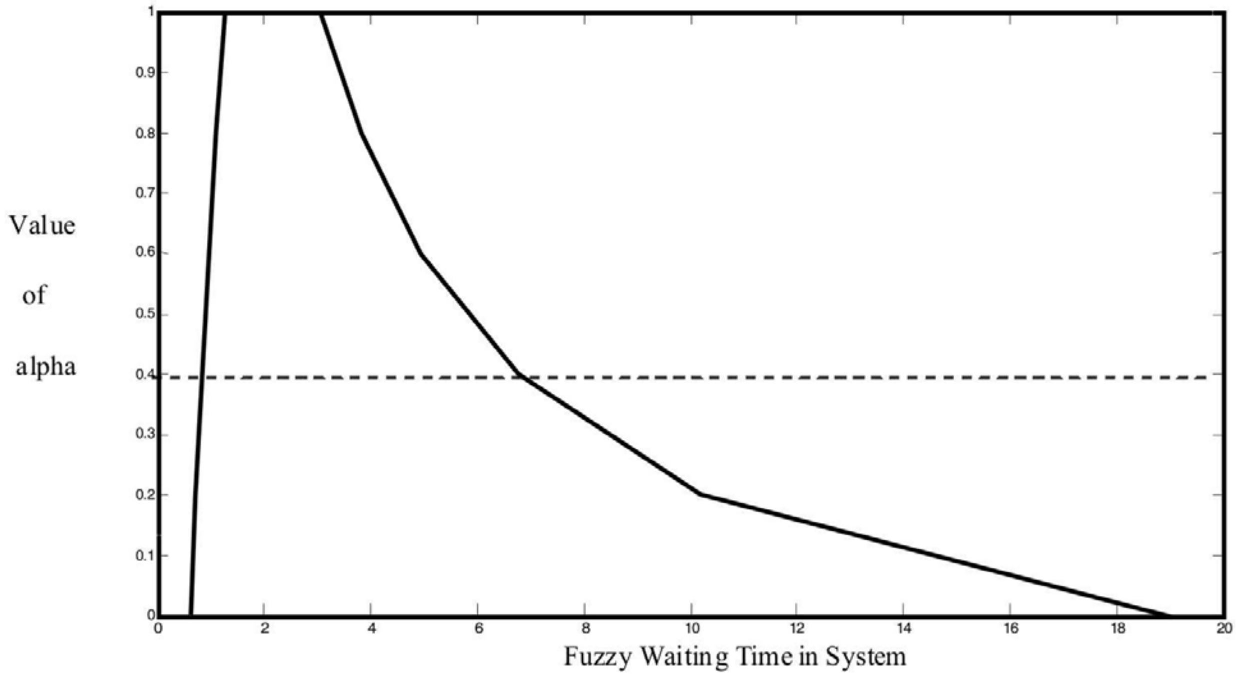

Figure 3: Alpha cut on waiting time in system

CONCLUSION:

We find that when arrival rates $\tilde{\lambda}_1$, $\tilde{\lambda}_2$ and service rate $\tilde{\mu}$ are in fuzzy numbers, the performance measures in the system are expressed by fuzzy number that completely conserve the fuzziness of input information. From the table & diagram, it is clear that as the values of α increase, the lower bound of queue length as well as waiting time of the system increases and corresponding upper bound decreases and therefore, uncertainty decreases and the value becomes closer to crisp value. The evaluation of threshold through α-cut avoid unnecessarily computations in order to eliminate small membership function (value).The effect of threshold creates dead zone and the α-cut thresh hold creates a sudden drop in evaluation of membership functions that is not desired. To avoid a sudden drop, we can suggest to a non linear threshold where the membership value goes down to zero in a slower pace. With these data the value of α=.4 can be considered as the most appropriate value where cut can be applied.

FURTHER EXTENSION:

We end this paper with suggestions:

- We have analysed our model for two parameters in detail while for three parameters the mathematical model has been given. Its analysis can also be interpreted on the same line. More parameters can be considered and behaviour of the model in changing environment can be discussed.
- Threshold effect for triangular fuzzy numbers can also be applied.
- Evaluation based on single input data as well as distributed data can be applied instead of evaluation by threshold effect and the result can be interpreted.

- Even a single change in any one parameter can make the mathematical formulation very complicated. The analytical approach will give unnecessarily computation burden. Simulation model is far better to apply.

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A RELIABILITY MODEL FOR A SINGLE UNIT SYSTEM USED FOR COMMUNICATION THROUGH SATELLITES

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ABSTRACT :
The present paper deals with the study of a reliability analysis of a model for a single unit system used for communication through satellites. The Satellite news gathering network using transportable VSATs (Very Small Aperture Terminals), sometimes called 'fly-away' stations, are transported by car or aircraft and set up at a location where news reporters can transmit video signals to a hub located near the company's studio. In the present study, stochastic analysis of a transportable VSAT used for voice communication and satellite news gathering has been carried out. On requirement, the system is unpacked and assembled. After the assembling process, it becomes operative for the purpose. System is disassembled when requirement is over. Various measures of system effectiveness have been obtained by making use of regenerative point technique.
Keywords ***: Reliability, Communication through Satellite, Regenerative point technique, Measures of system effectiveness, Profit analysis***

1. INTRODUCTION

The literature on reliability models contains a large number of papers dealing with models under different operational conditions and situations by various researchers such as Bhatia et al., Mathew et al., Parashar and Taneja, Singh D.V. etal. Singh T.P. etal. padmavathi etal, Dalip & Taneja [1-7]. These studies do not include the reliability models on systems used for communication through satellites. Our aim is to bridge in such a gap.

The Satellite news gathering network using transportable VSATs (Very Small Aperture Terminals), sometimes called 'fly-away' stations, are transported by car or aircraft and set up at a location where news reporters can transmit video signals to a hub located near the company's studio. On requirement, the system is unpacked and assembled. After the assembling process, it becomes operative for the purpose. System is disassembled when requirement is over.

In the present study, reliability and profit analysis of a transportable VSAT used for voice communication and satellite news gathering has been carried out after obtaining various measures of system effectiveness by making use of regenerative point technique.

2. NOTATION

$h_1(t)$: pdf of time required for unpacking the system
$h_2(t)$: pdf of time for assembling the system
$h_3(t)$: pdf of time during which the requirement is there
$h_4(t)$: pdf of disassembling time
$g(t)$: pdf of repair time
p : probability that there is requirement of system after completing of repair.
q: probability that system is not required after completion of repair
$f(t)$: pdf of time for going from operative to failed state

For other notations used in this paper, [4] may be referred to.

3. MODEL DESCRIPTION AND ASSUMPTIONS

A transition diagram showing the various states of transition of system is shown in **Fig 1**. The epochs of entry into states 0, 1, 2, 3 and 4 are regeneration points and thus these states are regenerative states. State 3 is failed state whereas states 1 and 4 are down states. State 0 is the state showing that the system is packed.

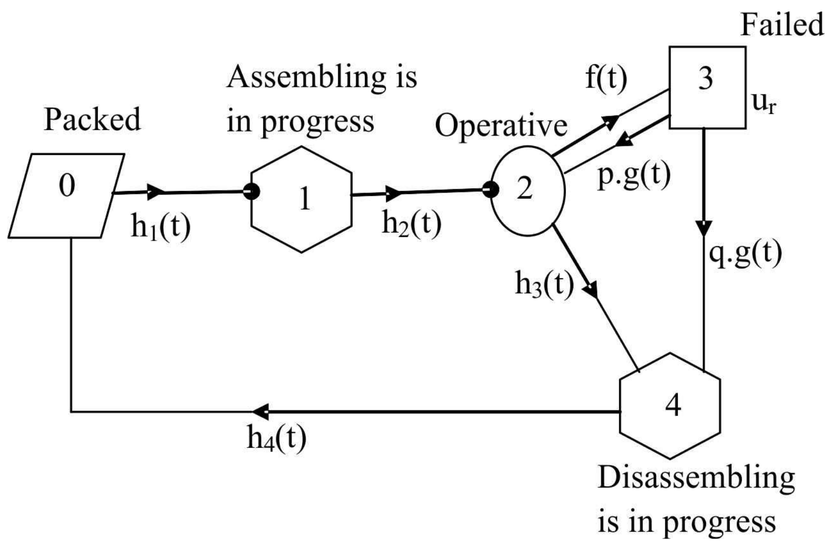

Fig. 1: State Transition diagram

4. STATE TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The **state transition probabilities** are:

$$q_{01}(t) = h_1(t), q_{12}(t) = h_2(t), q_{23}(t) = f(t)\overline{H_3}(t) = E_1(t) \quad \text{(say)}, \quad q_{24}(t) = h_3(t)\overline{F_3}(t) = E_2(t) \quad \text{(say)}$$

$$q_{32}(t) = p\,g(t), q_{34}(t) = q\,g(t), q_{40}(t) = h_4(t)$$

The corresponding non-zero element $p_{ij} = \lim_{s \to 0} q^*_{ij}(s)$ are given by

$$p_{01} = h^*_1(0) = 1,\ p_{12} = h^*_2(0) = 1,\ p_{23} = E^*_1(0),\ p_{24} = E^*_2(0),\ p_{32} = pg^*(0) = p,\ p_{40} = h^*_4(0) = 1$$

The **mean sojourn time** (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state i, then

$$\mu_i = E(T) = \Pr(T > t_i)$$

$$m_{01} = \int_0^\infty t.h_1(t)dt = \mu_0,\ m_{12} = \int_0^\infty t.h_2(t)dt = \mu_1$$

$$m_{23} + m_{24} = \int_0^\infty t\{q_{23}(t) + q_{24}(t)\}dt = \int_0^\infty t\{E_1(t) + E_2(t)\}dt = K_2 \text{(say)}$$

$$\mu_2 = \int_0^\infty \overline{F}(t).\overline{H_3}dt = \int_0^\infty E_3(t)dt = E^*_3(0)$$

where $\quad E_3(t) = \overline{F}(t).\overline{H_3}(t)$

$$\mu_3 = m_{32} + m_{34} = \int_0^\infty t\,\{p.g(t) + q.g(t)\}dt = \int_0^\infty t.g(t)dt\,,\ \mu_4 = m_{40} = \int_0^\infty t.h_4(t)dt$$

The unconditional time taken by the system to transit for any regenerative state 'j' when it is counted from the epoch of entrance into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty t.\,dQ_{ij}(t) = -q^*_{ij}(0)$$

5. THE MATHEMATICAL ANALYSIS OF MODEL

5.1 Reliability and Mean Time to System Failure

Let $\phi_i(t)$ be the c.d.f. of the first passage time from regenerative state i to a failed state. To determine the mean time to system failure(MTSF), regarding the failed state as absorbing state and employing the arguments used for regenerative processes, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t)$ Ⓢ $\phi_1(t)$

$\phi_1(t) = Q_{12}(t)$ Ⓢ $\phi_2(t)$

$\phi_2(t) = Q_{23}(t) + Q_{24}(t)$ Ⓢ $\phi_4(t)$

$\phi_4(t) = Q_{40}(t)$ Ⓢ $\phi_0(t)$

Solving the above equations for $\phi_0^{**}(s)$ by taking L.S.T. of the above equations, the Mean Time to system Failure (MTSF) when the system starts at the beginning of state 0, is $\quad MTSF = \lim_{s\to 0}\dfrac{1-\phi_0^{**}(s)}{s} = \dfrac{N}{D}$

where $\quad N = \mu_0 + \mu_1 + K_2 + \mu_4.\ p_{24}$ and $D = \mathbf{p_{23}}$.

The reliability of the system at time t is obtained by taking the inverse Laplace transform of $\dfrac{1-\phi_0^{**}(s)}{s}$.

5.2 Availability Analysis

By probabilistic arguments, we obtain the following recursive relations for $A_i(t)$

$A_0(t) = q_{01}(t)$ © $A_1(t)$

$A_1(t) = q_{12}(t)$ © $A_2(t)$

$A_2(t) = M_2(t) + q_{23}(t)$ © $A_3(t) + q_{24}(t)$ © $A_4(t)$

$A_3(t) = q_{34}(t)$ © $A_4(t) + q_{32}(t)$ © $A_2(t)$

$A_4(t) = q_{40}(t)$ © $A_0(t)$

where $M_2(t) = \overline{F}(t).\overline{H_3}(t)$

Taking Laplace Transform of above equations and solving the above equations by Cramer's Rule for $A_0^*(s)$, the availability in steady state is given by

$$A_0 = \lim_{s\to 0} s.A_0^{**}(s)$$
$$= \frac{N_1}{D_1}$$

where $N_1 = \mu_2$ and $\quad D_1 = (p_{24} + p_{23}p_{34})(\mu_0 + \mu_1 + \mu_4) + K_2 + p_{23}\mu_3$

5.3 Expected Time for Assembling/Disassembling

By probabilistic arguments, the recursive relations for $AD_i(t)$, probability that assembling/disassembling goes on at time t when the system started from state i at time t=0, are obtained as

$AD_0(t) = q_{01}(t)$ © $AD_1(t)$

$AD_1(t) = W_1(t) + q_{12}(t)$ © $AD_2(t)$

$AD_2(t) = q_{23}(t)$ © $AD_3(t) + q_{24}(t)$ © $AD_4(t)$

$AD_3(t) = q_{34}(t)$ © $AD_4(t) + q_{32}(t)$ © $AD_2(t)$

$AD_4(t) = W_4(t) + q_{40}(t)$ © $AD_0(t)$

where

$W_1(t) = \overline{H_2}(t)$ and $W_4(t) = \overline{H_4}(t)$.

The expected fraction of time during which the assembling/disassembling of the system goes on, is given by

$$AD_0 = \lim_{s \to 0} s.AD_0^*(s)$$
$$= \frac{N_2}{D_1}$$

where

$N_2 = (\mu_1 + \mu_4)(p_{23}p_{34} + p_{24})$ and D_1 is already specified

5.4 Expected Time during which the System Remains Packed

By probabilistic arguments, the recursive relations for $SP_i(t)$, probability that system is packed at time t when the system started from state i at time t=0, are obtained as

$SP_0(t) = W_0(t) + q_{01}(t)$ © $SP_1(t)$

$SP_1(t) = q_{12}(t)$ © $SP_2(t)$

$SP_2(t) = q_{23}(t)$ © $SP_3(t) + q_{24}(t)$ © $SP_4(t)$

$SP_3(t) = q_{34}(t)$ © $SP_4(t) + q_{32}(t)$ © $SP_2(t)$

$SP_4(t) = q_{40}(t)$ © $SP_0(t)$

where $W_0(t) = \overline{H_1}(t)$

The expected fraction of time during which the system remains packed is

$$SP_0 = \lim_{s \to 0} s.SP_0^{**}(s) = \frac{N_3}{D_1}$$

where $N_3 = \mu_0(1 - p_{23}p_{32})$ and $D_1$is already specified

5.5 Busy Period Analysis for Repair

By probabilistic arguments, the recursive relations for $B_i(t)$, the probability that the repairman is busy in repairing the system at time t given that the system started from state i at time t=0, are obtained as

$B_0(t) = q_{01}(t)$ © $B_1(t)$

$B_1(t) = q_{12}(t)$ © $B_2(t)$

$B_2(t) = q_{23}(t)$ © $B_3(t) + q_{24}(t)$ © $B_4(t)$

$B_3(t) = W_3(t) + q_{34}(t)$ © $B_4(t) + q_{32}(t)$ © $B_2(t)$

$B_4(t) = q_{40}(t)$ © $B_0(t)$

where $W_3(t) = \overline{G}(t)$

The expected fraction of time during which the repairman remains busy for repairing the failed unit is given by

$$B_0 = \lim_{s\to 0} s.B_0^{**}(s) = \frac{N_4}{D_1}$$

where

$N_4 = p_{23}\mu_3$ and $D_1$is already specified

5.6 Expected Number of Visits for Repair

By probabilistic arguments, the recursive relations for $V_i(t)$, the expected number of visits till time t given that the system started from state i at time t=0, are

$V_0(t) = Q_{01}(t) Ⓢ V_1(t)$

$V_1(t) = Q_{12}(t) Ⓢ V_2(t)$

$V_2(t) = Q_{23}(t) Ⓢ [1+V_3(t)] + Q_{24}(t) Ⓢ V_4(t)$

$V_3(t) = Q_{32}(t) Ⓢ V_2(t) + Q_{34}(t) Ⓢ V_4(t)$

$V_4(t) = Q_{40}(t) Ⓢ V_0(t)$

The expected number of visits of the repairman is

$$V_0 = \lim_{s\to 0} s.V_0^{**}(s) = \frac{N_5}{D_1}$$

where $N_5 = p_{23}$ and D_1 is already specified

5.7 Profit Function

In steady state, the profit per unit time incurred to the system is given by

Profit(P) = $C_0\, A_0 - C_1\, AD_0 - C_2\, SP_0 - C_3\, B_0 - C_4\, V_0$

where C_0 is revenue per unit up time,

C_1 is rent/expenses paid for assembling//disassembling per unit time,

C_2 is the cost incurred during the period when the system remains packed,

C_3 is the cost per unit time during which the system remains under repair,

C_4 is the cost per visit of the repairman.

$$P = \frac{C_0\mu_2 - (p_{23}p_{34} + p_{24})[C_1(\mu_1 + \mu_4) + C_2\mu_0] - p_{23}(C_3\mu_3 + C_4)}{(p_{24} + p_{23}p_{34})(\mu_0 + \mu_1 + \mu_4) + K_2 + p_{23}\mu_3}$$

6. DISCUSSION AND RESULTS

A particular case is discussed by assuming the repair rate/failure rate/assembling rate/unpacking rate as exponential distribution

$$f(t) = \lambda e^{-\lambda t},\ g(t) = \alpha e^{-\lambda t},\ h_1(t) = \beta_1 e^{-\beta_1 t}, h_2(t) = \beta_2 e^{-\beta_2 t},\quad h_3(t) = \beta_3 e^{-\beta_3 t},\ h_4(t) = \beta_4 e^{-\beta_4 t}$$

$$E_1(t) = f(t)\overline{H_3}(t) = \lambda e^{-(\lambda+\beta_3)t} \text{ and } E_2^*(t) = h_3(t)\overline{F}(t) = \beta_3 e^{-(\lambda+\beta_3)t}$$

Thus,

$$\text{MTSF} = \frac{\beta_2(\lambda+\beta_3)\beta_4 + \beta_1\beta_4(\lambda+\beta_3) + \beta_1\beta_2\beta_4 + \beta_1\beta_2\beta_3}{\lambda\beta_1\beta_2\beta_4}$$

$$\text{Availability } (A_0) = \frac{\alpha.\beta_1.\beta_2.\beta_4}{(\beta_3+\lambda.q)(\beta_2.\beta_4+\beta_1.\beta_4+\beta_1.\beta_2)\alpha+\beta_1.\beta_2.\beta_4.\alpha+\lambda.\beta_1.\beta_2.\beta_4}$$

$$AD_0 = \frac{\alpha.\beta_1\{\beta_4(\lambda+\beta_3-p\lambda)+\beta_2(\lambda q+\beta_3)\}}{(\beta_3+\lambda.q)(\beta_2.\beta_4+\beta_1.\beta_4+\beta_1.\beta_2)\alpha+\beta_1.\beta_2.\beta_4.\alpha+\lambda.\beta_1.\beta_2.\beta_4}$$

$$SP_0 = \frac{\beta_2\beta_4\alpha(\lambda+\beta_3-\lambda p)}{(\beta_3+\lambda.q)(\beta_2.\beta_4+\beta_1.\beta_4+\beta_1.\beta_2)\alpha+\beta_1.\beta_2.\beta_4.\alpha+\lambda.\beta_1.\beta_2.\beta_4}$$

$$B_0 = \frac{\beta_1.\beta_2.\beta_4(\lambda\alpha+\beta_3\alpha-\lambda)}{(\beta_3+\lambda.q)(\beta_2.\beta_4+\beta_1.\beta_4+\beta_1.\beta_2)\alpha+\beta_1.\beta_2.\beta_4.\alpha+\lambda.\beta_1.\beta_2.\beta_4}$$

$$V_0 = \frac{\lambda.\alpha.\beta_1.\beta_2.\beta_4}{(\beta_3+\lambda.q)(\beta_2.\beta_4+\beta_1.\beta_4+\beta_1.\beta_2)\alpha+\beta_1.\beta_2.\beta_4.\alpha+\lambda.\beta_1.\beta_2.\beta_4}$$

$$P = \frac{C_0\alpha\beta_1\beta_2\beta_4 - (\lambda q+\beta_3)[C_1(\beta_4+\beta_2)\beta_1 + C_2\beta_2\beta_4] - \lambda\beta_1\beta_2\beta_4(C_3+C_4\alpha)}{\alpha(\beta_3+\lambda q)(\beta_2\beta_4+\beta_1\beta_4+\beta_1\beta_2)+\beta_1\beta_2\beta_4(\alpha+\lambda)}$$

The behavior of the MTSF, Availability and profit with respect to failure rate is shown in **Figs. 2 to 4** and that of the profit with respect to revenue per unit up time is shown in **Fig. 5**. The values fixed for various parameters for plotting the graphs as shown in **Figs 2 to 4** are:

β_1=2, β_2=2, β_3=2, β_4=2, p=0,7, q=0.3, C_0=1000, C_1=500, C_2=100, C_3=100 and C_4=100 and α ranging from 0.001 to 0.005 with step 0.002.

For **Fig. 5**; λ=0.01, α =0.001,C_1=1000, 500 and 300 for series 1, 2 and 3 respectively. Other values has been fixed as taken in Figs 2 to 4.

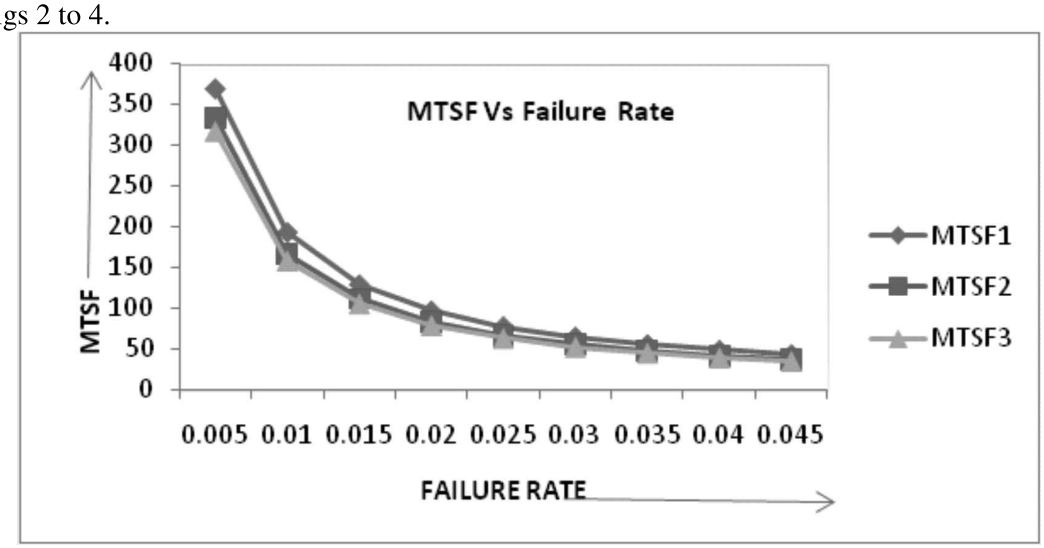

Fig. 2: MTSF Vs Failure Rate

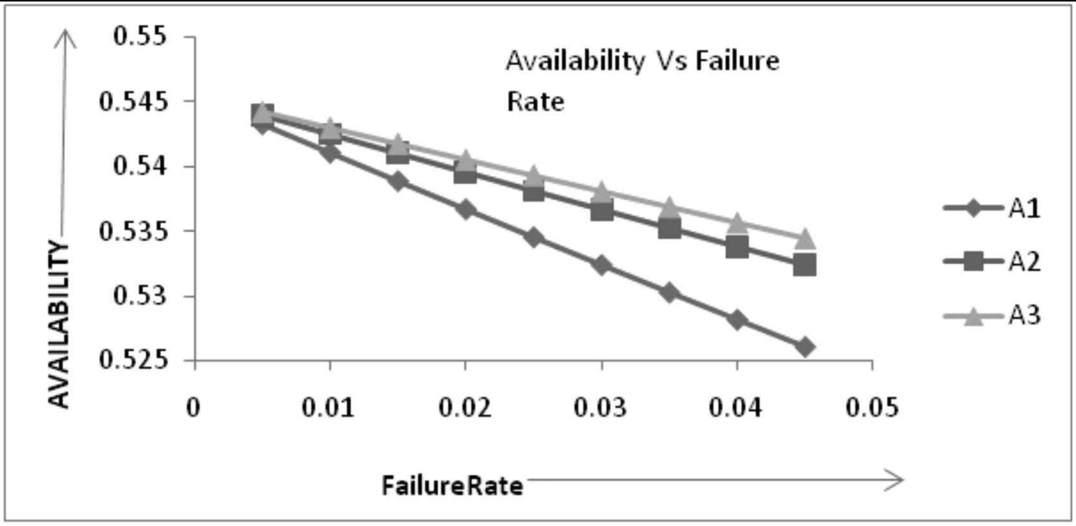

Fig.3: Availability Vs Failure Rate

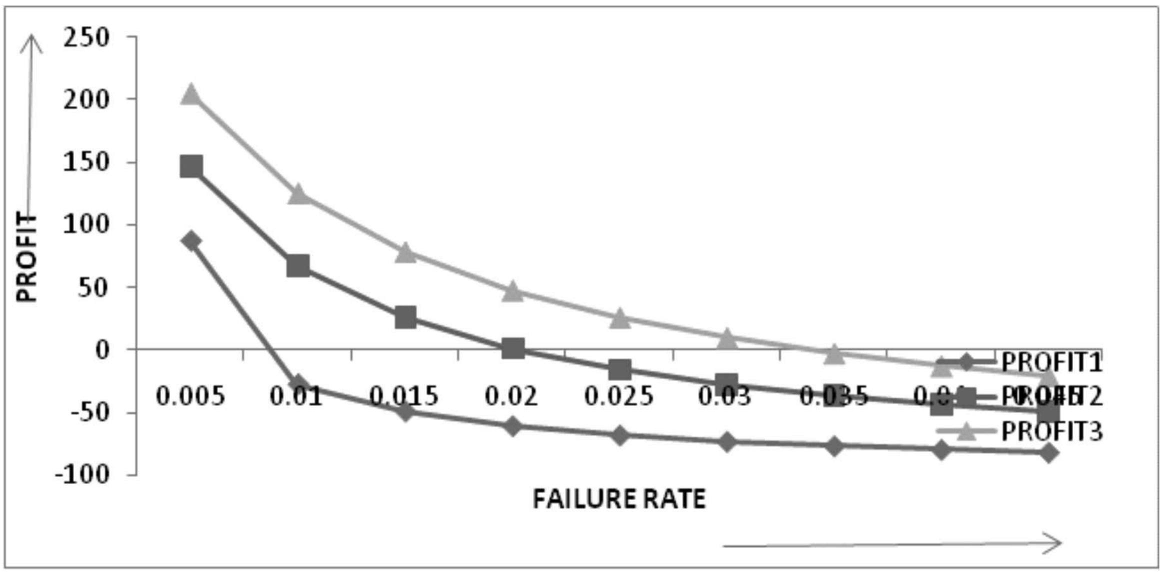

Fig. 4: Profit Vs Failure Rate

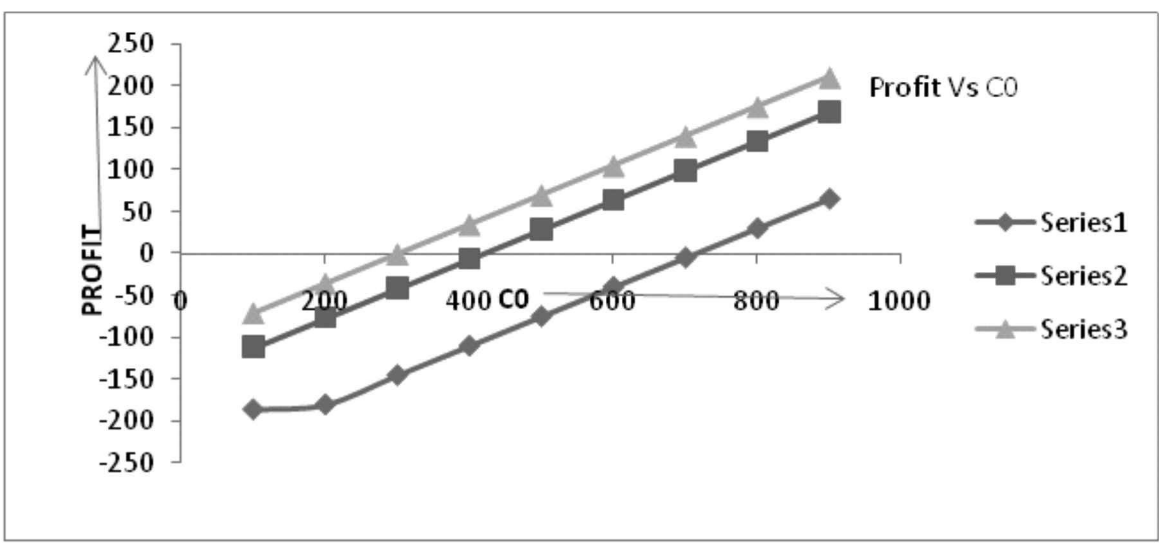

Fig. 5: Profit Vs C_0

7. CONCLUSION

- The MTSF, Availability and profit get decreased as failure rate of the system increases. However, they have the higher values for higher values of repair rate.
- The profit increases with increase in the values of revenue per unit up time. It is reveals that the revenue per unit up time should not be less than its value at cut-off point. This cut-off point helps the producer to fix the price of the electricity in such a way so as to get the positive profit

The above remarks are based on what computational work has been done in this study. However, if someone is interested in finding some other cut-off points related to the desired rates, costs involved, he/she can use the equations obtained for MTSF, measures of system effectiveness and the profit. Then, the expressions particularly for the system under consideration can be obtained putting the numerical values of various rates/costs experienced therein which will be helpful in taking important decisions so far as the the profitability of the system is concerned

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SOME COMMON FIXED POINT THEOREMS FOR THREE MAPPINGS IN G-METRIC SPACES

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ABSTRACT :
Our objects in this paper is to obtain some common fixed point theorems for three mappings using the setting of generalized metric spaces. It is worth mentioning that our results that do not rely on the notion of continuity, weakly commuting or compatibility of mappings involved therein. These results generalized well known comparable results in the literature.
Keywords: ***Metric space, G-metric space, Common fixed point.***

1. Introduction:

In 1963, Gahler [1] introduced the concept of 2-metric spaces and claimed that a 2-metric is a generalization of the usual notion of a metric, but some authors proved that there is no relation between these two functions. It is clear that in 2-metric, $d(x, y, z)$ is to be taken as the area of the triangle with vertices x, y and $z$in R^2.Rao K.P.R etal (11, 12) studied the common fixed point theorem for 3 maps on a cone matrix space and two pairs of hybrid mappings.And recently,Giniswamy etal (13) Studied fixed point theorem on a f-reciprocal continuity but in their results the notions of continuity are involved.

In 1992, Dhage [2] introduced the concept of a D-metric space. Geometrically, a D-metric D(x, y, z) represents the perimeter of the triangle with vertices x, y, and z in R^2. Recently, Mustafa and Sims [3] have shown that most of the results concerning Dhage's D-metric spaces are invalid (also see [4, 5, 6]). Therefore, they introduced an improved version of the generalized metric space structure, which they called G-metric spaces. Our results generalize of Mustafa et. al [7].

In 2006, Mustafa and Sims [8] introduced the concept of G-metric spaces as follows :

Definition 1.1[8] Let X be a nonempty set, and let$G : X \times X \times X \to R^+$be a function satisfying the following axioms:

(G1) $G(x, y, z) = 0$if $x = y = z$,

(G2) $0 < G(x, x, y)$for all $x, y \in X$with $x \neq y$,

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$with $z \neq y$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric or , more specifically, a G-metric on X and the pair (X, G) is called a G-metric space.

Definition 1.2[8] Let (X, G) a G-metric space and let $\{x_n\}$ be a sequence of points in X. A point x in X is said to be the limit of the sequence $\{x_n\}$, $\lim_{m,n\to\infty} G(x, x_n, x_m) = 0$,and one says that the sequence $\{x_n\}$ is G-convergent to x.

Thus, $x_n \to x, n \to \infty$ or $\lim_{n\to\infty} x_n = x$ in a G-metric space (X, G) if for each $\varepsilon > 0$, there exists a positive integer N such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq N$.

Proposition 1.1 [8] Let (X, G) be a G-metric space then the following are equivalent:

1. $\{x_n\}$ is G-convergent to x,
2. $G(x_n, x_n\, x) \to 0$ as $n \to \infty$,
3. $G(x_n, x, x) \to 0\ as\ n \to \infty$,
4. $G(x_m, x_n, x) \to 0\ \ as\ m, n \to \infty$.

Definition 1.3 [8] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if, for each $\varepsilon > 0$, there exists a positive integer N such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq N$. i.e., if $G(x_n, x_m, x_l) \to 0$as $n, m, l \to \infty$.

Proposition 1.2 [9]If (X, G) is a G-metric space, then the following are equivalent:

1. the sequence $\{x_n\}$ is G-Cauchy,
2. For each $\varepsilon > 0$, there exists a positive integer N such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq N$.

Proposition 1.3 [8] Let (X, G) be a G-metric space then the function $G(x, y, z)$ is jointly continuous in all three variables.

Definition 1.4 [8] A G-metric space (X, G) is called a symmetric G-metric space if

$G(x, y, y) = G(y, x, x)$for all x, yin X.

Proposition 1.4 [10]Every G-metric space (X, G) will define a metric space (X, d_G) by

1. $d_G(x, y) = G(x, y, y) + G(y, x, x)$for all x, y in X.

If (X, G) is asymmetric G-metric space, then

2. $d_G(x, y) = 2G(x, y, y)$for all x, y in X.

However, if (X, G) is not symmetric, then it follows from the G-metric properties that

3. $\frac{3}{2} G(x, y, y) \leq d_G(x, y) \leq 3G(x, y, y)$for all x, y in X.

Definition 1.5 [10]A G-metric space (X, G) is said to be G-complete if every G-Cauchy sequence in (X, G) is G-convergent in X.

Proposition 1.5 [10] A G-metric space (X, G) is said to be G-complete if every G-Cauchy sequence in (X, d_G) is complete metric space.

Proposition 1.6 [9] Let (X, G) be a G-metric space. Then, for any $x, y, z, a \in X$, it follows that

1. If $G(x, y, z) = 0\ then\ x = y = z$,
2. $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,
3. $G(x, y, y) \leq 2G(y, x, x)$,
4. $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,
5. $G(x, y, z) \leq \frac{2}{3}(G(x, y, a) + G(x, a, z) + G(a, y, z))$,
6. $G(x, y, z) \leq (G(x, a, a) + G(y, a, a) + G(z, a, a))$.

2. Main results:

Theorem 2.1. Let f, g and h be self-maps on a complete G-metric space X satisfying

$$G(fx, gy, hz) \leq\ a\{G(x, gy, gy) + G(y, fx, fx)\} + b\{G(y, hz, hz) + G(z, gy, gy)\}$$

$$+c\{G(z,fx,fx)+G(x,hz,hz)\}+dG(x,y,z) \tag{1}$$

For all $x,y,z\in X$, where $a,b,c,d>0$ and $0<a+b+3c+d<1$. Then f,g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Proof: Our proof will going in two steps.

Step 1. We prove that any fixed point of f is a fixed point of g and h. Consider that $s\in X$ is such that $fs=s$. Now we prove that $s=gs=hs$. If it is not then for $s\neq gs$ and $s\neq hs$, we get

$$\begin{aligned}G(s,gs,hs)=G(fs,gs,hs)&\leq a\{G(s,gs,gs)+G(s,fs,fs)\}+b\{G(s,hs,hs)+G(s,gs,gs)\}\\&+c\{G(s,fs,fs)+G(s,hs,hs)\}+dG(s,s,s)\\&=a\{G(s,gs,gs)+G(s,s,s)\}+b\{G(s,hs,hs)+G(s,gs,gs)\}\\&+c\{G(s,s,s)+G(s,hs,hs)\}+dG(s,s,s)\\&=aG(s,gs,gs)+b\{G(s,hs,hs)+G(s,gs,gs)\}+cG(s,hs,hs)\end{aligned}$$

$\leq(a+2b+c)G(s,gs,hs)$ is a contradiction.

Analogously, for $s\neq gs$ and $s=hs$ or for $s\neq hs$ and $s=gs$, following the similar arguments to those given above, we obtain a contradiction. Hence in all the cases we conclude that $s=gs=hs$. The same conclusion holds if $s=gs$ or $s=hs$.

Step 2. We prove that f, g and h have a unique common fixed point. Let x_0 is an arbitrary point in X. Define $\{x_n\}$ by $x_{3n+1}=fx_{3n}$, $x_{3n+2}=gx_{3n+1}$, $x_{3n+3}=hx_{3n+2}$, $n=0,1,2\ldots$ if $x_n=x_{n+1}$ for some n, with $n=3m$, then $s=x_{3n}$ is a fixed point of f and by the step 1, s is a common point for f, g and h. The same holds if $n=3m+1$ or $n=3m+2$. Now let us suppose that $x_n\neq x_{n+1}$ for all $n\in N$ then, we have

$$\begin{aligned}&G(x_{3n+1},x_{3n+2},x_{3n+3})=G(fx_{3n},gx_{3n+1},hx_{3n+2})\\&\leq a\{G(x_{3n},gx_{3n+1},gx_{3n+1})+G(x_{3n+1},fx_{3n},fx_{3n})\}\\&\quad+b\{G(x_{3n+1},hx_{3n+2},hx_{3n+2})+G(x_{3n+2},gx_{3n+1},gx_{3n+1})\}\\&\quad+c\{G(x_{3n+2},fx_{3n},fx_{3n})+G(x_{3n},hx_{3n+2},hx_{3n+2})\}+dG(x_{3n},x_{3n+1},x_{3n+2})\\&=a\{G(x_{3n},x_{3n+2},x_{3n+2})+G(x_{3n+1},x_{3n+1},x_{3n+1})\}+b\{G(x_{3n+1},x_{3n+3},x_{3n+3})+G(x_{3n+2},x_{3n+2},x_{3n+2})\}\\&\quad+c\{G(x_{3n+2},x_{3n+1},x_{3n+1})+G(x_{3n},x_{3n+3},x_{3n+3})\}+dG(x_{3n},x_{3n+1},x_{3n+2})\\&\leq aG(x_{3n},x_{3n+1},x_{3n+2})+bG(x_{3n+1},x_{3n+2},x_{3n+3})+c\{G(x_{3n+2},x_{3n+1},x_{3n})+G(x_{3n},x_{3n+3},x_{3n+3})\}\\&\quad+dG(x_{3n},x_{3n+1},x_{3n+2})\\&=(a+c+d)G(x_{3n},x_{3n+1},x_{3n+2})+bG(x_{3n+1},x_{3n+2},x_{3n+3})+cG(x_{3n},x_{3n+3},x_{3n+3})\end{aligned}$$

$$\begin{aligned}&(1-b)G(x_{3n+1},x_{3n+2},x_{3n+3})\\&\leq(a+c+d)G(x_{3n},x_{3n+1},x_{3n+2})+c\{G(x_{3n},x_{3n+1},x_{3n+1})+G(x_{3n+1},x_{3n+3},x_{3n+3})\}\end{aligned}$$

$$(1-b)G(x_{3n+1},x_{3n+2},x_{3n+3}) \leq (a+c+d)G(x_{3n},x_{3n+1},x_{3n+2}) + c\{G(x_{3n},x_{3n+1},x_{3n+2}) + G(x_{3n+1},x_{3n+2},x_{3n+3})\}$$

$$(1-b-c)G(x_{3n+1},x_{3n+2},x_{3n+3}) \leq (a+2c+d)G(x_{3n},x_{3n+1},x_{3n+2})$$

$$G(x_{3n+1},x_{3n+2},x_{3n+3}) \leq \frac{a+2c+d}{1-b-c}G(x_{3n},x_{3n+1},x_{3n+2})$$

$$G(x_{3n+1},x_{3n+2},x_{3n+3}) \leq kG(x_{3n},x_{3n+1},x_{3n+2})$$

Where $k=\frac{a+2c+d}{1-b-c}$. Obviously $0<k<1$. Similarly it can be shown that

$$G(x_{3n+2},x_{3n+3},x_{3n+4}) \leq kG(x_{3n+1},x_{3n+2},x_{3n+3})$$

and

$$G(x_{3n+3},x_{3n+4},x_{3n+5}) \leq kG(x_{3n+2},x_{3n+3},x_{3n+4})$$

Therefore, for all n,

$$G(x_{n+1},x_{n+2},x_{n+3}) \leq kG(x_n,x_{n+1},x_{n+2}) \\ \leq \cdots \leq k^{n+1}G(x_0,x_1,x_2)$$

Now for any $l,m,n$with $l>m>n$,

$$G(x_n,x_m,x_l) \leq G(x_n,x_{n+1},x_{n+2}) + G(x_{n+1},x_{n+2},x_{n+3}) + \cdots + G(x_{l-2},x_{l-1},x_l)$$
$$G(x_n,x_m,x_l) \leq [k^n+k^{n+1}+\cdots+k^{l-2}]G(x_0,x_1,x_2)$$
$$G(x_n,x_m,x_l) \leq \frac{k^n}{1-k}G(x_0,x_1,x_2)$$

The same holds for $l=m>n$ and $l>m=n$. Letting as $n,m,l\to\infty$, we have $G(x_n,x_m,x_l)=0$. Thus $\{x_n\}$ is G-Cauchy sequence in X. Since (X, G) is complete G-metric space, therefore, there exists a point $p\in X$ such that $\{x_n\}$ is converges to $p$as $n\to\infty$. We claim that $fp=p$ if not, then consider

$$\begin{aligned} G(fp,x_{3n+2},x_{3n+3}) &= G(fp,gx_{3n+1},hx_{3n+2}) \\ &\leq a\{G(p,gx_{3n+1},gx_{3n+1}) + G(x_{3n+1},fp,fp)\} + b\{G(x_{3n+1},hx_{3n+2},hx_{3n+2}) + G(x_{3n+2},gx_{3n+1},gx_{3n+1})\} \\ &\quad + c\{G(x_{3n+2},fp,fp) + G(p,hx_{3n+2},hx_{3n+2})\} + dG(p,x_{3n+1},x_{3n+2}) \\ &= a\{G(p,x_{3n+2},x_{3n+2}) + G(x_{3n+1},fp,fp)\} + b\{G(x_{3n+1},x_{3n+3},x_{3n+3}) + G(x_{3n+2},x_{3n+2},x_{3n+2})\} \\ &\quad + c\{G(x_{3n+2},fp,fp) + G(p,x_{3n+3},x_{3n+3})\} + dG(p,x_{3n+1},x_{3n+2}) \end{aligned}$$

Now taking limit as $n\to\infty$, we have

$$G(fp,p,p) \leq (a+c)G(fp,p,p)$$

a contradiction. Hence $fp = p$. Similarly it can be shown that $gp = p$ and $hp = p$.

For uniqueness, let us consider that q is another common fixed point of $f, g$and h, then

$$\begin{aligned} G(p,q,q) &= G(fp,gq,hq) \\ &\leq a\{G(p,gq,gq) + G(q,fp,fp)\} + b\{G(q,hq,hq) + G(q,gq,gq)\} \\ &\quad + c\{G(q,fp,fp) + G(p,hq,hq)\} + dG(p,q,q) \\ &= a\{G(p,q,q) + G(q,p,p)\} + b\{G(q,q,q) + G(q,q,q)\} + c\{G(q,p,p) + G(p,q,q)\} + dG(p,q,q) \\ &\leq a\{G(p,q,q) + 2G(p,q,q)\} + c\{2G(p,q,q) + G(p,q,q)\} + dG(p,q,q) \end{aligned}$$

{by proposition 1.6}

$$G(p,q,q) \leq (3a + 3c + d)G(p,q,q)$$

Which gives that $G(p,q,q) = 0$ this implies that $p = q$. Thus p is unique common fixed point of $f, g$and h.

Corollary 2.2. Let f, g and h be self-maps on a complete G-metric space X satisfying

$$G(f^m x, g^m y, h^m z) \leq a\{G(x, g^m y, g^m y) + G(y, f^m x, f^m x)\} + b\{G(y, h^m z, h^m z) + G(z, g^m y, g^m y)\}$$
$$+c\{G(z, f^m x, f^m x) + G(x, h^m z, h^m z)\} + dG(x,y,z) \quad (2)$$

For all $x, y, z \in X$, where $a, b, c, d > 0$ and $0 < a + b + 3c + d < 1$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Theorem 2.3. Let f, g and h be self-maps on a complete G-metric space X satisfying

$$G(fx,gy,hz) \leq aG(x,y,z) + b\{G(x,gy,gy) + G(y,fx,fx) + G(z,hz,hz)\} + c\{G(y,hz,hz) + G(z,gy,gy) + G(x,fx,fx)\} + d\{G(z,fx,fx) + G(x,hz,hz) + G(y,gy,gy)\} \quad (3)$$

For all $x, y, z \in X$, where $a, b, c, d > 0$ and $0 < a + 2b + 2c + 4d < 1$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

Proof: Our proof will going in two steps.

Step 1. We prove that any fixed point of f is a fixed point of $g$and h. Consider that $s \in X$ is such that $fs = s$. Now we prove that $s = gs = hs$. If it is not then for $s \neq gs$and $s \neq hs$, we get

$$\begin{aligned} G(s,gs,hs) &= G(fs,gs,hs) \\ &\leq aG(s,s,s) + b\{G(s,gs,gs) + G(s,fs,fs) + G(s,hs,hs)\} + c\{G(s,hs,hs) + G(s,gs,gs) \\ &\quad + G(s,fs,fs\} + d\{G(s,fs,fs) + G(s,hs,hs) + G(s,gs,gs\} \\ &= aG(s,s,s) + b\{G(s,gs,gs) + G(s,s,s) + G(s,hs,hs)\} + c\{G(s,hs,hs) + G(s,gs,gs) + G(s,s,s)\} \\ &\quad + d\{G(s,s,s) + G(s,hs,hs) + G(s,gs,gs)\} \\ &= b\{G(s,gs,gs) + G(s,hs,hs)\} + c\{G(s,hs,hs) + G(s,gs,gs)\} + d\{G(s,hs,hs) + G(s,gs,gs)\} \end{aligned}$$

$\leq 2(b + c + d)G(s,gs,hs)$is a contradiction.

Analogously, for $s \neq gs$ and $s = hs$ or for $s \neq hs$ and $s = gs$, following the similar arguments to those given above, we obtain a contradiction. Hence in all the cases we conclude that $s = gs = hs$. The same conclusion holds if $s = gs$or $s = hs$.

Step 2. We prove that f, g and h have a unique common fixed point. Let x_0 is an arbitrary point in X. Define $\{x_n\}$ by $x_{3n+1} = fx_{3n}$, $x_{3n+2} = gx_{3n+1}$, $x_{3n+3} = hx_{3n+2}$, $n = 0, 1, 2$ ... if $x_n = x_{n+1}$ for some n, with $n = 3m$, then $s = x_{3n}$ is a fixed point of f and by the step 1, s is a common point for f, g and h. The same holds if $n = 3m + 1$or $n = 3m + 2$. Now let us suppose that $x_n \neq x_{n+1}$ for all $n \in N$ then, we have

$$\begin{aligned}
G(x_{3n+1}, x_{3n+2}, x_{3n+3}) &= G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\
&\leq aG(x_{3n}, x_{3n+1}, x_{3n+2}) \\
&\quad + b\{G(x_{3n}, gx_{3n+1}, gx_{3n+1}) + G(x_{3n+1}, fx_{3n}, fx_{3n}) + G(x_{3n+2}, hx_{3n+2}, hx_{3n+2})\} \\
&\quad + c\{G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+2}, gx_{3n+1}, gx_{3n+1}) + G(x_{3n}, fx_{3n}, fx_{3n})\} \\
&\quad + d\{G(x_{3n+2}, fx_{3n}, fx_{3n}) + G(x_{3n}, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+1}, gx_{3n+1}, gx_{3n+1})\} \\
&= aG(x_{3n}, x_{3n+1}, x_{3n+2}) + b\{G(x_{3n}, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, x_{3n+1}, x_{3n+1}) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})\} \\
&\quad + c\{G(x_{3n+1}, x_{3n+3}, x_{3n+3}) + G(x_{3n+2}, x_{3n+2}, x_{3n+2}) + G(x_{3n}, x_{3n+1}, x_{3n+1})\} \\
&\quad + d\{G(x_{3n+2}, x_{3n+1}, x_{3n+1}) + G(x_{3n}, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2})\} \\
&\leq aG(x_{3n}, x_{3n+1}, x_{3n+2}) + b\{G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} + c\{G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \\
&\quad + G(x_{3n}, x_{3n+1}, x_{3n+2})\} \\
&\quad + d\{G(x_{3n+1}, x_{3n+2}, x_{3n+3}) + G(x_{3n}, x_{3n+1}, x_{3n+1}) + G(x_{3n+1}, x_{3n+3}, x_{3n+3}) \\
&\quad + G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} \\
&\leq aG(x_{3n}, x_{3n+1}, x_{3n+2}) + b\{G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+3})\} + c\{G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \\
&\quad + G(x_{3n}, x_{3n+1}, x_{3n+2})\} \\
&\quad + d\{G(x_{3n+1}, x_{3n+2}, x_{3n+3}) + G(x_{3n}, x_{3n+1}, x_{3n+2}) + G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \\
&\quad + G(x_{3n+1}, x_{3n+2}, x_{3n+3})\}
\end{aligned}$$

$$(1 - b - c - 3d)G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq (a + b + c + d)G(x_{3n}, x_{3n+1}, x_{3n+2})$$

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq \frac{a + b + c + d}{1 - b - c - 3d} G(x_{3n}, x_{3n+1}, x_{3n+2})$$

$$G(x_{3n+1}, x_{3n+2}, x_{3n+3}) \leq kG(x_{3n}, x_{3n+1}, x_{3n+2})$$

Where $k = \frac{a+b+c+d}{1-b-c-3d}$. Obviously $0 < k < 1$. Similarly it can be shown that

$$G(x_{3n+2}, x_{3n+3}, x_{3n+4}) \leq kG(x_{3n+1}, x_{3n+2}, x_{3n+3})$$

and

$$G(x_{3n+3}, x_{3n+4}, x_{3n+5}) \leq kG(x_{3n+2}, x_{3n+3}, x_{3n+4})$$

Therefore, for all n,

$$\begin{aligned}
G(x_{n+1}, x_{n+2}, x_{n+3}) &\leq kG(x_n, x_{n+1}, x_{n+2}) \\
&\leq \cdots \leq k^{n+1}G(x_0, x_1, x_2)
\end{aligned}$$

Now for any $l, m, n$with $l > m > n$,

$$G(x_n, x_m, x_l) \leq G(x_n, x_{n+1}, x_{n+2}) + G(x_{n+1}, x_{n+2}, x_{n+3}) + \cdots + G(x_{l-2}, x_{l-1}, x_l)$$
$$G(x_n, x_m, x_l) \leq [k^n + k^{n+1} + \cdots + k^{l-2}]G(x_0, x_1, x_2)$$
$$G(x_n, x_m, x_l) \leq \frac{k^n}{1-k} G(x_0, x_1, x_2)$$

The same holds for $l = m > n$ and $l > m = n$. Letting as $n, m, l \to \infty$, we have $G(x_n, x_m, x_l) = 0$. Thus $\{x_n\}$ is G-Cauchy sequence in X. Since (X, G) is complete G-metric space, therefore, there exists a point $p \in X$ such that $\{x_n\}$ is converges to $p$as $n \to \infty$. We claim that $fp = p$ if not then consider

$$\begin{aligned} G(fp, x_{3n+2}, x_{3n+3}) &= G(fp, gx_{3n+1}, hx_{3n+2}) \\ &\leq aG(p, x_{3n+1}, x_{3n+2}) \\ &\quad + b\{G(p, gx_{3n+1}, gx_{3n+1}) + G(x_{3n+1}, fp, fp) + G(x_{3n+2}, hx_{3n+2}, hx_{3n+2})\} \\ &\quad + c\{G(x_{3n+1}, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+2}, gx_{3n+1}, gx_{3n+1}) + G(p, fp, fp)\} \\ &\quad + d\{G(x_{3n+2}, fp, fp) + G(p, hx_{3n+2}, hx_{3n+2}) + G(x_{3n+1}, gx_{3n+1}, gx_{3n+1})\} \\ &= aG(p, x_{3n+1}, x_{3n+2}) + b\{G(p, x_{3n+2}, x_{3n+2}) + G(x_{3n+1}, fp, fp) + G(x_{3n+2}, x_{3n+3}, x_{3n+3})\} \\ &\quad + c\{G(x_{3n+1}, x_{3n+3}, hx_{3n+3}) + G(x_{3n+2}, x_{3n+2}, x_{3n+2}) + G(p, fp, fp)\} \\ &\quad + d\{G(x_{3n+2}, fp, fp) + G(p, x_{3n+3}, x_{3n+3}) + G(x_{3n+1}, x_{3n+2}, x_{3n+2})\} \end{aligned}$$

Now taking limit as $n \to \infty$, we have

$$G(fp, p, p) \leq (a + c)G(fp, p, p)$$

a contradiction. Hence $fp = p$. Similarly it can be shown that $gp = p$and $hp = p$.

For uniqueness, let us consider that q is another common fixed point of $f, g$and h, then

$$\begin{aligned} G(p, q, q) &= G(fp, gq, hq) \\ &\leq aG(p, q, q) + b\{G(p, gq, gq) + G(q, fp, fp) + G(q, hq, hq)\} + c\{G(q, hq, hq) + G(q, gq, gq) \\ &\quad + G(p, fp, fp)\} + d\{G(q, fp, fp) + G(p, hq, hq) + G(q, gq, gq)\} \\ &= aG(p, q, q) + b\{G(p, q, q) + G(q, p, p) + G(q, q, q)\} + c\{G(q, q, q) + G(q, q, q) + G(p, p, p)\} \\ &\quad + d\{G(q, p, p) + G(p, q, q) + G(q, q, q)\} \\ &= aG(p, q, q) + b\{G(p, q, q) + G(q, p, p)\} + d\{G(q, p, p) + G(p, q, q) + G(q, q, q)\} \\ &= aG(p, q, q) + b\{G(p, q, q) + 2G(p, q, q)\} + d\{2G(p, q, q) + G(p, q, q)\} \end{aligned}$$

{by proposition 1.6}

$$G(p, q, q) \leq (a + 3b + 3d)G(p, q, q)$$

Which gives that $G(p, q, q) = 0$ this implies that $p = q$. Thus p is unique common fixed point of $f, g$and h.

Corollary 2.4. Let f, g and h be self-maps on a complete G-metric space X satisfying

$$G(f^m x, g^m y, h^m z) \leq aG(x, y, z) + b\{G(x, g^m y, g^m y) + G(y, f^m x, f^m x) + G(z, h^m z, h^m z)\} + c\{G(y, h^m z, h^m z) + G(z, g^m y, g^m y) + G(x, f^m x, f^m x)\} + d\{G(z, f^m x, f^m x) + G(x, h^m z, h^m z) + G(y, g^m y, g^m y)\} \quad (4)$$

For all $x, y, z \in X$, where $a, b, c, d > 0$ and $0 < a + 2b + 2c + 4d < 1$. Then f, g and h have a unique common fixed point in X. Moreover, any fixed point of f is a fixed point of g and h and conversely.

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A RESULT ON WATSON'S SUMMATION THEOREM FOR THE SERIES $_3F_2$

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ABSTRACT :
In the theory of hypergeometric and generalized hypergeometric series, Watson's summation theorem for the series $_3F_2$ plays an important role. The aim of this short research note is to add one more result in the literature.
Key Words and Phrases ***Generalized hypergeometric series, Watson's, Dixon's and Whipple's summation theorems, Thomae transformation.***

1. Introduction

We start, with classical Watson's summation theorem [1].

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ \frac{1}{2}(a+b+1),2c \end{matrix} \;\middle|\; 1\right] = \frac{\Gamma\left(\frac{1}{2}\right)\ \Gamma\left(c+\frac{1}{2}\right)\ \Gamma\left(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}\right)\ \Gamma\left(c-\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}a+\frac{1}{2}\right)\ \Gamma\left(\frac{1}{2}b+\frac{1}{2}\right)\ \Gamma\left(c-\frac{1}{2}a+\frac{1}{2}\right)\ \Gamma\left(c-\frac{1}{2}b+\frac{1}{2}\right)} \quad \ldots(1.1) $$

{provided $\Re$e (2c – a – b) > -1}

Dixon' summation theorem [1]

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ 1+a-b,1+a-c \end{matrix} \;\middle|\; 1\right] = \frac{\Gamma\left(1+\frac{1}{2}a\right)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma\left(1+\frac{1}{2}a-b-c\right)}{\Gamma(1+a)\ \Gamma\left(1+\frac{1}{2}a-b\right)\ \Gamma\left(1+\frac{1}{2}a-c\right)\ \Gamma(1+a-b-c)} \quad \ldots(1.2) $$

{provided $\Re$e (a – 2b – 2c) > -1}

Whipple's summation theorem [1]

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ e,f \end{matrix} \;\middle|\; 1\right] = \frac{\pi\ \ \Gamma(e)\ \Gamma(f)}{2^{2c-1}\ \ \Gamma\left(\frac{1}{2}a+\frac{1}{2}e\right)\ \Gamma\left(\frac{1}{2}a+\frac{1}{2}f\right)\ \Gamma\left(\frac{1}{2}b+\frac{1}{2}e\right)\ \Gamma\left(\frac{1}{2}b+\frac{1}{2}f\right)} \quad \ldots(1.3) $$

{provided a + b = 1, e + f = 1 + 2c and $\Re$e (e + f – a – b – c) > 0}

and Thomae transformation formula [5]

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ e,f \end{matrix} \;\middle|\; 1\right] = \frac{\Gamma(e)\;\Gamma(f)\;\Gamma(s)}{\Gamma(a)\;\Gamma(s+b)\;\Gamma(s+c)}\;{}_3F_2\left[\begin{matrix} e-a,f-a,s \\ s+b,s+c \end{matrix} \;\middle|\; 1\right] \qquad \ldots(1.4) $$

where $\Re$e (s) > 0 with s = e + f – a – b – c

in the literature we have the Watson's summation theorem (1.1) can be established with the help of Dixons' summation theorem (1.2) and Thomae transformation formula (1.4). Also, Whipple's theorem can be established by employing Thomae transformation formula (1.4) together with Watson's summation theorem (1.1).

in this research paper we establish a new result by employing Thomae transformation (1.4) together with generalized Whipple's summation theorem.

2. Main Results

$$ {}_3F_2\left[\begin{matrix} A,B,C \\ \frac{1}{2}(A+B+4),2C-2 \end{matrix} \;\middle|\; 1\right] $$

$$ = \frac{\Gamma(\frac{1}{2}A+\frac{1}{2}B+2)\;\Gamma(2C-2)\;\Gamma\left(\frac{1}{2}A-\frac{1}{2}B-1\right)\;\Gamma\left(C-\frac{1}{2}A-\frac{1}{2}B-1\right)}{2^{2C-A-B+1}\;\Gamma(A)\;\Gamma(B)\;\Gamma(C)\;\Gamma\left(\frac{1}{2}A-\frac{1}{2}B+2\right)} $$

$$ \left\{(2(AC-AB-A+3BC-2C-B^2-3B+2)\frac{\Gamma\left(\frac{1}{2}A+\frac{1}{2}\right)\;\Gamma\left(\frac{1}{2}B\right)}{\Gamma\left(C-\frac{1}{2}B-\frac{1}{2}\right)\;\Gamma\left(C-\frac{1}{2}A-1\right)}\right. $$

$$ \left.+2(-3AC+A^2+2C+3A+AB-BC+B-2)\frac{\Gamma\left(\frac{1}{2}A\right)\;\Gamma\left(\frac{1}{2}B+\frac{1}{2}\right)}{\Gamma\left(C-\frac{1}{2}B-1\right)\;\Gamma\left(C-\frac{1}{2}A-\frac{1}{2}\right)}\right\} $$

$\Re$e (2C-A-B) > 0

…(2.1)

3. Result Required

(a) Thomae transformation [5]

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ e,f \end{matrix} \;\middle|\; 1\right] = \frac{\Gamma(e)\;\Gamma(f)\;\Gamma(s)}{\Gamma(a)\;\Gamma(s+b)\;\Gamma(s+c)}\;{}_3F_2\left[\begin{matrix} e-a,f-a,s \\ s+b,s+c \end{matrix} \;\middle|\; 1\right] \qquad \ldots(3.1) $$

where $\Re$e (s) > 0 with s = e + f – a – b – c

(b) Generalized Whipple's theorem [3]

$$
{}_3F_2\left[\begin{matrix}a,b,c\\ e,f\end{matrix}\;\Big|\;1\right]
$$

$$
=\frac{\Gamma(e)\;\Gamma(f)\;\Gamma\left(c-\frac{1}{2}(j+|j|)\right)\;\Gamma\left(e-c-\frac{1}{2}(i+|i|)\right)\;\Gamma\left(a-\frac{1}{2}(i+j+|i+j|)\right)}{2^{2a-i-j}\;\Gamma(e-a)\;\Gamma(f-a)\;\Gamma(e-c)\;\Gamma(a)\;\Gamma(f)}
$$

$$
\left\{A_{i,j}\frac{\Gamma\left(\frac{1}{2}e-\frac{1}{2}a+\frac{1}{4}(1-(-1)^i)\right)\;\Gamma\left(\frac{1}{2}f-\frac{1}{2}a\right)}{\Gamma\left(\frac{1}{2}e+\frac{1}{2}a-\frac{1}{2}i+\left[-\frac{1}{2}j\right]\right)\;\Gamma\left(\frac{1}{2}f+\frac{1}{2}a-\frac{1}{2}i+\left((-1)^j\frac{1}{4}\right)\left((-1)^i-1\right)+\left[-\frac{1}{2}j\right]\right)}\right.
$$

$$
\left.+B_{i,j}\frac{\Gamma\left(\frac{1}{2}e-\frac{1}{2}a+\frac{1}{4}\left(1+(-1)^i\right)\right)}{\Gamma\left(\frac{1}{2}e+\frac{1}{2}a-\frac{1}{2}-\frac{1}{2}i+\left[-\frac{1}{2}j+\frac{1}{2}\right]\right)}\;\frac{\Gamma\left(\frac{1}{2}f-\frac{1}{2}a+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}f+\frac{1}{2}a-\frac{1}{2}-\frac{1}{2}i+\left((-1)^j\frac{1}{4}\right)\left(1-(-1)^i\right)+\left[-\frac{1}{2}j+\frac{1}{2}\right]\right)}\right\}
$$

…(3.2)

Where a + b = 1 + i + j, e + f = 2 c + 1 + i

4. Derivation of Watson's Summation Theorem

In order to establish Watson's result (2.1), we proceed as follows. For this, consider the Thomae transformation formula (3.1) in the form

$$
{}_3F_2\left[\begin{matrix}e-a,f-a,s\\ s+b,s+c\end{matrix}\;\Big|\;1\right]=\frac{\Gamma(a)\;\Gamma(s+b)\;\Gamma(s+c)}{\Gamma(e)\;\Gamma(f)\;\Gamma(s)}\;{}_3F_2\left[\begin{matrix}a,b,c\\ e,f\end{matrix}\;\Big|\;1\right]
$$

…(4.1)

where s = e + f – a – b – c

On substitutions

e – a = A …(4.1.1)

f – a = B …(4.1.2)

e + f – a – b – c = C …(4.1.3)

e + f – a – c = $\frac{1}{2}$ (A + B + 4) …(4.1.4)

e + f – a – b = 2C – 2 …(4.1.5)

On solving (4.1.1) to (4.1.5) we get

$a=-\frac{1}{2}A-\frac{1}{2}B+C$ $\qquad$ $b=\frac{1}{2}A+\frac{1}{2}B-C+2$

c = C – 2 $\qquad$ $e=\frac{1}{2}A-\frac{1}{2}B+C$ $\qquad$ $f=-\frac{1}{2}A+\frac{1}{2}B+C$

On putting the values of a, b, c, e and f

$$ {}_3F_2\left[\begin{matrix} A,B,C \\ \frac{1}{2}(A+B+4),2C-2 \end{matrix} \;\middle|\; 1\right] = \frac{\Gamma(C-\frac{1}{2}A-\frac{1}{2}B)\ \Gamma\left(\frac{1}{2}A+\frac{1}{2}B+2\right)\ \Gamma(2C-2)}{\Gamma\left(C+\frac{1}{2}A-\frac{1}{2}B\right)\ \Gamma\left(C-\frac{1}{2}A+\frac{1}{2}B\right)\ \Gamma(C)} $$

$$ {}_3F_2\left[\begin{matrix} C-\frac{1}{2}A-\frac{1}{2}B,\frac{1}{2}A+\frac{1}{2}B-C+2,C-2 \\ C+\frac{1}{2}A-\frac{1}{2}B,C-\frac{1}{2}A+\frac{1}{2}B \end{matrix} \;\middle|\; 1\right] $$

…(4.2)

Now, we observe that the ${}_3F_2$ appearing on the right-hand side can now be evaluated with the help of Whipple's summation theorem (3.2) for i = 3, j = -2 by letting

$a = C-\frac{1}{2}A-\frac{1}{2}B,\ \ b=\frac{1}{2}A+\frac{1}{2}B+\frac{1}{2}-C+2,$

c = C-2, $e = C+\frac{1}{2}A-\frac{1}{2}B,\ \ f = C-\frac{1}{2}A+\frac{1}{2}B$ and after some simplification, we easily arrive at the right-hand side of Watson's summation theorem (2.1). This completes the proof of Watson's summation theorem.

Various workers [2,4,5,6,7] also gave some valuable results.

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PERFORMANCE AND COST-BENEFIT ANALYSIS OF A SYSTEM HAVING BATH-TUB CURVE SHAPED FAILURE PATTERN WITH PROVISION OF TWO TYPES OF REPLACEMENT

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ABSTRACT :
The present paper deals with a stochastic model for a system having bath-tub curve shaped failure patternt. The system having bath-tub curve shaped failure pattern is catogarised in to three stages of operation which have different failure rates, i. e. corresponding to testing/ burn-in process, useful life period and wear-out period. After burn-in stage, on failure of the system, the available repairman first inspects whether fault is minor or major and if the fault is minor he repairs the system. In case, fault is major then he carries out further inspection of the system to judge whether replacement has to be done by an old one unit of same operational stage or by a new one and accordingly carries out the replacement. Various measures of the system effectiveness are obtained at different stages of operation of the system using Markov process and regenerative point techniques. The system is analysed with respect to its reliability, availabilty and profit. Various conclusions have been drawn for the system on the basis of graphical study.
Keywords: ***Bath-tub curve shaped failure, burn-in period, useful life period, wear-out period, mean time to system failure, availabilty, profit, Markov process and regenerative point techniques.***

Introduction

In the field of reliability modeling, many researchers in the past analysed a large number of systems considering various aspects such as different failure modes, maintenaces, repairs, replacements, inspections, degradation etc. Nakagawa and Osaki (1975) considered a two-unit standby redundant system with repair and preventive maintenance. Lesanovosky (1980) investigated availability of two unit cold standby system with degraded state. Gupta and Goel (1989) discussed a two-unit priority standby system with administrative delay in repair. Tuteja et al. (1991) analysed stochastic behaviour of a two unit system with two types of repairman and subject to random inspection. Kumar et al. (1996) investigated two-unit cold standby system introducing the concept of instruction time while Kumar et al. (2001) discussed a two unit redundant system with degradation and replacement. Taneja and Nanda (2003) analysed a two-unit cold standby system with resume and repeat repair policies.

Recently some others aspects have also been taken in this area of research by a few authors. Kumar et al. (2010) carried out analysis of a three stage operational single unit system under warranty and two types of services facility. Mathew et. al. (2011) analysed the reliability of two unit parallel cc plant with full installed capacity. Kumar and Kapoor (2012) studied cost-benefit analysis of a base transceiver system considering hardware/ software faults and congestion of calls. Kumar and Bhatia (2013) carried out probabilistic analysis of a model for a centrifuge system considering neglected faults and halt of the system. Singh and Taneja (2014) performed reliability and economic analysis of a power plant with scheduled inspection.Anita (2014) obtained profit evaluation of a two unit system with inspection for repair or replacement

Infact a system in which failure rate follows bath-tub curve shaped pattern may be catogarised in to three stages of operation on the basis of different failure rates, i. e. the stages corresponding to testing/ burn-in process, useful life period and wear-out period, as taken in Kumar et al. (2010). During any of these operational stages, the system may have failures due to minor or major faults. The system manufacturer/ provider carries out inspections, repairs or replacements, if any during testing/ burn-in period of the system and usually does free of charges installation of the system for the users through its service engineer. But after proper installation of the system, i.e. in other opeartional stages of the system, the system user has to get repair/repalcement if any at its own level from the available repairman on payement basis in case warranty/gaurantee of the system is not provided/availed.

In practice, numerous systems having bath-tub curve shaped failure pattern are encountered in industries/ firms/ households wherein on a major failure replacement by an old unit of same operational stage is done in place of a new one. This may be due to multiplicity of reasons. Sometimes, a new unit is not available in the inventory or market. Moreover, replacement by a new unit for such systems requires re-installation and thus, in that case replacement by an old unit of same operational stage result in to save in time and money. Taking this practical aspects in to account, analysis has not been reported in the literature of reliability.

Keeping the above fact in view, the present paper is an attempt to develop and analyze a stochastic model for a single-unit system having bath-tub curve shaped failure pattern considering above aspects. In the model, at three operational stages of the system viz. burn in period, useful life period and wear out period, different failure as well as repair rates are considered. Also, an improvement rate with which system reaches useful life period from burn in period and a deterioration rate with which system reaches wear-out period are taken. After burn-in stage, on failure of the system, the available repairman first inspects whether fault is minor or major. In case the fault is minor, he repairs the system, otherwise he carries out further inspection of the system to judge whether replacement has to be done by an old unit of same operational stage or by a new one. Accordingly he carries out the replacement. Various measures of the system performance are obtained at different stages of operation of the system using Markov process and regenerative point techniques. The system is analysed with respect to its reliability, availability and profit. Lastly, various conclusions drawn for the system on the basis of graphical study is presented.

Notations

$\lambda_1/\lambda_2/\lambda_3$	failure rate during burn-in / useful life / wear-out period
η_1/η_2	improvement /deterioration rate of the system
$a_1/a_2/a_3$	probability that there is a minor fault during burn-in /useful life /wear-out period
$b_1/b_2/b_3$	probability that there is a major fault during burn-in /useful life /wear-out period
x_1/x_2	probability that replacement is done by a new unit during useful life/wear-out
y_1/y_2	probability that replacement is done by an old same operational unit during useful life /wear-out period; $y_1=1-x_1$, $y_2=1-x_2$
$g_1(t)/g_2(t)/g_3(t)$	p.d.f. of repair time during burn-in /useful life /wear-out period
$G_1(t)/G_2(t)/G_3(t)$	c.d.f. of repair time during burn-in /useful life /wear-out period
$h_1(t)/h_2(t)/h_4(t)$	p.d.f. of replacement time by a new unit during burn-in/ useful life /wear-out period
$H_1(t)/H_2(t)/H_4(t)$	c.d.f. of replacement time during by a new unit burn-in/ useful life /wear-out period
$h_3(t)/h_5(t)$	p.d.f. of replacement time by old same operational stage unit during useful life /wear-out period
$H_3(t)/H_5(t)$	c.d.f. of replacement time by old same operational stage unit during useful life /wear-out period
$i_1(t)/ i_2(t)/i_4(t)$	p.d.f. of inspection time for type of fault during burn-in / useful life/wear-out period
$I_1(t)/I_2(t)/I_4(t)$	c.d.f. of inspection time for type of fault during burn-in / useful life /wear-out period
$i_3(t)/i_5(t)$	p.d.f. of inspection time for type of replacement during useful life /wear-out period
$I_3(t)/I_5(t)$	c.d.f of inspection time of major faults during burn-in /useful life /wear-out period

State of the System

Si	state of the system i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,13,14,15
$O_I/O_{II}/O_{III}$	system is operating during burn-in / useful life / wear-out period
$F_{Ii1}/F_{IIi2}/F_{IIIi4}$	system is in failed state and is under inspection during burn-in /useful life wear-out period
F_{IIi3}/F_{IIIi5}	system is in failed state and is under inspection for type of replacement during useful life /wear-out period
$F_{Ir}/F_{IIr}/F_{IIIr}$	system is under repair on minor fault during burn-in /useful life /wear- out period
F_{Irp}	system is in failed state and is under replacement during burn-in period
F_{IIrp1}/F_{IIIrp1}	system is in failed state and is under replacement by a new unit during useful life /wear-out period
F_{IIrp2}/F_{IIIrp2}	system is in failed state and is under replacement by a same operational unit during useful life /wear-out period

Other Assumptions

1. The system manufacturer/ provider carries out inspections, repairs or replacements, if any during testing/ burn-in period of the system and does free of charges installation of the system for the users through its service engineer.
2. An occurrence of minor/major fault at any operational stage leads to failure of the sytem. On failure of the system, the system is inspected for type of fault-minor/ major.
3. An occurrence of a minor fault requires repair whereas a major fault requires repalcement of the unit at any operational stage of the system.
4. The unit is as good as new after repair/ replacement in that particular operational stage.
5. The service engineer/ available repairman takes negligible time to reach the system.
6. The times to failure, improvement and deterioration are taken exponential distributions while the other time distributions are considered arbitrary.
7. All the random variables are mutually independent.

Transition Probabilities and Mean Sojourn Times

A state-transition diagram in fig. 1 shows various states of transitions of the system. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 are regeneration points and thus these are regenerative states. The states 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14 and 15 are failed states.

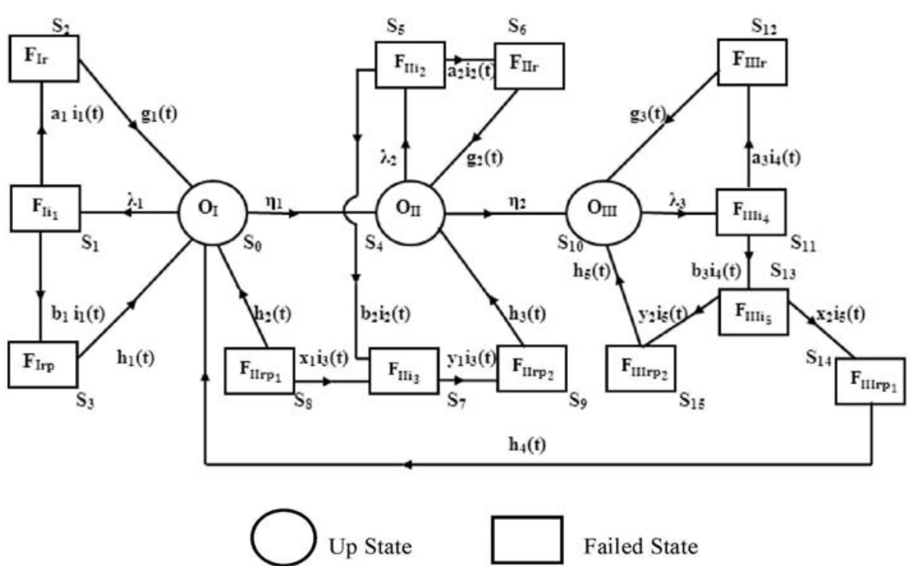

Fig. 1: State Transition Diagram

The non-zero elements p_{ij} obtained as $p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$ are

$p_{01} = \frac{\lambda_1}{\lambda_1+\eta_1}$ $p_{12} = a_1$ $p_{13} = b_1$ $p_{04} = \frac{\eta_1}{\lambda_1+\eta_1}$

$p_{45} = \frac{\lambda_2}{\lambda_2+\eta_2}$ $p_{4,10} = \frac{\eta_2}{\lambda_2+\eta_2}$ $p_{56} = a_2$ $p_{57} = b_2$

$p_{78} = x_1$ $p_{79} = y_1$ $p_{11,12} = a_3$ $p_{11,13} = b_3$

$p_{13,14} = x_2$ $p_{13,15} = y_2$

By these transitions probabilities it can be verified that

$p_{01}+p_{04} = 1$ $p_{45}+p_{4,10} = 1$ $p_{12}+p_{13} = 1$

$p_{56}+p_{57} = 1$ $p_{78}+p_{79} = 1$ $p_{11,12}+p_{11,13}=1$

$p_{13,14}+p_{13,15} = 1$ $p_{20} = p_{30} = p_{64} = p_{80} = p_{94} = p_{10,11} = p_{12,10} = p_{14,0} = p_{15,10} = 1$

Also, the mean sojourn times in regenerative states i (μ_i) are

$\mu_0 = \frac{1}{\lambda_1 + \eta_1}$ $\mu_1 = -i_1^{*'}(0)$ $\mu_2 = -g_1^{*'}(0)$ $\mu_3 = -h_1^{*'}(0)$

$\mu_4 = \frac{1}{\lambda_2 + \eta_2}$ $\mu_5 = -i_2^{*'}(0)$ $\mu_6 = -g_2^{*'}(0)$ $\mu_7 = -i_3^{*'}(0)$

$\mu_8 = -h_2^{*'}(0)$ $\mu_9 = -h_3^{*'}(0)$ $\mu_{10} = \frac{1}{\lambda_3}$ $\mu_{11} = -i_4^{*'}(0)$

$\mu_{12} = -g_3^{*'}(0)$ $\mu_{13} = -i_5^{*'}(0)$ $\mu_{14} = -h_4^{*'}(0)$ $\mu_{15} = -h_5^{*'}(0)$

The unconditional mean time taken by the system to transit for any state j when it is counted from the epoch of entrance into the state i, is mathematically stated as

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}^{*'}(s)$$

Thus,

$m_{01} + m_{04} = \mu_0$ $m_{12}+m_{13} = \mu_1$ $m_{20} = \mu_2$

$m_{30} = \mu_3$ $m_{45}+m_{4,10} = \mu_4$ $m_{56}+m_{57} = \mu_5$

$m_{64} = \mu_6$ $m_{78}+m_{79} = \mu_7$ $m_{80} = \mu_8$

$m_{94} = \mu_9$ $m_{10,11} = \mu_{10}$ $m_{11,12}+m_{11,13} = \mu_{11}$

$m_{12,10} = \mu_{12}$ $m_{13,14}+m_{13,15} = \mu_{13}$ $m_{14,0} = \mu_{14}$

$m_{15,10} = \mu_{15}$

Other Measures of System Performance

Using the probabilistic arguments for regenerative process, several recursive relations for various measures of the system performance for different stages of operation are obtained and on solving them using Laplace/Laplace-Shieltjes transforms, we get the following:

Mean Time to System Failure

In the steady-state, mean time to system failure is given by

$$T_0 = \frac{N_1}{D_1},$$

where

$N_1 = \mu_0 + \mu_4 p_{04} + \mu_{10} p_{04} p_{4,10}$ and $D_1 = 1$

Availability Analysis

In the steady-state, availability of the system is given by

$$A_0 = \frac{N_2}{D_2},$$

where

$$N_2 = \mu_0\, p_{11,13}p_{13,14}(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9}) + \mu_4\, p_{0,4}\, p_{11,13}\, p_{13,14} + \mu_{10}\, p_{0,4}p_{4,10}$$

and

$$D_2 = p_{11,13}p_{13,14}[(\mu_0 + p_{0,1}\,\mu_1 + \mu_2\, p_{0,1}p_{1,2} + \mu_3\, p_{0,1}p_{1,3})(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9}) +$$
$$p_{0,4}(\mu_4 + p_{4,5}\,\mu_5 + \mu_6\, p_{4,5}p_{5,6} + \mu_7\, p_{4,5}p_{5,7} + \mu_8\, p_{4,5}p_{5,7}\,p_{7,8} + \mu_9\, p_{4,5}p_{5,7}\,p_{7,9}) +$$
$$p_{0,4}\, p_{4,10}\,(\mu_{10} + \mu_{11} + p_{11,12}\,\mu_{12} + \mu_{13}\, p_{11,13} + \mu_{14}\, p_{11,13}p_{13,14} + \mu_{15}\, p_{11,13}p_{13,15})]$$

Busy Period of the Service Engineer/ Repairman

At Burn-in Period

Expected busy period of the service engineer repair time only (BR_0) $= \frac{N_3}{D_2}$

Expected busy period of the service engineer inspection time only (BI_0) $= \frac{N_4}{D_2}$

Expected number of the replacement by the service engineer (Rp_0) $= \frac{N_5}{D_2}$

Expected number of the visit by the service engineer (V_0) $= \frac{N_6}{D_2}$

At Useful-life Period

Expected busy period of the available repairman repair time only (BR_4) $= \frac{N_7}{D_2}$

Expected busy period of the available repairman inspection time only (BI_4) $= \frac{N_8}{D_2}$

Expected number of the replacement by the available repairman by

(i) new unit (Rpn_4) $= \frac{N_9}{D_2}$

(ii) old same operational unit (Rps_4) $= \frac{N_{10}}{D_2}$

Expected number of the visit by the available repairman (V_4) $= \frac{N_{11}}{D_2}$

At Wear-out Period

Expected busy period of the available repairman repair time only (BR_{10}) $= \frac{N_{12}}{D_2}$

Expected busy period of the available repairman inspection time only(BI_{10}) $= \frac{N_{13}}{D_2}$

Expected number of the replacement by available repairman by

i. new unit (Rpn_{10}) $= \frac{N_{14}}{D_2}$

ii. old same operational unit (Rps_{10}) $= \frac{N_{15}}{D_2}$

Expected number of the visit by the available repairman (V_{10}) $= \frac{N_{16}}{D_2}$,

where

$$N_3 = \mu_2\, p_{0,1}\, p_{1,2}\, p_{11,13}\, p_{13,14}(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9})$$
$$N_4 = \mu_1\, p_{11,13}\, p_{13,14}(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9})$$
$$N_5 = p_{0,1}\, p_{1,3}\, p_{11,13}\, p_{13,14}(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9})$$
$$N_6 = p_{0,1}\, p_{11,13}\, p_{13,14}(1-p_{4,5}p_{5,6}-p_{4,5}p_{5,7}\,p_{7,9})$$

$N_7 = \mu_6 p_{0,4} p_{4,5} p_{5,6} p_{11,13} p_{13,14}$

$N_8 = p_{0,4} p_{4,5} p_{11,13} p_{13,14} [\mu_5 + \mu_7 p_{5,7}]$

$N_9 = p_{0,4} p_{4,5} p_{5,7} p_{7,8} p_{11,13} p_{13,14}$

$N_{10} = p_{0,4} p_{4,5} p_{5,7} p_{7,9} p_{11,13} p_{13,14}$

$N_{11} = p_{0,4} p_{4,5} p_{11,13} p_{13,14}$

$N_{12} = \mu_{12} p_{0,4} p_{4,10} p_{11,12}$

$N_{13} = p_{0,4} p_{4,10} [\mu_{11} + \mu_{13} p_{11,13}]$

$N_{14} = p_{0,4} p_{4,10} p_{11,13} p_{13,14}$

$N_{15} = p_{0,4} p_{4,10} p_{4,5} p_{11,13} p_{13,15}$

$N_{16} = p_{0,4} p_{4,10}$

and D_2 is already specified.

Profit Analysis of the System

(A) Expected Profit for System User (P_1) is given by

$$P_1 = C_0[A_0]-C_1[BI_4+BI_{10}]-C_2[BR_4+BR_{10}]-C_3[BRpn_4+BRpn_{10}]-C_4[BRps_4+BRps_{10}] - C_5[V_4+V_{10}]\ ,$$

where

C_0 = revenue per unit up time of the system

C_1 = cost per unit time of inspection by the available repairman

C_2 = cost per unit time of repair by the available repairman

C_3 = cost per unit replacement by the new unit

C_4 = cost per unit replacement by the old same operational unit

C_5 = cost per visit of the available repairman

(B) Expected Profit for the System Provider (P_2) is given by

$$P_2 = (SP\text{-}CP)-C_6[BI_0]-C_7[BR_0]-C_8[BR_{P0}]- C_9[V_0]\ ,$$

where

SP/CP = sale price/ cost price of the system

C_6 = cost per unit time of inspection by the service engineer

C_7 = cost per unit time of repair by the service engineer

C_8 = cost per unit replacement by the service engineer

C_9 = cost per visit of the service engineer

Graphical Interpretations and Conclusions

The following particular case is considered for the graphical analysis of the system for each stages of its operation:

$g_1(t) = \beta_1 e^{-\beta_1 t}$, $g_2(t) = \beta_2 e^{-\beta_2 t}$, $g_3(t) = \beta_3 e^{-\beta_3 t}$,

$h_1(t) = \gamma_1 e^{-\gamma_1 t}$, $h_2(t) = \gamma_2 e^{-\gamma_2 t}$, $h_3(t) = \gamma_3 e^{-\gamma_3 t}$,

$h_4(t) = \gamma_4 e^{-\gamma_4 t}$, $h_5(t) = \gamma_5 e^{-\gamma_5 t}$, $i_1(t) = \alpha_1 e^{-\alpha_1 t}$,

$i_2(t) = \alpha_2 e^{-\alpha_2 t}$, $i_3(t) = \alpha_3 e^{-\alpha_3 t}$, $i_4(t) = \alpha_4 e^{-\alpha_4 t}$

$i_5(t) = \alpha_5 e^{-\alpha_5 t}$

Various graphs are plotted for the mean time to system failure, availability and profit of the system user and system provider for the different operational stages of the syetm for various values of failure rates (λ_1, λ_2, λ_3), repair rates (β_1, β_2, β_3), replacement rates (γ_1, γ_2, γ_3, γ_4 ,γ_5) and inspection rates (α_1, α_2, α_3, α_4, α_5). The following conclusions have been drawn from the graphs:

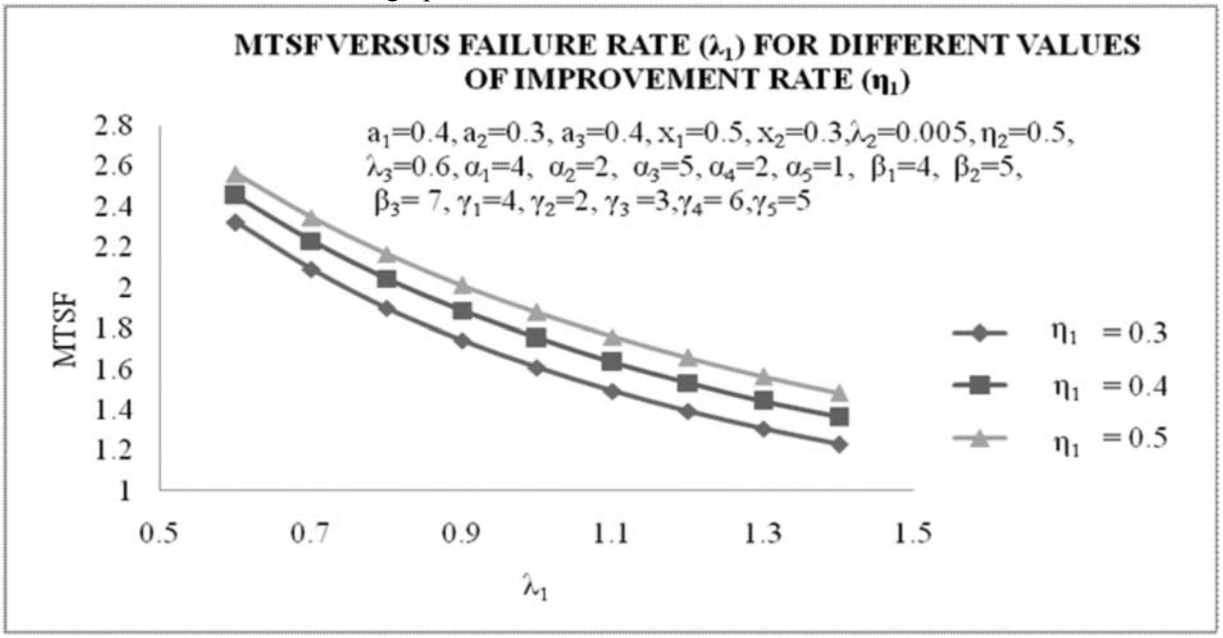

Fig. 2

Fig. 2 shows the behavior of mean time to system failure (MTSF) with respect to failure rate (λ_1) of the system during burn in perod for different values of improvement rate (η_1). It can be concluded from the graph that MTSF decreases with the increase in the values of λ_1 and has higher values for higher values of η_1 when other parameters are fixed.

The curves in fig. 3 reveal the pattern of mean time to system failure (MTSF) with respect to failure rate (λ_2) of the system during useful life perod for different values of improvement rate (η_1). It is concluded from the graph that MTSF decreases with the increase in the values of λ_2 and has higher values for higher values of η_1 when other parameters are fixed.

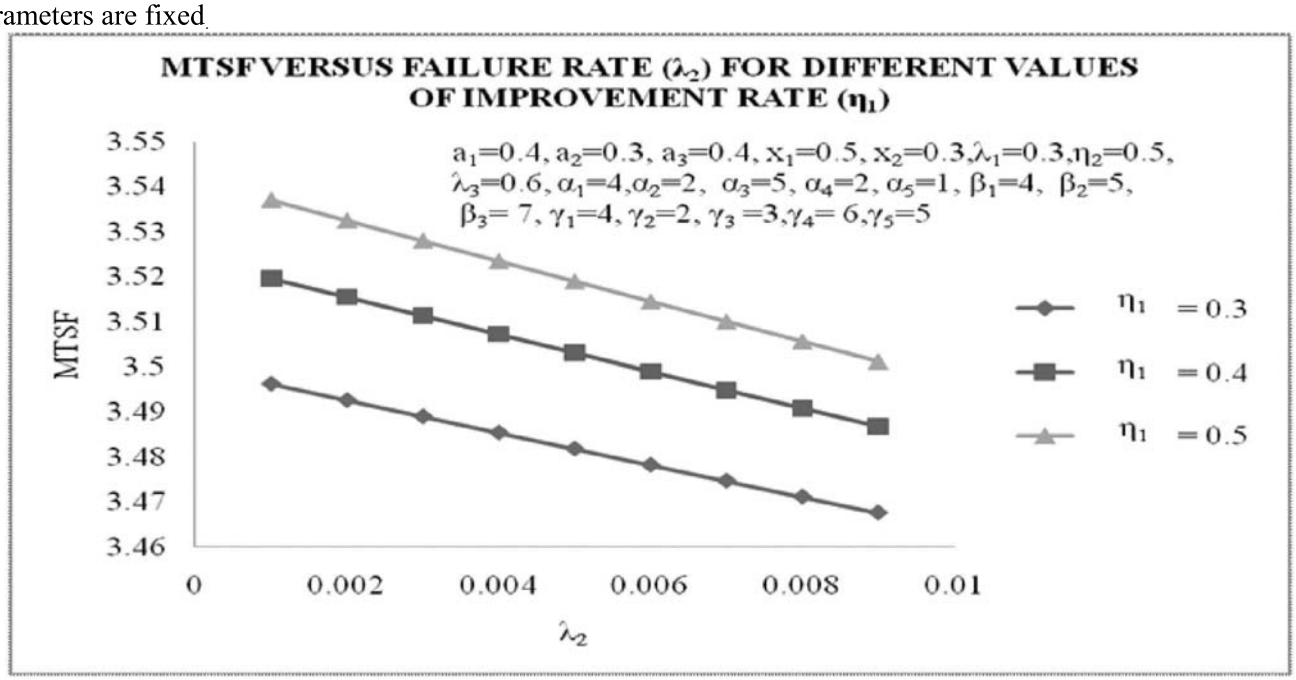

Fig. 3

Fig. 4 depicts the behavior of mean time to system failure (MTSF) with respect to failure rate (λ_3) of the system during wear-out perod for different values of deterioration rate (η_2). It can be interpreted from the graph that MTSF decreases with the increase in the values of λ_3 when other parameters are fixed and has lower values for higher values of η_2

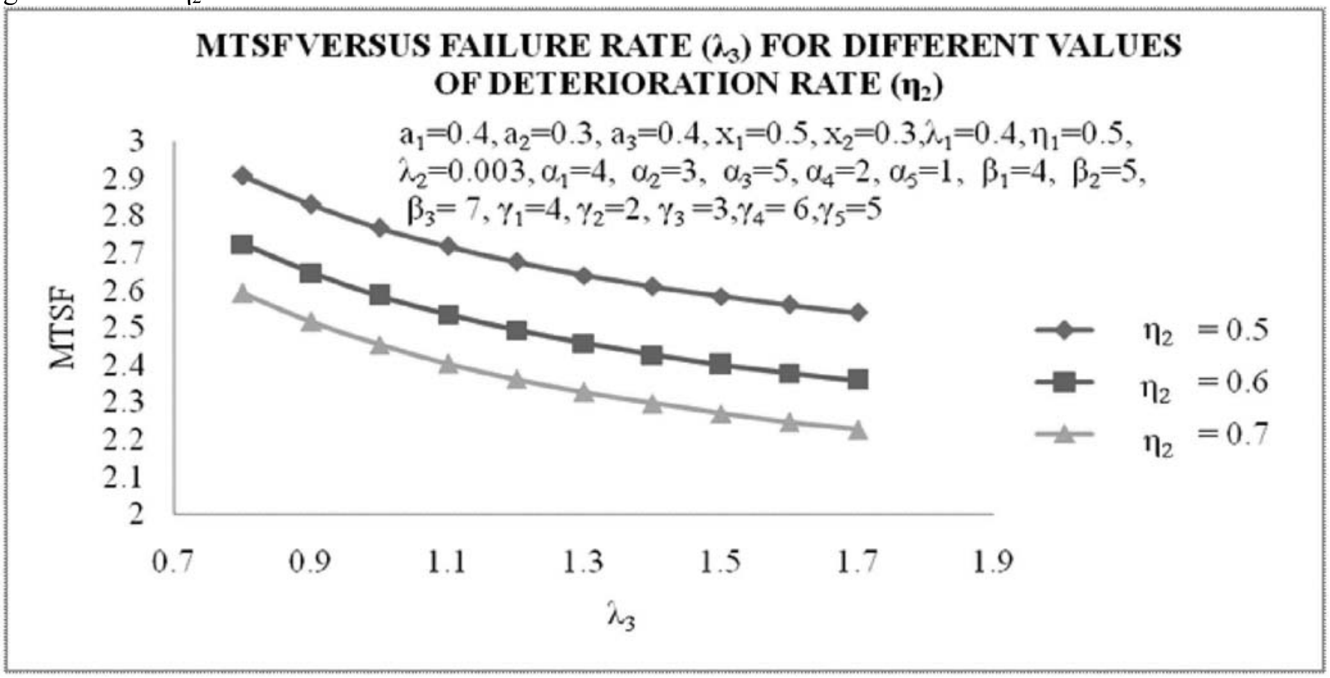

Fig. 4

The graph in fig. 5 shows the behavior of availability (A_0) with respect to failure rate (λ_1) of the system during useful life perod for different values of improvement rate (η_1). It is concluded from the graph that A_0 decreases with the increase in the values of λ_1 and has higher values for higher values of η_1 when other parameters are fixed .

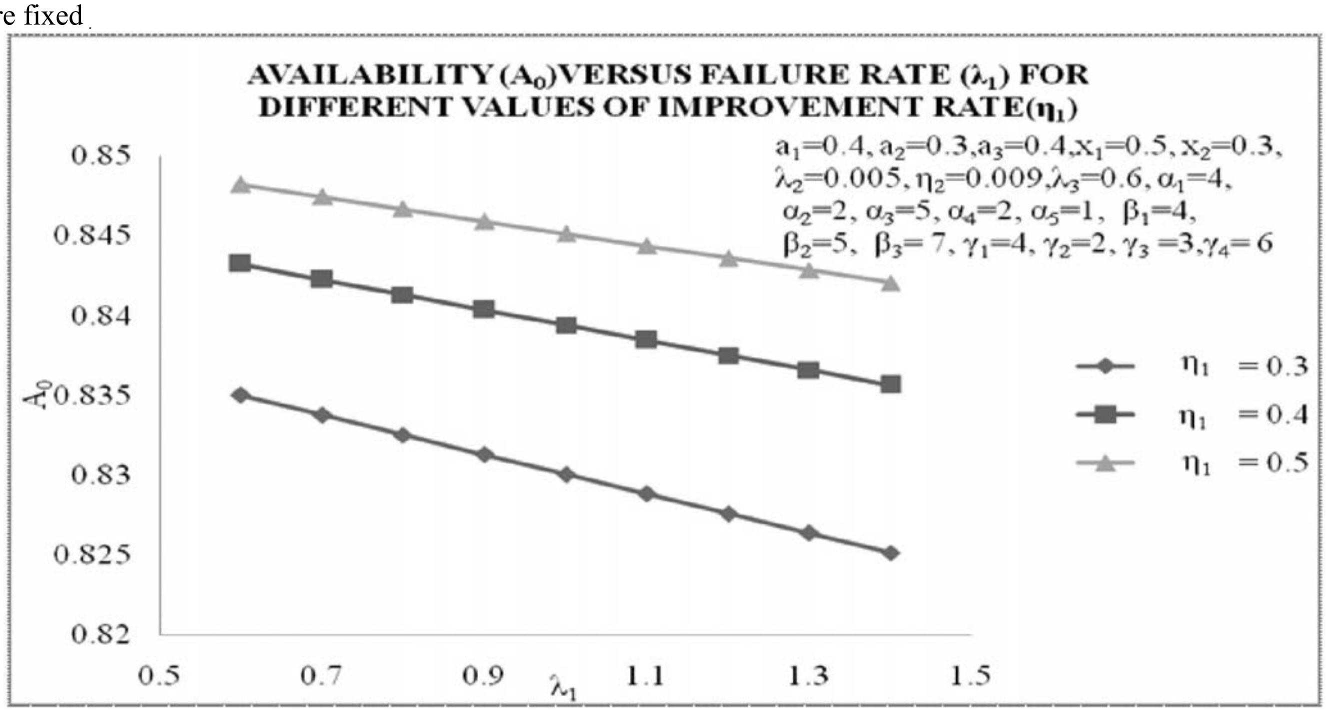

Fig. 5

Fig. 6 presents the behavior of availability(A_0) with respect to failure rate (λ_2) of the system during useful life perod for different values of improvement rate (η_1). It can be concluded from the graph that A_0 decreases with the increase in the values of λ_2 when other parameters are fixed and has higher values for higher values of η_1 .

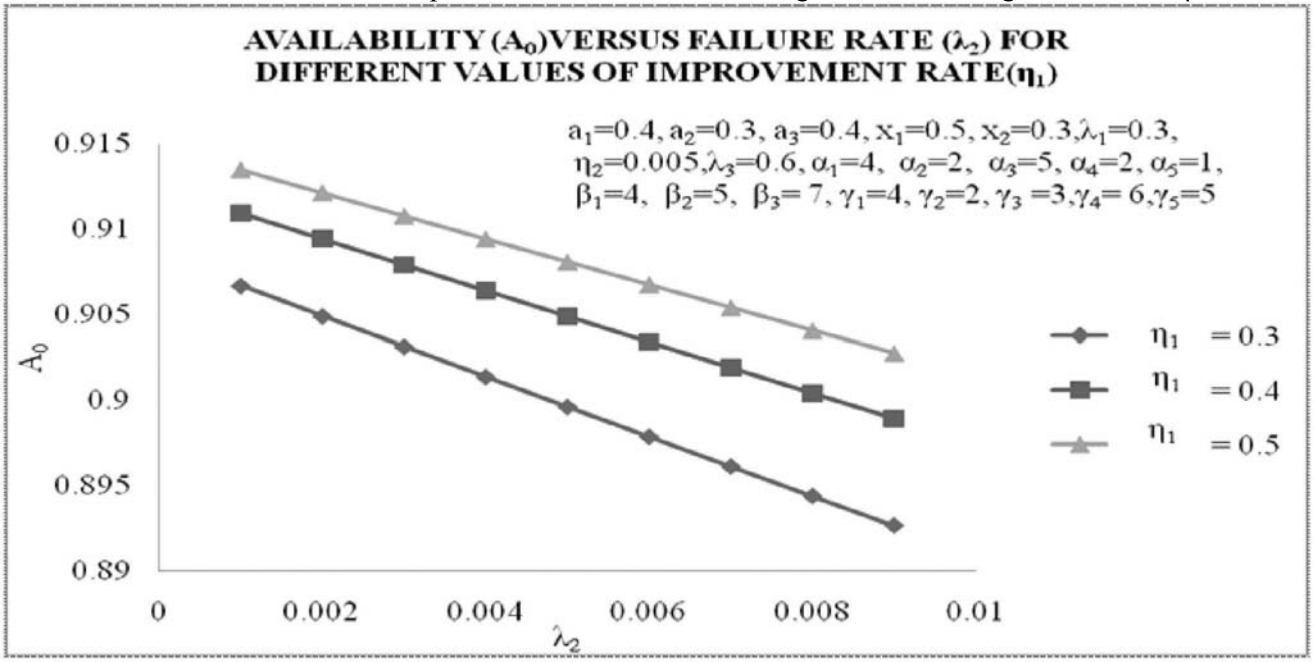

Fig. 6

The curves in fig. 7 shows the pattern of availability(A_0) with respect to failure rate (λ_3) of the system during wear-out perod for different values of deterioration rate (η_2). It is concluded from the graph that A_0 decreases with the increase in the values of λ_3 and has lower values for higher values of η_2 when other parameters are fixed.

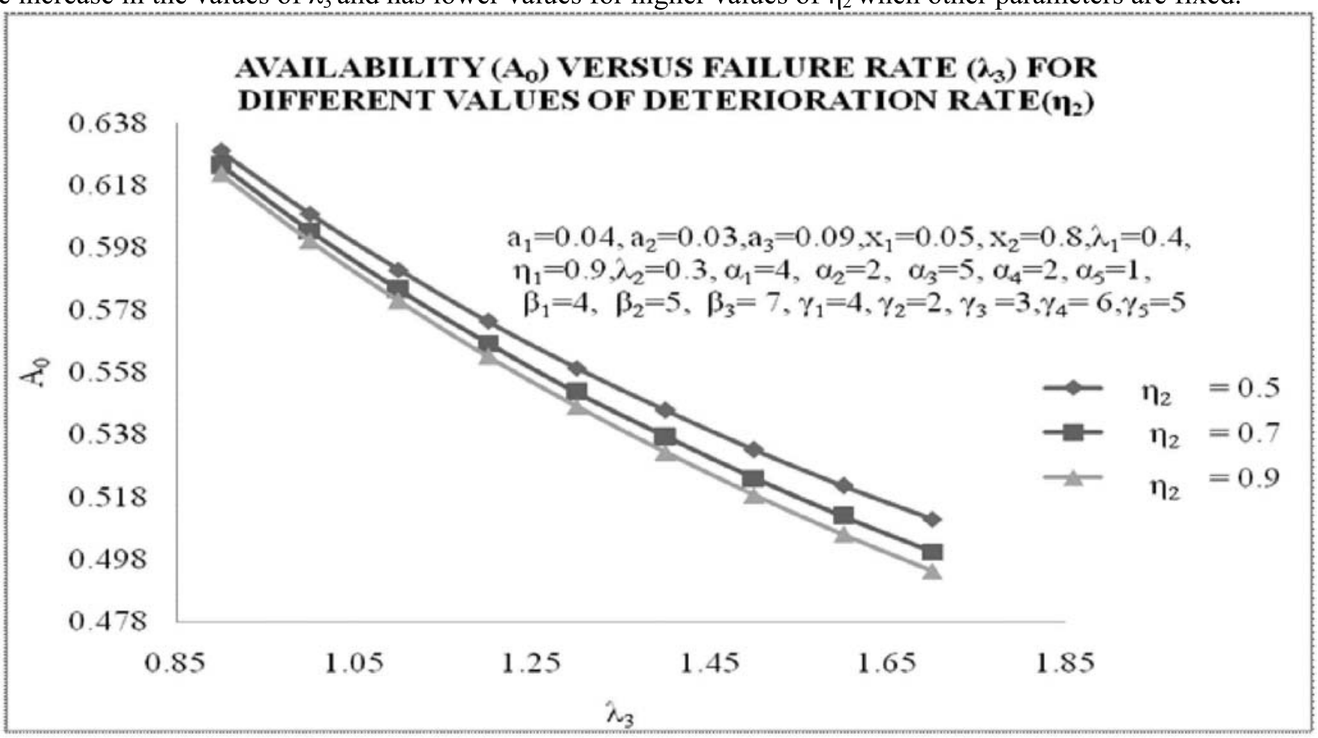

Fig. 7

The curves in the fig. 8 highlights the behaviour of profit P_1 which decreases with the increase in the values of deterioration rate (η_2) and has lower values for higher values of failure rate λ_2. From the fig. 8, it can also be observed that for λ_2 =0.03, P_1 is positive or zero or negative as η_2 < or = or > 0.537430 and thus in this case, the system is profitable whenever deterioration rate should be fixed less than 0.537430. Similarly for λ_2 = 0.05 and λ_2 = 0.07, the system user is profitable whenever η_2 < 0.406172 and 0.298171 respectively.

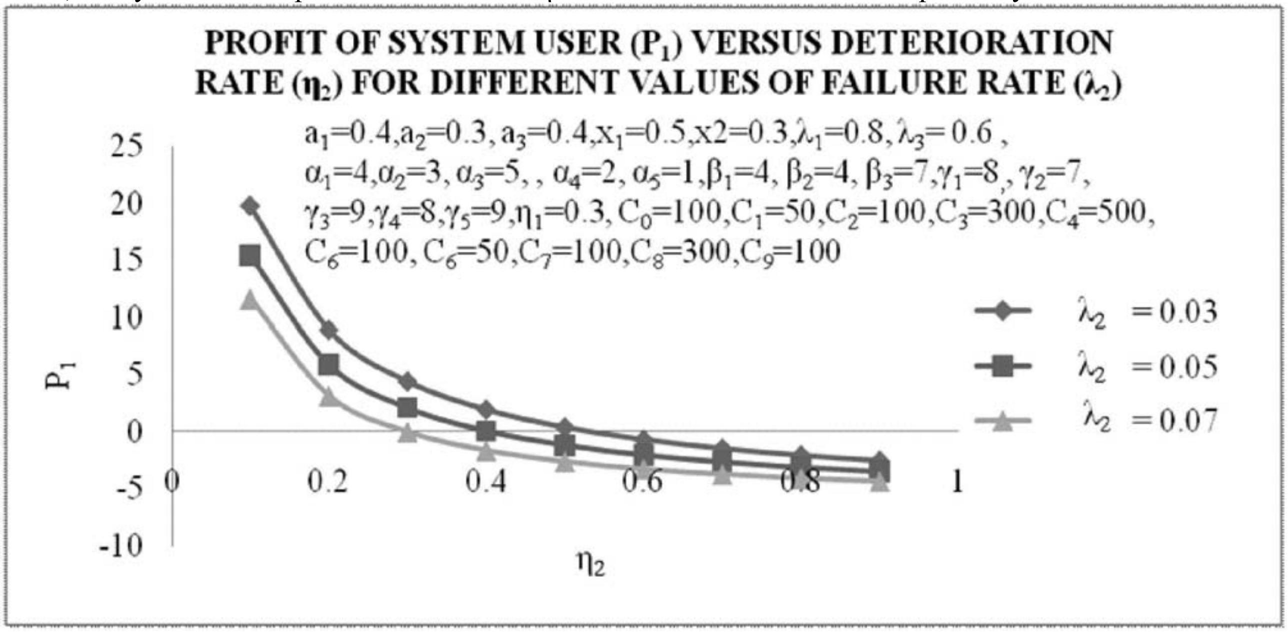

Fig. 8

Fig. 9 represents the behaviour of profit of system user (P_1) with respect to revenue per unit up time (C_0) for different values of cost per visit (C_5) of the service engineer. It can be concluded that profit of system user increases with the increase in the values of revevue per unit uptime and has lower values for higher values of cost per visit of the service engineer. From the fig. 9, it can also be observed that for C_5 = Rs.100, P_1 is positive or zero or negative as C_0 > or = or < Rs.108.57 and thus in this case, the system is profitable whenever revenue per unit up time is greater than Rs. 108.57. Similarly for C_5 = Rs.200 and C_5 = Rs.300, the system user is profitable whenever C_0 > Rs.150.27 and Rs.149.23 respectively.

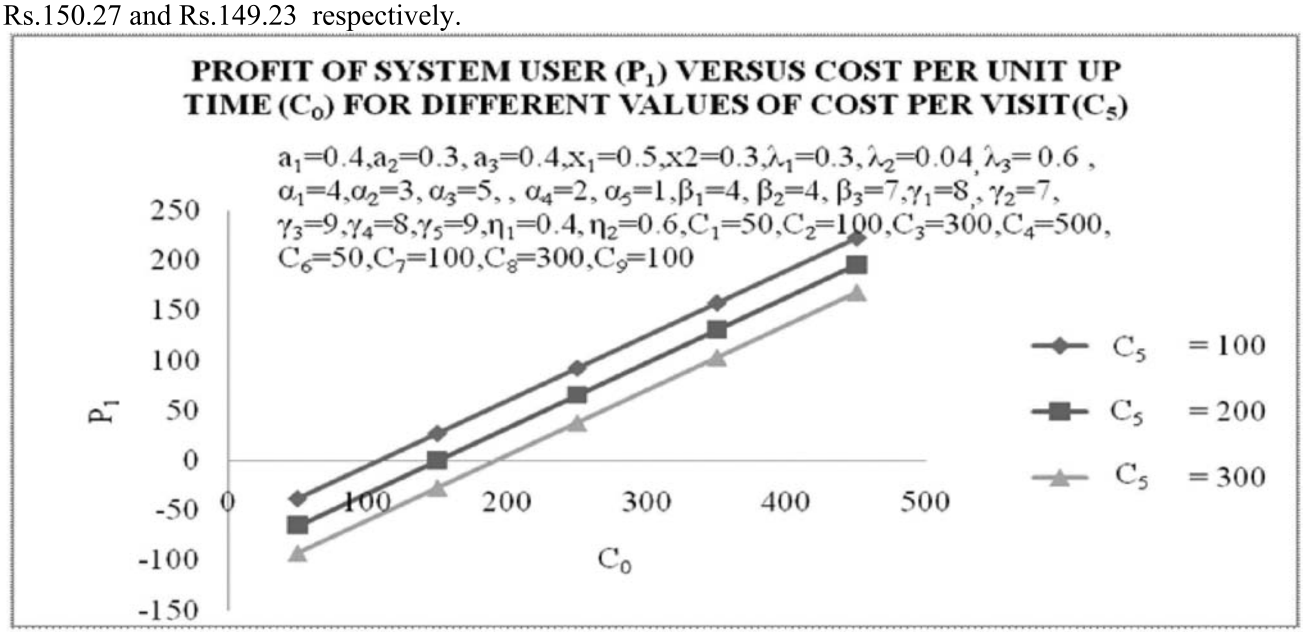

Fig. 9

Fig. 10 depicts the behaviour of profit of system provider (P_2) with respect to cost per visit (C_9) for different values of failure rate (λ_2). It is concluded from the graph that P_2 decreases with the increase in the values of C_9 and has lower values for higher values of λ_2. From the fig. 10, it can also be observed that for $\lambda_2 = 0.1$, P_2 is positive or zero or negative as C_9 < or = or > Rs.4775.49 and thus in this case, the system is profitable whenever C_9 should be fixed less than Rs.4775.49. Similarly for $\lambda_2 = 0.5$ and $\lambda_2 = 0.9$, the system provider is profitable whenever C_9 < Rs.2310.36 and Rs.1789.57 respectively.

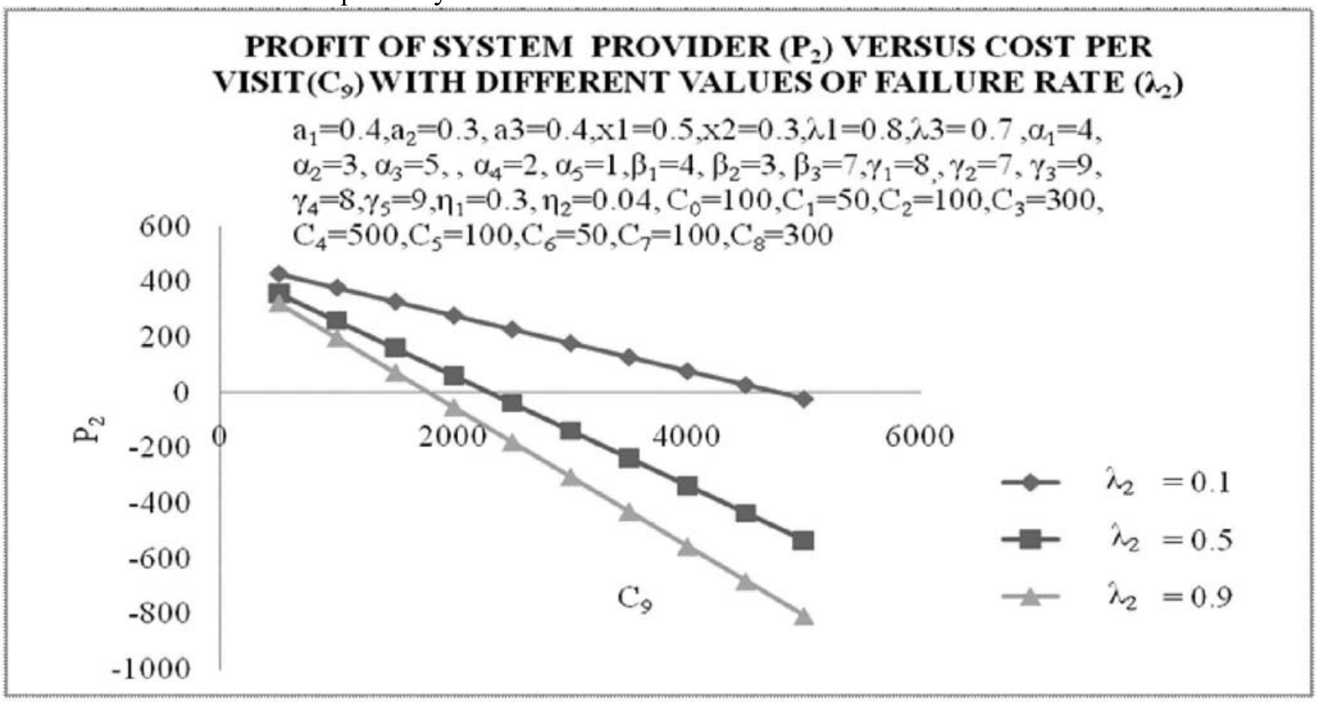

Fig.10

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MATHEMATICAL MODEL OF GAMMA FAMILY ON CORTISOL ASSOCIATED WITH CHRONIC BACK PAIN

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ABSTRACT :
One of the most important way to get a new probability distribution in applied probability is the mixture distribution. Because of this reason, we get a new mixed distribution namely MGG distribution and it is obtained by mixing GG and LGG. The MGG distribution is useful of modelling bathtub-shaped hazard rate, and also it includes some special sub-models of gamma and exponential family. In application part, the hormonal level of Cortisol due to chronic back pain is discussed. It has been identified that maladaptive physiological responses of the organism with a recurrent stressor maintain the chronic pain. The associations between basal levels of Cortisol collected over seven successive days, the hippocampal volumes & brain activation to thermal stimulations is also examined by administering in 16 patients with chronic back pain (CBP) and 18 healthy control subjects. Finally, it is found that patients with CBP have higher levels of Cortisol than control subjects.

Keywords : ***MGG, hazard rate, Cortisol, chronic back pain, stress.***
Mathematics Subject Classification: ***60E05, 62P10.***

1. NOTATIONS

α , β – Shape parameters
λ – Scale parameter
$\Gamma(\alpha)$ – The gamma function
MGG - Mixture Generalised Gamma distribution
LGG - Length biased Generalised Gamma distribution
GG - Generalised Gamma distribution

2. INTRODUCTION

The generalised gamma (GG) was introduced by Stacy [13] and it included special sub-models such as exponential, Weibull, gamma and Rayleigh distribution among other distributions. The GG distribution is assumed for modelling data with dissimilar types of hazard rate. This is used to estimate individual hazard rate and both relative hazard rate and relative times [1, 5]. The concept of length biased distribution has been found useful in various lifetime data such as family history disease and survival events. Simultaneously, LGG distribution is more flexible in modelling data. When the original family of distributions broadened with length biased distributions then one class of distributions was obtained by the two component mixture model. One of the most crucial ways of getting new probability distributions in applied probability and several research areas is the mixture distribution. As looking for a flexible alternative GG distribution so, a new mixture generalised gamma distribution was obtained by mixing the GG distribution with the LGG distribution. While analysing its properties it is found more flexible in fitting lifetime data. It has itself many well known lifetime special sub models. Comparison analysis has been proposed among the GG, LGG, and MGG [11, 14]. The probability density function of GG distribution is,

$$g(x) = \frac{\lambda\beta}{\Gamma(\alpha)} (\lambda x)^{\alpha\beta-1} e^{-(\lambda x)^{\beta}} \text{ , x > 0, α, β, λ > 0.} \quad (1)$$

Length biased Generalised Gamma [1] has the density function as,

$$g_L(x) = \frac{\lambda\beta}{\Gamma(\alpha+\frac{1}{\beta})} (\lambda x)^{\alpha\beta} e^{-(\lambda x)^{\beta}} \text{ , x > 0; α, β, λ > 0.} \quad (2)$$

3. MATHEMATICAL MODEL

3.1. MIXTURE GENERALISED GAMMA DISTRIBUTION

A new mixture distribution is proposed here to create extensively flexible distribution and some special cases are considered [11, 12].

Definition

Let $g(x)$ and $g_L(x)$ are respectively, the probability function and length biased probability function of the random variable X, where x > 0, 0≤ p ≤ 1 then the mixture length biased distribution of X is obtained in the form as $g(x)$ + (1-p) $g_L(x)$ by mixing $g(x)$ and $g_L(x)$.

THEOREM 1

Let X is the random variable of MGG (α, β, λ, p). Then the probability density function is

$$f(x) = \left[\frac{p}{\Gamma(\alpha)} + \frac{(1-p)\lambda x}{\Gamma(\alpha+\frac{1}{\beta})}\right] \lambda\beta\, (\lambda x)^{\alpha\beta-1} e^{-(\lambda x)^{\beta}} \text{ , for x > 0; α, β, λ > 0.} \quad (3)$$

And cumulative distribution function is,

$$F(x) = 1- \frac{p\Gamma(\alpha,(\lambda x)^{\beta})}{\Gamma(\alpha)} - \frac{(1-p)\,\Gamma(\alpha+\frac{1}{\beta}, (\lambda x)^{\beta})}{\Gamma(\alpha+\frac{1}{\beta})} \quad (4)$$

Proof

If X is distributed as mixture generalised gamma distribution with α, β, λ and mixing parameter p, (3) is called the two- component mixture distribution, if its pdf , is getting by replacing (1) and (2) in definition ,is as follows

$$f(x) = p\left[\frac{\lambda\beta}{\Gamma(\alpha)} (\lambda x)^{\alpha\beta-1} e^{-(\lambda x)^{\beta}}\right] + (1-p)\left[\frac{\lambda\beta}{\Gamma(\alpha+\frac{1}{\beta})} (\lambda x)^{\alpha\beta} e^{-(\lambda x)^{\beta}}\right]$$

$$= \left[\frac{p}{\Gamma(\alpha)} + \frac{(1-p)\lambda x}{\Gamma(\alpha+\frac{1}{\beta})}\right] \lambda\beta\, (\lambda x)^{\alpha\beta-1} e^{-(\lambda x)^{\beta}} .$$

Let F (x) is the generalised class of distribution and is generated from MGG as follows

$$F(x) = \int_0^x [p\, g(t) + (1-p) g_L(t)] dt$$

$$= p\int_0^x g(t)dt + (1-p)\int_0^x g_L(t)dt$$

$$= pG(x) + (1-p)G_L(x)$$

Then

$$F(x) = p\left[1 - \frac{\Gamma(\alpha,x)}{\Gamma(\alpha)}\right] + (1-p)\left[1 - \frac{\Gamma\left(\alpha+\frac{1}{\beta},\ x\right)}{\Gamma(\alpha+\frac{1}{\beta})}\right]$$

$$= 1 - \frac{p\Gamma(\alpha\,,(\lambda x)^{\beta})}{\Gamma(\alpha)} - \frac{(1-p)\Gamma(\alpha+\frac{1}{\beta}\,,(\lambda x)^{\beta})}{\Gamma(\alpha+\frac{1}{\beta})}$$

3.2. RESULTS

1. The MGG distribution is reduced to LGG distribution with the parameters α, β and λ When p = 0.
2. The MGG distribution is deduced to GG distribution when p = 1.
3. The MGG distribution is reduced to exponential distribution when α = β = 1and p = 1.

3.3. HAZARD RATE

The hazard rate function of random variable X is defined with pdf f(x) and cdf F(x) as, $h(x) = \frac{f(x)}{1-F(x)}$.

After substituting (3) and (4), the hazard rate of MGG distribution could be defined as,

$$h(x) = \frac{\left[\frac{p}{\Gamma(\alpha)} + \frac{(1-p)\lambda x}{\Gamma(\alpha+\frac{1}{\beta})}\right]\lambda\beta\,(\lambda x)^{\alpha\beta-1}e^{-(\lambda x)^{\beta}}}{1 - \left\{1 - \frac{p\Gamma(\alpha,x)}{\Gamma(\alpha)} - \frac{(1-p)\,\Gamma(\alpha+\frac{1}{\beta},x)}{\Gamma(\alpha+\frac{1}{\beta})}\right\}} \tag{5}$$

$$= \frac{\left[p\Gamma\left(\alpha+\frac{1}{\beta}\right) + (1-p)\lambda x\Gamma(\alpha)\right]\ \lambda\beta\,(\lambda x)^{\alpha\beta-1}e^{-(\lambda x)^{\beta}}}{p\Gamma(\alpha,x)\Gamma\left(\alpha+\frac{1}{\beta}\right) + (1-p)\Gamma(\alpha)\Gamma\left(\alpha+\frac{1}{\beta},x\right)}$$

For different values of parameters, the hazard rate of MGG would become the hazard rate of different distributions.

- The hazard rate of MGG would be LGG when p = 0.
- The hazard rate of MGG would be GG when p = 1.
- The hazard rate of MGG would be exponential distribution when α = β = 1 and p = 1.

4. APPLICATION

Chronic pain is a self-increasing state in which no changes found in the stress system and it give patients suffering & pain–related disability. The level of metabolic activity to adapt to environmental demands is modified by the organism that may finally lead to maladaptive responses including a series of stress–related pathophysiological strain, when facing prolonged, uncertain and uncontrollable threat. Such state has been referred to as allostatic load and moreover, the triggering, the amplification and/or the endurance of the pain state are contributed by this allostatic load. In particular, a weakening of brain grey matter volume or changes in cortical thickness and functional reorganisation of pain- related brain networks including those related to hippocampal formation has been identified as related to chronic pain. In addition, the major adaption changed by the pain state that is known to hippocampal structure and functions, reflected by the patients with chronic pain and also a dysreglation of the HPA axis often presented by them. The persistence of chronic pain states and individual differences in the intensity of clinical pain are explained by contributing the interconnections between maladaptive stress, chronic pain, and hippocampal functions [4]. Sensitivity to the destroying effect of sustained high levels of glucocorticoids caused by over activation of HPA axis and the successive structural and functional changes in the hippocampal formation, it is identified as one of the potential consequences of allostatic load [6].

The prolonged pain is found that it may constitute an allostatic load in individuals showing more stress vulnerability, including long-lasting changes that initiate a falling down of the patient's condition. A multivariate study was conducted for patients with chronic pain and age-and sex-matched healthy controls to examine the associations between the basal levels of Cortisol, the structural volumetric morphology of the hippocampal and functional brain activity to phasic thermal noxious stimuli [2, 3]. Depends on the model of allostatic load in chronic pain it is assumed that basal levels of Cortisol measured over seven consecutive days would be higher in patients with chronic back pain. Then it is considered that patients with chronic back pain show higher levels of basal Cortisol would have smaller hippocampii and stronger pain evoked activity measured by using blood oxygen level dependent(BOLD)-functional MRI in the anterior hippocampal formation [7, 8, 16].

4.1. SALIVARY CORTISOL

Throughout a full week basal salivary Cortisol levels were measured. Participants were asked to fill pure saliva in a small plastic vial using a straw. An instruction also given to them to collect five samples /day over seven consecutive days, starting exactly the day after the brain scanning. Samples were collected by the participants on each day at their awakening, 30 min after awakening, at noon, in the afternoon, and at bedtime. Eventually patients with chronic back pain show higher levels of Basal Cortisol than matched healthy control subjects.

4.2. PAIN PROCEDURE

Two runs of thermal pain applied to the lower leg of the participants and two separate scans during which the participant observed images displaying pain-evoking situations were consisted in the brain imaging session. Pain-evoking images data will be presented in a separate report. Each functional scan consisted of eight noxious and eight innocuous thermal stimulation applied in a pseudorandom order making the intensity of the stimulation unpredictable [10]. BOLD responses evoked by painful and non-painful thermal stimulation administered to the lower leg of the participant were compared to assess pain-related brain activations in patients with chronic back pain and healthy participants. But there are no notable results in pain-related brain activation among groups.

4.3. HIPPOCAMPAL VOLUMES

The cortical reconstruction and the volumetric segmentation of the hippocampus were performed. The procedure includes the motion correction, the removal of the skull using watershed/ surface deformation procedure etc., the volume of each hemispheric hippocampus was extracted for each subject. Finally, patients with chronic back pain of smaller hippocampal volumes have higher levels of basal Cortisol [9].

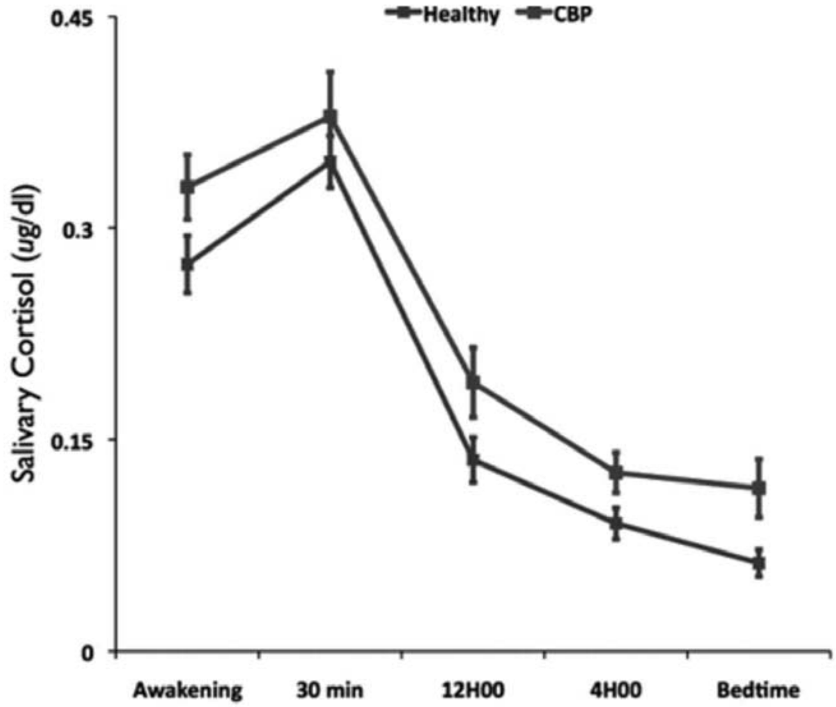
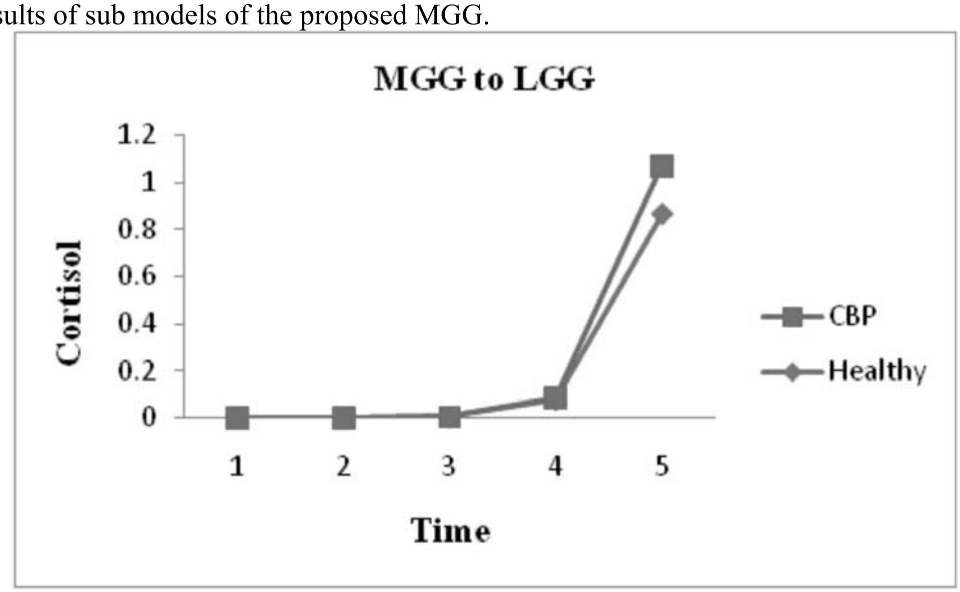
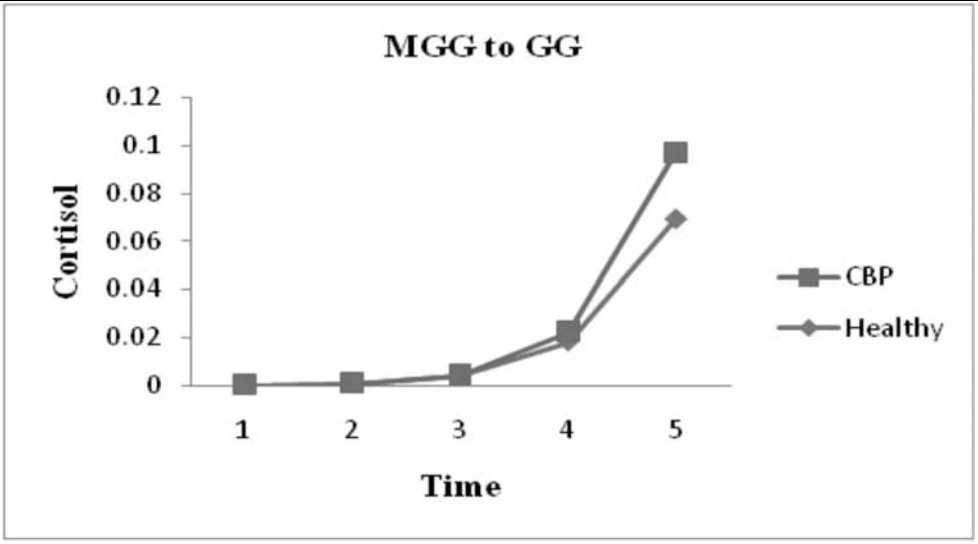


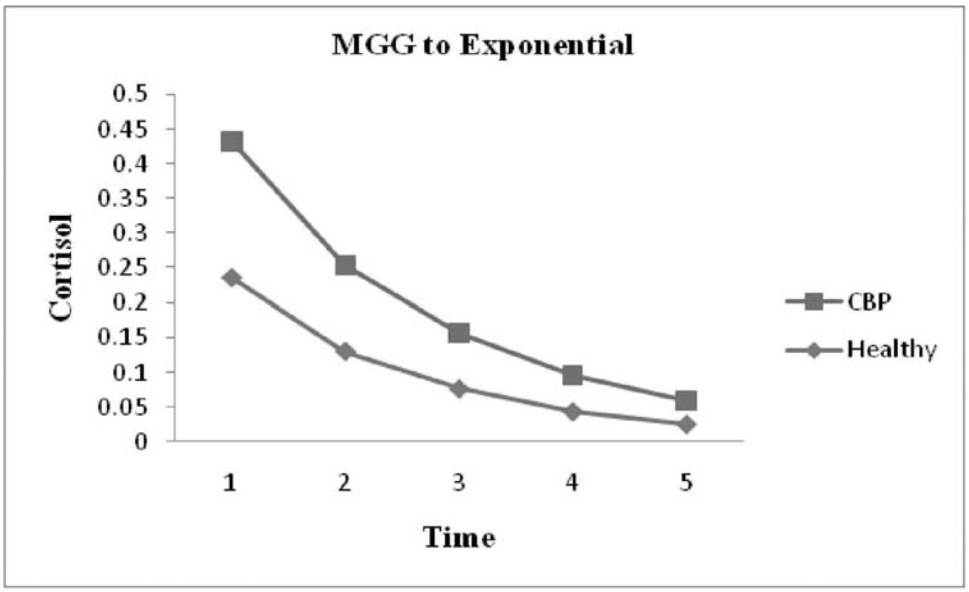
Figure 1: Diurnal basal glucocorticoid activity averaged over seven consecutive days. The patients with chronic back pain (CBP) had significantly increased stress hormone activity compared with the healthy control subjects.

5. MATHEMATICAL RESULTS

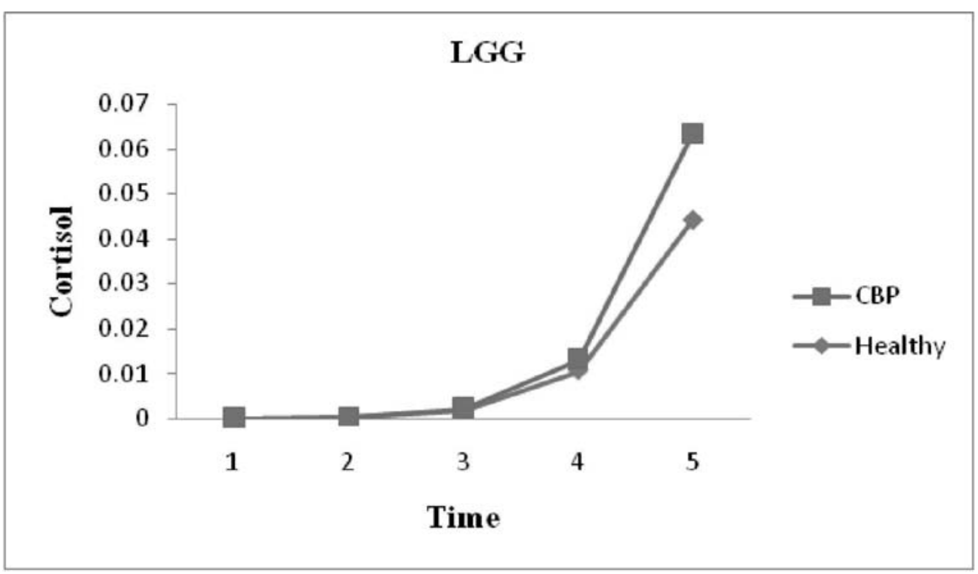
Mathematical results of sub models of the proposed MGG.

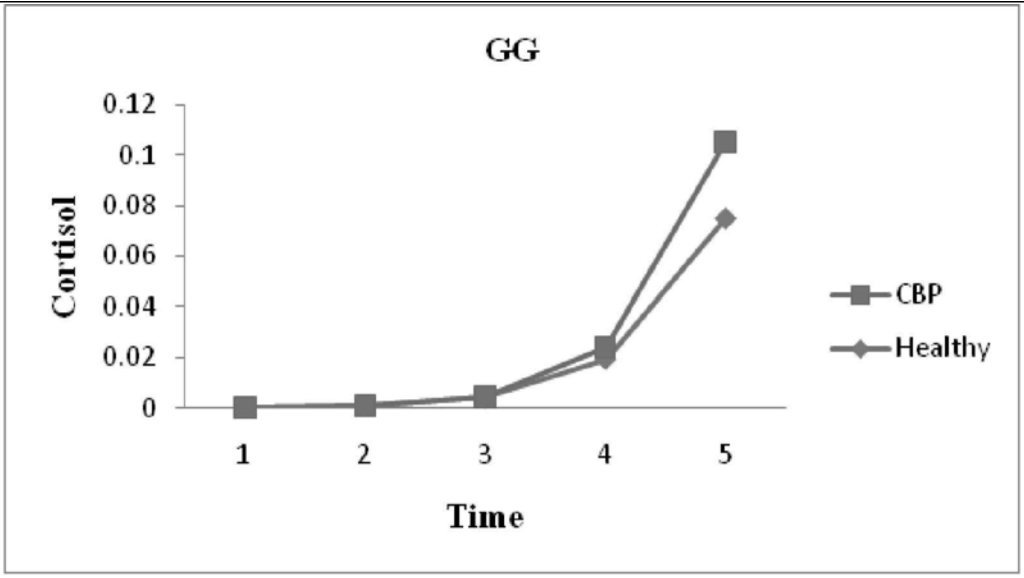


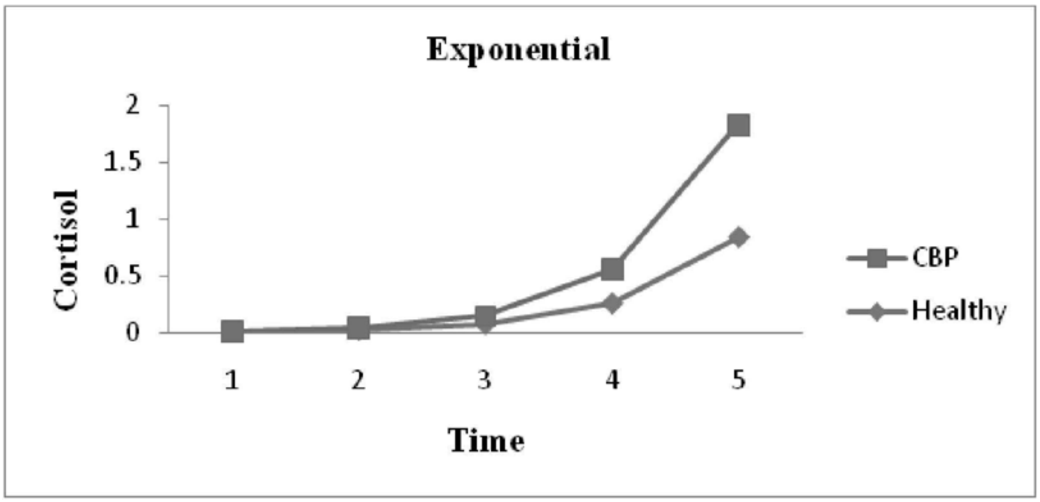




Mathematical results of Hazard rate of LGG, GG and Exponential distributions.







6. CONCLUTION

In this paper we proposed a new family of GG distribution that is the MGG distribution, and is obtained by mixing GG distribution with LGG distribution. When compare to LGG and GG distribution it is found that MGG distribution provides a considerably better fit than LGG and GG based on p values. Likewise, in application part, it is found that chronic back pain was associated with increased pain-related responses in anterior hippocampal formation. Moreover, it showed that chronic back pain (CBP) have been associated with higher basal Cortisol levels in patients than healthy controls. When applying this result to the sub models of MGG and their hazard rates we conclude that, patients with CBP have higher levels of Cortisol than healthy controls.

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A NOTE ON UNDEFINED INTEGRAL

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ABSTRACT :
In the literature various workers obtained the values of some typical integration with the help of Watson, Whipple and Dixon theorem. In this research note we obtain the value of an undefined integral with the help of a result of Watson's theorem.

Key Words and Phrases. ***Integral representation, Watsons' theorem.***

1. Introduction

MacRobert [3, p. 450] gave us a very interesting integral

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[1+cx+d(1-x)]^{-\alpha-\beta}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{(1+c)^{\alpha}(1+d)^{\beta}\Gamma(\alpha+\beta)} \qquad \ldots(1.1)$$

Provided $\Re e\,(\alpha) > 0$, $\Re e\,(\beta) > 0$ and the constants c and d are such that none of the expressions 1 + c, 1 + d and [1 + cx + d (1-x)]

Where $0 \le x \le 1$, are zero.

Also watson's theorem [4] plays an important role in the summation of series. Rathie gave us some valuable generalization of Watson's theorem[1].One generalization of Watson's theorem on the sum of a ${}_3F_2$ viz.

Watson's Summation Theorem[2]

$${}_3F_2\left[\begin{matrix} a, b, c \\ \frac{1}{2}(a+b+i+1), 2c \end{matrix} \;\middle|\; 1\right]$$

$$= \frac{\Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2})\quad\Gamma(\frac{1}{2})\quad\Gamma(\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}i+\frac{1}{2})}{\Gamma(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}|i|+\frac{1}{2})}$$

$$\sum_{m=0}^{\infty} \frac{(\frac{1}{2}a)_m \quad (\frac{1}{2}a+\frac{1}{2})_m \quad (\frac{1}{2}b)_m(\frac{1}{2}b+\frac{1}{2})_m}{(c+\frac{1}{2})_m \quad m!}$$

$$\left\{\frac{A_i{}'}{\Gamma(\frac{1}{2}a+\frac{1}{2})\quad\Gamma(\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}-\left[\frac{1+i}{2}\right])\quad(\frac{1}{2}a+\frac{1}{2})_m\quad(\frac{1}{2}b+\frac{1}{2}i-\left[\frac{1+i}{2}\right]+\frac{1}{2})_m}\right.$$

$$\left.\frac{B_i{}'}{\Gamma(\frac{1}{2}a)\ \ \Gamma(\frac{1}{2}b+\frac{1}{2}i-\left[\frac{i}{2}\right])\ \ (\frac{1}{2}a)_m\ \ (\frac{1}{2}b+\frac{1}{2}i-\left[\frac{i}{2}\right])_m}\right\} \qquad \ldots(1.2)$$

For i = 0, ± 1, ± 2, ± 3, ± 4, ± 5, and for $\Re e$ (a) ≠ $\Re e$ (b). The coefficients A_i' and B_i' can be obtained from the tables of A_i and B_i of generalized gauss's second theorem by changing a to a + 2m and b to b + 2m, respectively.

2. Results Required

The following results will be required in our present investigation [7]

$$\int_0^1 x^{\mu-1}(1-x)^{\sigma-1}[1+cx+d(1-x)]^{-\mu-\sigma}\ {}_2F_1\left(a,b;\lambda;\frac{(1+c)x}{1+cx+d(1-x)}\right)dx$$

$$=\frac{\Gamma(\mu)\Gamma(\sigma)}{(1+c)^{\mu}(1+d)^{\sigma}\Gamma(\mu+\sigma)}\,{}_3F_2(a,b,\mu;\lambda,\mu+\sigma;1) \qquad \ldots(2.1)$$

Provided $\Re e$ (μ) > 0, $\Re e$ (σ) > 0, $\Re e$ (λ + σ - a - b)) > 0 and the constants c and are such that none of the expressions 1 + c, 1 + d and 1 + cx + d (1-x) where 0 ≤ x ≤ 1, are zero.

Put i = 3 in equation (1.2)

$${}_3F_2\left[\begin{matrix} a,b,c \\ \frac{1}{2}(a+b+4),2c \end{matrix}\ \middle|\ 1\right]$$

$$=\frac{\Gamma(\frac{1}{2}a+\frac{1}{2}b+2)\quad\Gamma(2c)\quad\Gamma(\frac{1}{2}a-\frac{1}{2}b-1)\quad\Gamma(c-\frac{1}{2}a-\frac{1}{2}b-1)}{2^{2c-a-b+3}\Gamma(a)\Gamma(b)\Gamma(c)\Gamma(\frac{1}{2}a-\frac{1}{2}b+2)}$$

$$\left\{(6bc-4c+2ac-a^2-3b^2+4)\frac{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b)}{\Gamma(c-\frac{1}{2}b+\frac{1}{2})\quad\Gamma(c-\frac{1}{2}a)}\right.$$

$$\left.+(4c-6ac+3a^2-2bc+b^2-4)\frac{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2})}{\Gamma(c-\frac{1}{2}b)\quad\Gamma(c-\frac{1}{2}a+\frac{1}{2})}\right\} \qquad \ldots(2.2)$$

$\Re e$ (2c – a – b) > -4.

3. Main Results

The following results will be established in this section

$$\int_0^1 x^{e-1}(1-x)^{e-1}[1+cx+d(1-x)]^{-2e}\ {}_2F_1\left[\begin{matrix} a,b \\ \frac{1}{2}(a+b+4)\end{matrix}\Big|\ \frac{(1+c)x}{1+cx+d(1-x)}\right]dx$$

$$=\frac{\Gamma(e)\Gamma(\frac{1}{2}a+\frac{1}{2}b+2)\Gamma(\frac{1}{2}a-\frac{1}{2}b-1)\Gamma(e-\frac{1}{2}a-\frac{1}{2}b-1)}{(1+c)^e(1+d)^e\quad 2^{2e-a-b+3}\Gamma(a)\Gamma(b)\Gamma(\frac{1}{2}a-\frac{1}{2}b+2)}$$

$$\left\{6eb-4e+2ae-a^2-3b^2+4)\frac{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b)}{\Gamma(e-\frac{1}{2}b+\frac{1}{2})\Gamma(e-\frac{1}{2}a)}\right.$$

$$\left.+(4e-6ae+3a^2-2be+b^2-4)\frac{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2})}{\Gamma(e-\frac{1}{2}b)\Gamma(e-\frac{1}{2}a+\frac{1}{2})}\right\} \qquad \ldots(3.1)$$

Proof. We start with the following integral

$$\int_0^1 x^{\mu-1}(1-x)^{\sigma-1}[1+cx+d(1-x)]^{-\mu-\sigma}\ {}_2F_1\left(a,b;\lambda;\frac{(1+c)x}{1+cx+d(1-x)}\right)dx$$

$$=\frac{\Gamma(\mu)\Gamma(\sigma)}{(1+c)^{\mu}(1+d)^{\sigma}\Gamma(\mu+\sigma)}{}_3F_2(a,b,\mu;\lambda,\mu+\sigma;1)$$

Provided $\Re e\ (\mu) > 0$, $\Re e\ (\sigma) > 0$, $\Re e\ (\lambda + \sigma - a - b)) > 0$ and the constants c and are such that none of the expressions $1 + c$, $1 + d$ and $1 + cx + d\ (1-x)$ where $0 \le x \le 1$, are zero.

If we take $\mu = e$, $\sigma = e$ and $\lambda = \frac{1}{2}\ (a + b + 4)$ the we obtain integral

$$\int_0^1 x^{e-1}(1-x)^{e-1}[1+cx+d(1-x)]^{-2e}\ {}_2F_1\left(a,b;\frac{1}{2}(a+b+4);\frac{(1+c)x}{1+cx+d(1-x)}\right)dx$$

$$=\frac{\Gamma(e)\Gamma(e)}{(1+c)^e(1+d)^e\Gamma(2e)}{}_3F_2(a,b,e;\frac{1}{2}(a+b+4),2e;1)$$

Clearly in right hand side ${}_3F_2$ is the result of Watson's theorem for i = 3 so use equation (2.2) and after little simplification, we arrive at required result. Various workers [5,6] have been given their valuable work for solving of typical integration.

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A COMPARATIVE STUDY OF FUZZIFIED CPU SCHEDULING POLICIES IN REAL TIME SYSTEM

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ABSTRACT :
Several scheduling policies like FCFS,SJF, Priority, RR, SRTN etc. have been studied so far. The scheduling decision from these policies are generally based on crisp parameters. However, practically it has been observed that in many real world situations these parameters are vague or imprecise in nature. Due to the vagueness behavior, the scheduler uses fuzzy logic in scheduling algorithms that incorporates fuzziness of CPU burst time. A comparative study has been made regarding performance measures based on bubble sort of various CPU scheduling policies. The study helps the designer in selecting the best scheduling policy at high abstraction level.
Keywords: ***Round Robin, multiprogramming, CPU scheduling, defuzzification, etc.***

1. Introduction:

In a real time system, each computational entity e.g. a task, a process, a thread, or a truncation has a fixed deadline when submitted to the system. These entities must be scheduled and processed in such a way that they are completed well in time according to their corresponding deadlines. There are several scheduling algorithms with different properties that are available in literature. The choice of a particular algorithm may prefer one set of process over another. Various criteria have been explored for comparing CPU scheduling algorithms and to take decision for the selection of the best one. Some of the criteria include

(a) CPU utilization throughput,
(b) turnaround time,
(c) waiting time,
(d) Response time.

The objective of our study on one hand is to minimize turnaround time, waiting time to avoid starvation, so that CPU scheduler may not remain idle. i.e. its utilization and throughput be maximized, on the other hand to compare the output parameters in these scheduling policies for the optimal one to be adopted.

Various scheduling algorithms are available in the literature e.g. FCFS, SJF, SRTN,(Shortest Remaining time next), round robin, preemptive priority, multilevel feedback queue scheduling, etc. In these scheduling algorithms, generally the parameters used are crisp, however, in many practical circumstances these parameters are vague in nature. To exploit this vagueness we use fuzzy logic in our proposed scheduling algorithm. In this paper, the CPU burst time of these processes are considered to be fuzzy in nature. This paper considers 12 homogeneous processes that execute bubble sort algorithm with different number of elements and are aligned in a ready queue. We have defuzzified the CPU burst time with 2 different cases via triangular and trapezoidal membership function. The defuzzified lowest time has been used to calculate average turnaround time used in different scheduling policies.

Following introductory part the rest of the paper is organized as: Section2 highlights the literature survey. Section 3 applies bubble sorting technique in order to sort various random numbers and to calculate their respective CPU burst time. Section 4 explores various scheduling policies which are applied on the CPU burst time in order to calculate the average waiting time and average turnaround time. Section 5 makes an analysis of the result. Section 6 draws the conclusion.

2. Literature Survey:

A wealth of literature can be found in area of scheduling .From the mathematical point of view, the scheduling problems was solved by various researchers as Johnson[1954], Chen S.H & Hseih(1999), Maggu and Das[1977], Szwarc [1974], Singh T.P [1985,2006,2010].Most of these researchers applied heuristic technique for solving n jobs 2 machines, n jobs 3 machines and n jobs n machines problems under different parameters and criteria. Liu and Leyland[1973] for the first time studied priority driven algorithm in multiprogramming under real world situation. Yashuwanth et al.[2010] designed a real time scheduler simulator. Garg and Vikram Singh[2010] developed a simulator for real time operating system in stochastic environment. Mehdi Neshat et al.[2012] explored a scheduling policy which optimized the average waiting time and response time for system process. Raheja and Supriya [2012] introduced the concept of time quantum using linguistic synthesis for round robin CPU scheduling algorithm. Singh et al[2013] developed a heuristic algorithm for general priority job scheduling in fuzzy environment and obtained optimal solution to the general machine scheduling. Recently, Silky and T.P Singh [2014] extended the work of earlier researchers on the basis of quick sort by considering the CPU burst time in fuzzy environment. The present study is further extended the work of Raheja et al. & Silky Miglani et al. in the sense that the study has been made on the basis of BUBBLE SORT and the CPU burst time has been considered in triangular as well as trapezoidal fuzzy in nature. Some new concepts, modification and graphs have been shown to evaluate better analysis of the system. The average waiting time and turnaround time of the scheduling policies are analyzed for a real world environment.

Section 3:**Fuzzy triangular and trapezoidal membership function.**

Fuzzy logic was developed by Zadadh (1965).

Defuzzification of Triangular Fuzzy number- Let A=(a,b,c) be a triangular fuzzy number,then its associated crisp number is given by Yager's formula,

$A=(3b+(c-a))/3$

Defuzzification of Trapezoidal Fuzzy number- Let A=(a,b,c,d) be a trapezoidal fuzzy number ,then its associated crisp number is given by Chen's formula,

$A=(a+2b+2c+d)/6$

3(a): Designing of Bubble sort program in C++ to calculate CPU Burst Time.

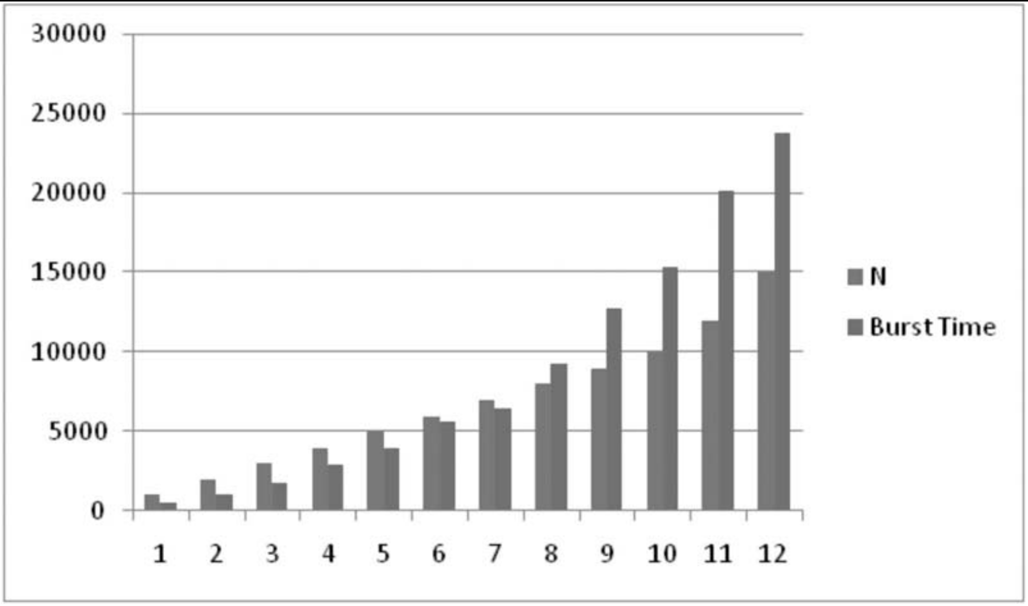
```
#include<iostream.h>
#include<stdlib.h>
#include<time.h>
#define size 14000
void bubble_sort(long list[], long n)
{
  long c, d, t;
  for (c = 0 ; c < ( n - 1 ); c++)
  {
   for (d = 0 ; d < n - c - 1; d++)
    {
    if (list[d] > list[d+1])
  {
/* Swapping */
           t         = list[d];
             list[d]   = list[d+1];
             list[d+1] = t;
      }
    }
```

```
  }
}
int main()
 {
 long i,randnum[15000];
 double total_time;
 clock_t start,end;
 start=clock();
 srand((unsigned)clock());
 cout<<"start generating random numbers"<<endl;
 for(i=1;i<=size;i++)
 {
 randnum[i]=rand();
 cout<<randnum[i]<<"\t";
 }
 bubble_sort(randnum,size);
  for(i=1;i<=size;i++)
  {
 cout<<randnum[i]<<"\t";
 }
 end=clock();
 cout<<"cps"<<CLOCKS_PER_SEC<<"\n";
 total_time=((double)(end-start))/85899345;
 cout<<"start time---\n"<< start;
 cout<<"end time\n"<<end;
 cout<<"time taken to print "<<size <<" random numbers of given size"<<total_time;
 return (0);
 }
```

3 (b):Calculation of CPU burst time based on bubble sort.

Table 1:CPU Burst time(Triangular Membership function)

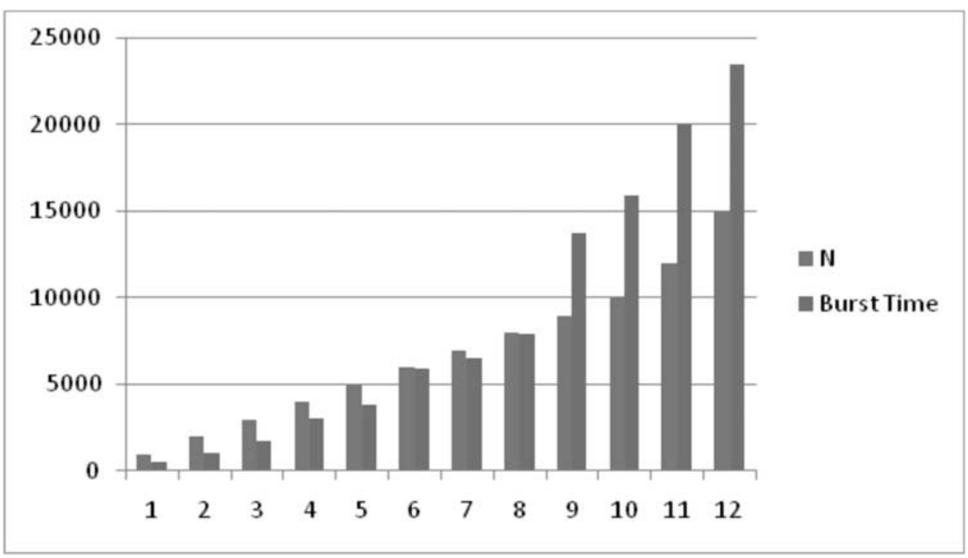
S. No.	N	Burst time T (a)-ns	Burst time T (b)-ns	Burst time T (c)-ns	Final values(Nanoseconds) CPU BURST (TRIANGULAR)
1	1000	384.170	407.453	582.076	473.5555
2	2000	954.6056	931.3226	1164.153	1001.17173
3	3000	1653.098	1676.381	1804.438	1726.82767
4	4000	2735.76	3003.515	2852.175	2941.4266667
5	5000	3573.59	3853.347	3958.121	3981.524
6	6000	4994.217	5261.973	6170.012	5653.9046667
7	7000	5809.125	6193.295	6612.39	6461.05
8	8000	4330.65	7986.098	8381.903	9336.5156667
9	9000	9790.529	10838.27	15646.22	12790.167
10	10000	13527.46	13911.63	17997.81	15401.7466667
11	12000	19057.19	19825.53	20232.98	20217.46
12	15000	22398.31	23351.74	23818.58	23825.1633333



Graph 1 showing burst time for triangular membership function.

Table 2:CPU Burst time(Trapezoidal Membership function)

S.No.	N	Burst time T (a)-ns	Burst time T (b)-ns	Burst time T (c)-ns	Burst time T (d)-ns	Final values(Nanoseconds) CPU BURST(TRAPEZOIDAL)
1	1000	384.170	407.453	582.076	593.718	492.812667
2	2000	954.6056	931.3226	1164.153	1199.078	1057.4391333
3	3000	1653.098	1676.381	1804.438	1874.287	1748.1705
4	4000	2735.76	3003.515	2852.175	3713.649	3026.7981667
5	5000	3573.59	3853.347	3958.121	3966.479	3860.5008333
6	6000	4994.217	5261.973	6170.012	7520.43	5896.4361667
7	7000	5809.125	6193.295	6612.39	7706.694	6521.1981667
8	8000	4330.65	7986.098	8381.903	10721.85	7964.750333
9	9000	9790.529	10838.27	15646.22	19802.25	13760.2931667
10	10000	13527.46	13911.63	17997.81	18137.51	15913.975
11	12000	19057.19	19825.53	20232.98	20861.63	20005.9733333
12	15000	22398.31	23351.74	23818.58	24563.63	23550.43



Graph 2 showing burst time for trapezoidal membership function.

3(c) CPU Burst Time Arrangement

The average waiting and turnaround time on the basis of bubble sort for various scheduling algorithms are calculated and compared by assuming the 12 processes in queue with different arrangements of CPU Burst Time.Keeping CPU Burst Time same,their priorities have been numbered in ascending,descending or random order w.r.to their burst times.

1.) **Burst time in ascending order** :The list of 12 processes with their burst arranged in ascending order is shown in Table 3.

2.) **Burst time in random order** :The list of 12 processes with their burst arranged in random order has been shown in Table 4.There can be many other combinations of the processes in random order that can be arranged in queue.

3.) **Burst time in descending order**: The processes can be arranged in the decreasing order of their burst time in the queue keeping the longest process at the first position and moving towards the process with the shortest burst time as shown in Table5.The difference in the behavior of the scheduling algorithms is observed by changing the order of burst time of the processes in the queue.

Table3: Ascending Order

S.No.	N(size)	Triangular	Trapezoidal	Priority
1	1000	473.5555	492.812667	1
2	2000	1001.17173	1057.4391333	2
3	3000	1726.82767	1748.1705	3
4	4000	2941.4266667	3026.7981667	4
5	5000	3981.524	3860.5008333	5
6	6000	5653.9046667	5896.4361667	6
7	7000	6461.05	6521.1981667	7
8	8000	9336.5156667	7964.750333	8
9	9000	12790.167	13760.2931667	9
10	10000	15401.7466667	15913.975	10
11	12000	20217.46	20005.9733333	11
12	15000	23825.1633333	23550.43	12

Table 4:Random Order

S.No.	N	Triangular	Trapezoidal	Priority
1	15000	23825.163	23550.43	12
2	1000	473.555	492.8126	11
3	12000	20217.46	20005.973	10
4	2000	1001.1713	1057.4391	9
5	10000	15401.746	15913.975	8
6	3000	1726.8276	1748.1705	7
7	9000	12790.167	13760.2931	6
8	4000	2941.4266	3026.7981	5
9	8000	9336.515	7964.7503	4
10	5000	3981.524	3860.5008	3
11	7000	6461.05	6521.1981	2
12	6000	5653.05	5896.4361	1

Table 5:Descending Order

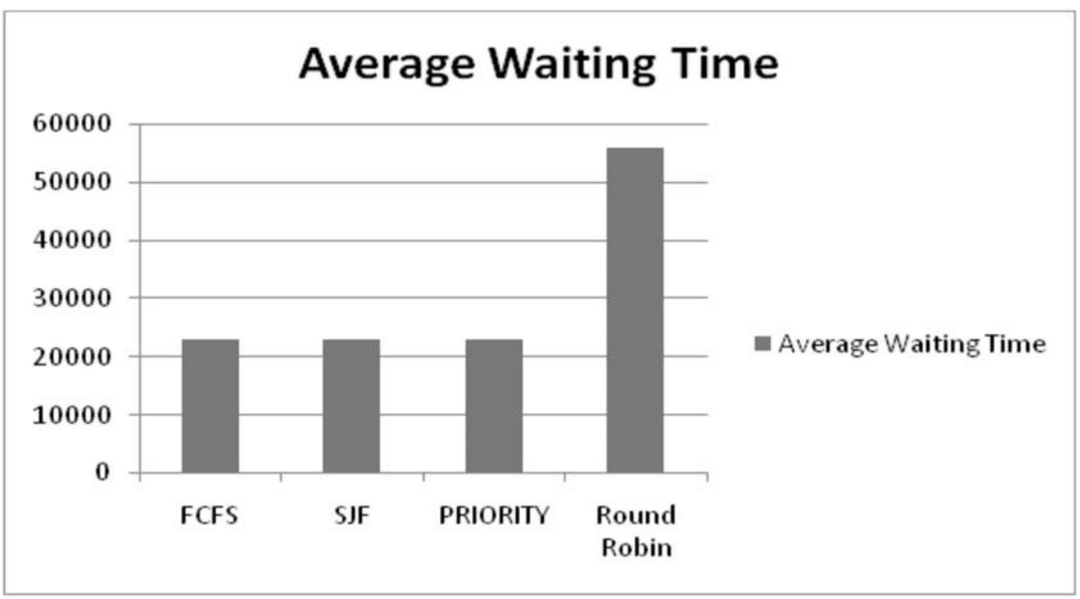
S.No.	N	Triangular	Trapezoidal	Priority
1	15000	23825.1633333	23550.43	12
2	12000	20217.46	20005.9733333	11
3	10000	15401.7466667	15913.975	10
4	9000	12790.167	13760.2931667	9
5	8000	9336.5156667	7964.750333	8
6	7000	6461.05	6521.1981667	7
7	6000	5653.9046667	5896.4361667	6
8	5000.	3981.524	3860.5008333	5
9	4000	2941.4266667	3026.7981667	4
10	3000	1726.82767	1748.1705	3
11	2000	1001.17173	1057.4391333	2
12	1000	473.5555	492.812667	1

4. Comparative Study:

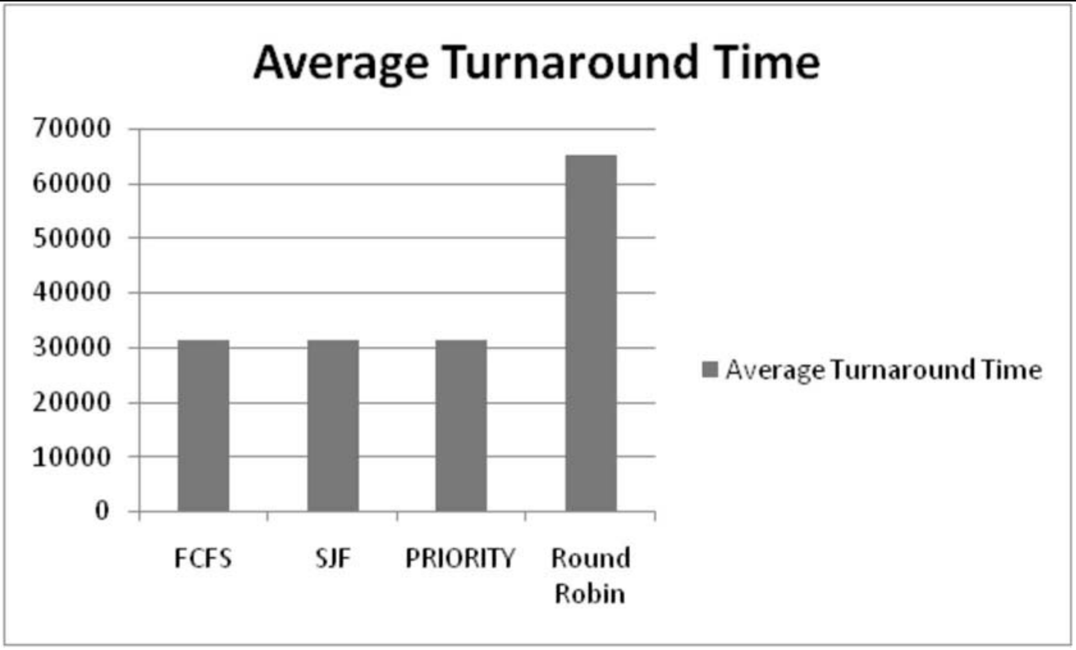
Priority scheduling algorithms, First come First serve (FCFS) and Shortest Job First (SJF) have been implemented and the comparative study is made and in this paper we have discussed the benchmarks among these algorithms and the concluding benchmarks are made on these basis. The results obtained after comparison of these algorithms are shown in the Tables 6-8.The analysis are done for both triangular and trapezoidal membership functions for the defuzzified values of the burst time.

Table 6: Comparison of Average Waiting & Turnaround Time in Triangular Fuzzy Environment Total Processes=12(ASCENDING SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-50ns)
Average Waiting Time(ns)	22927.536458	22927.53645	22927.544271	55764.56250
Average Turnaround Time(ns)	31578.40	31578.419922	31578.41	65165.437502



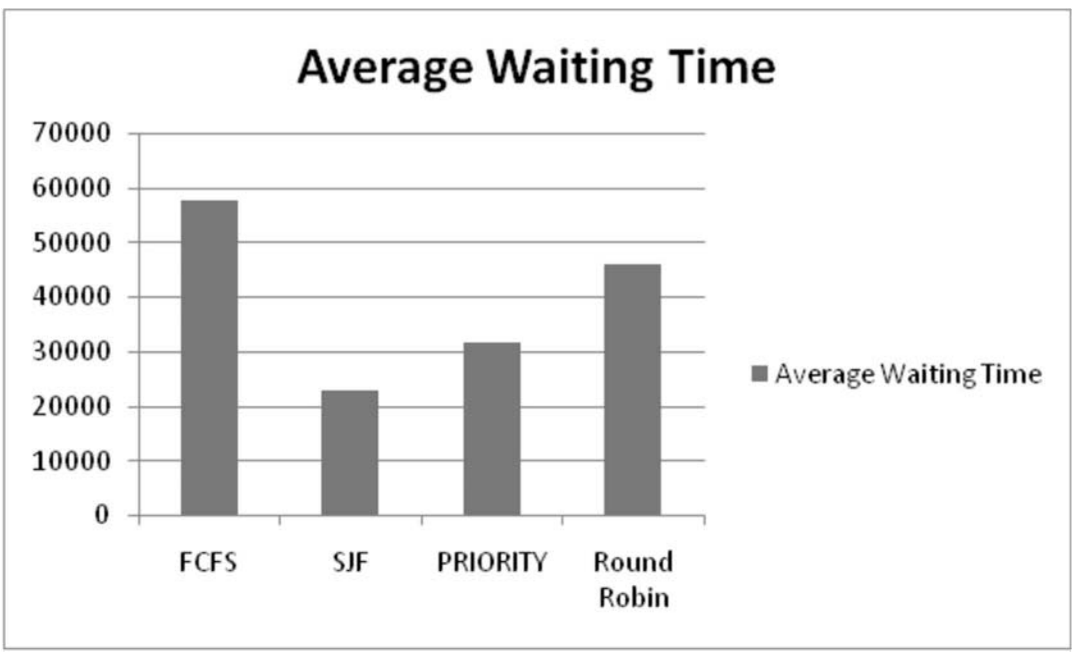
Graph3:Average waiting Time for triangular in ascending order



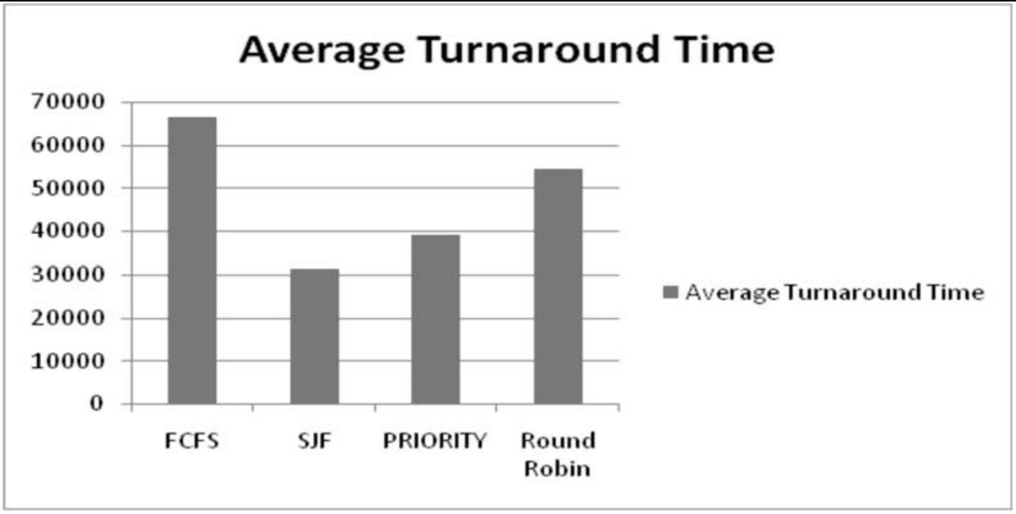
Graph4:Average Turnaround Time for triangular in ascending order

Table 7: Comparison of Average Waiting & Turnaround Time in Triangular Fuzzy Environment
Total Processes=12(RANDOM SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-50ns)
Average Waiting Time(ns)	57758.519531	22927.544922	31640.330078	45885.453125
Average Turnaround Time(ns)	66409.320312	31578.41992	39331.136719	54536.253906



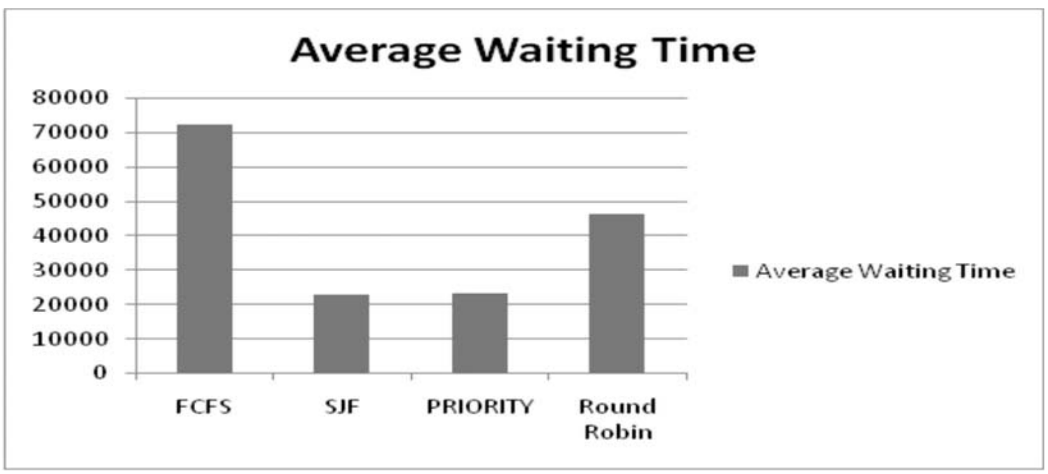
Graph5:Average Waiting Time for triangular in random order



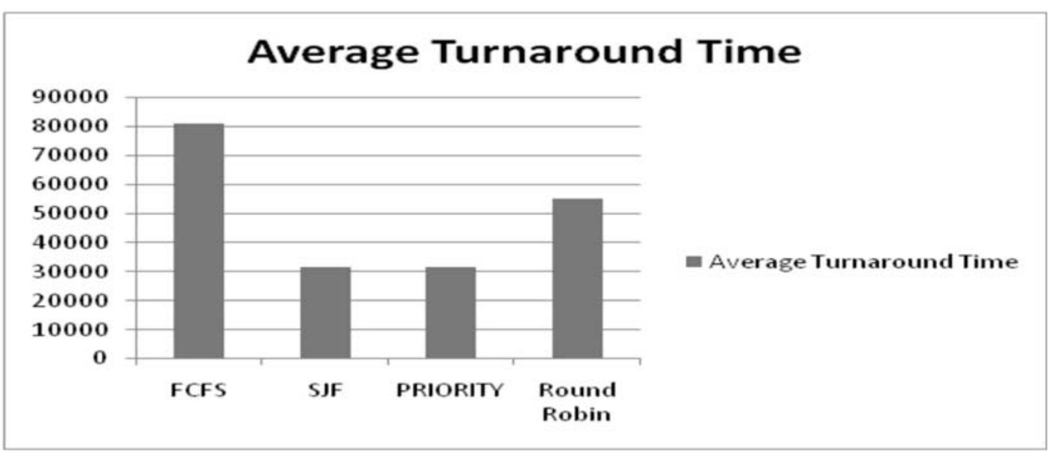
Graph6:Average Turnaround Time for triangular in random order

Table 8: Comparison of Average Waiting & Turnaround Time in Triangular Fuzzy Environment
Total Processes=12(DESCENDING SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-50ns)
Average Waiting Time (ns)	72232.085938	22927.544922	23149.703125	46158.7968
Average Turnaround Time(ns)	80882.960938	31578.41992	31800.578125	54809.6757



Graph7:Average Waiting Time for triangular in descending order

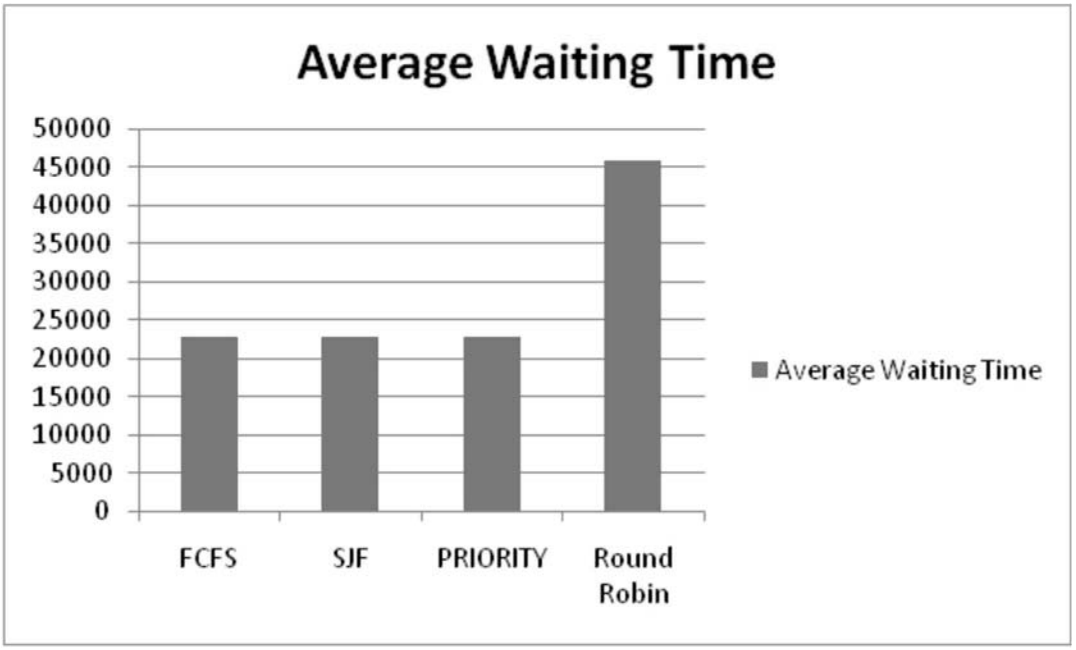


Graph 8 :Average Turnaround Time for triangular in descending order

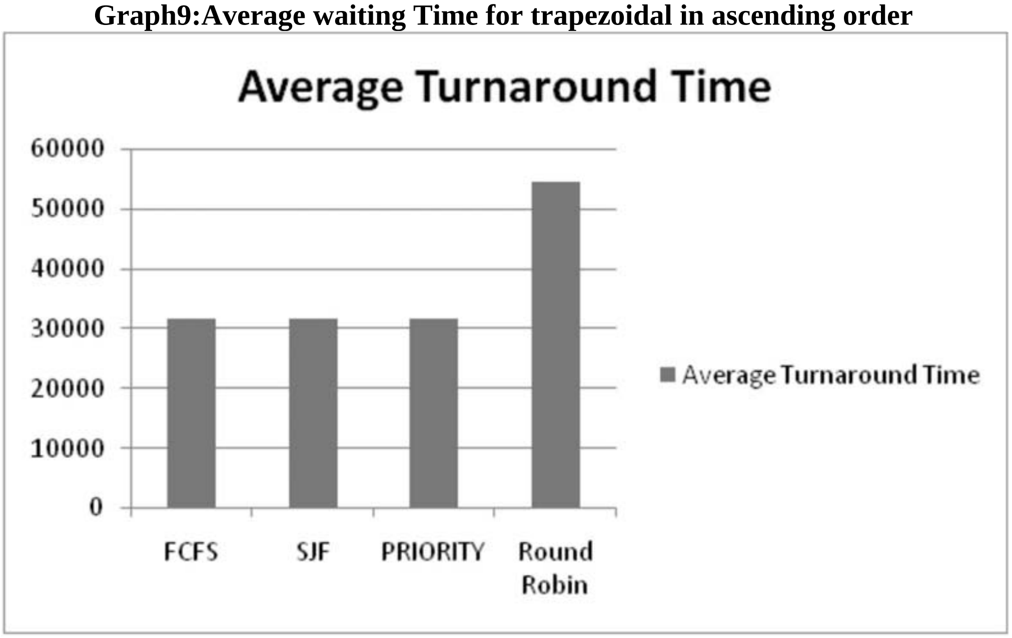
Table 9: Comparison of Average Waiting & Turnaround Time in Trapezoidal Fuzzy Environment

Total Processes=12(ASCENDING SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-75ns)
Average Waiting Time(ns)	22993.711484	22993.763672	22993.76367	45812.515625
Average Turnaround Time(ns)	31643.669922	31643.662109	31643.662109	54462.417969



Graph9:Average waiting Time for trapezoidal in ascending order

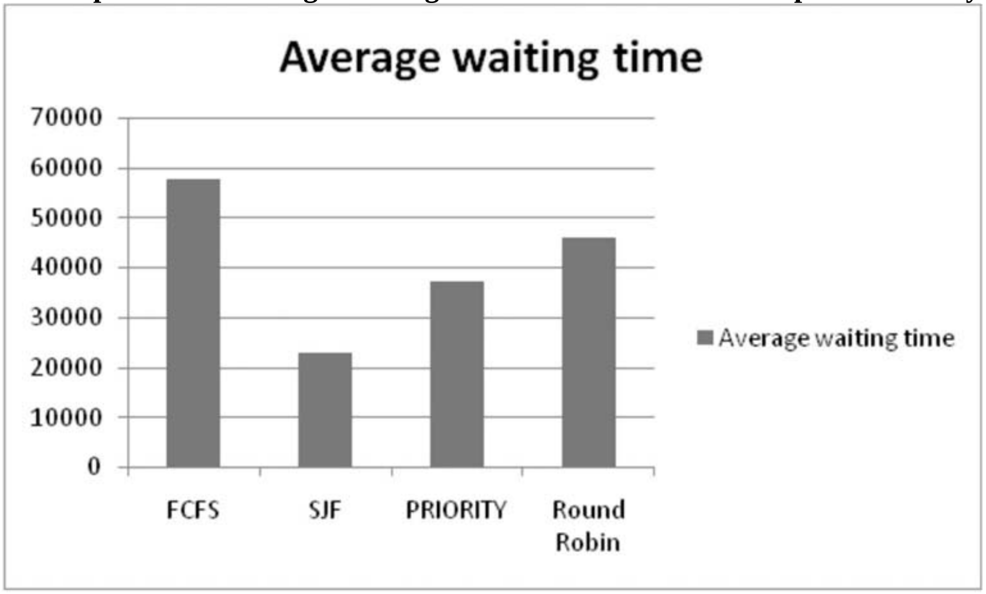


Graph10:Average Turnaround Time for trapezoidal in ascending order

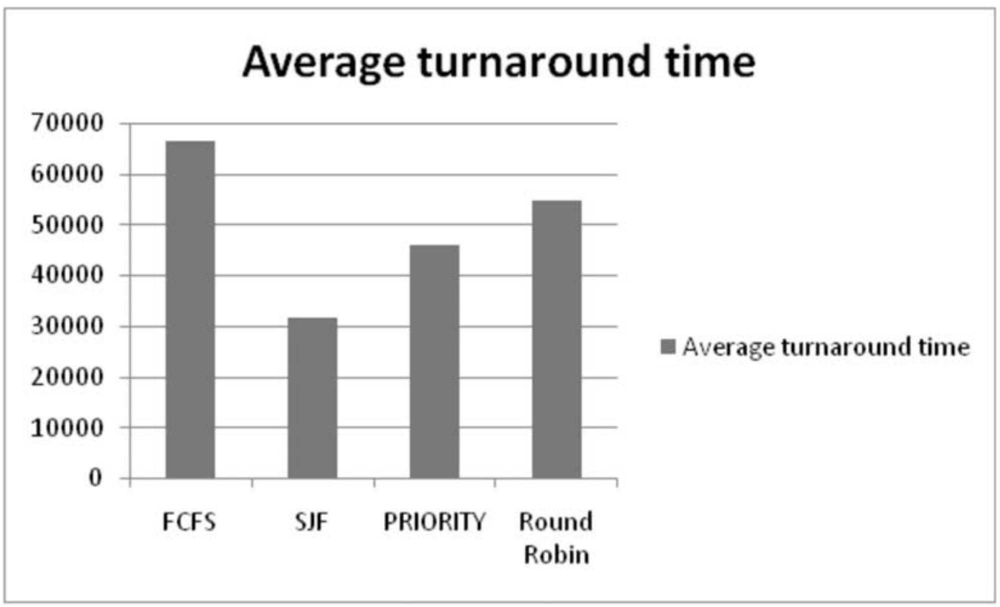
Table 10: Comparison of Average Waiting & Turnaround Time in Trapezoidal Fuzzy Environment
Total Processes=12(RANDOM SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-25ns)
Average Waiting Time(ns)	57785.667969	22993.763672	37363.207031	45985.433594
Average Turnaround Time(ns)	66435.562500	31643.662109	46013.105469	54635.324219

Table 11: Comparison of Average Waiting & Turnaround Time in Trapezoidal Fuzzy system



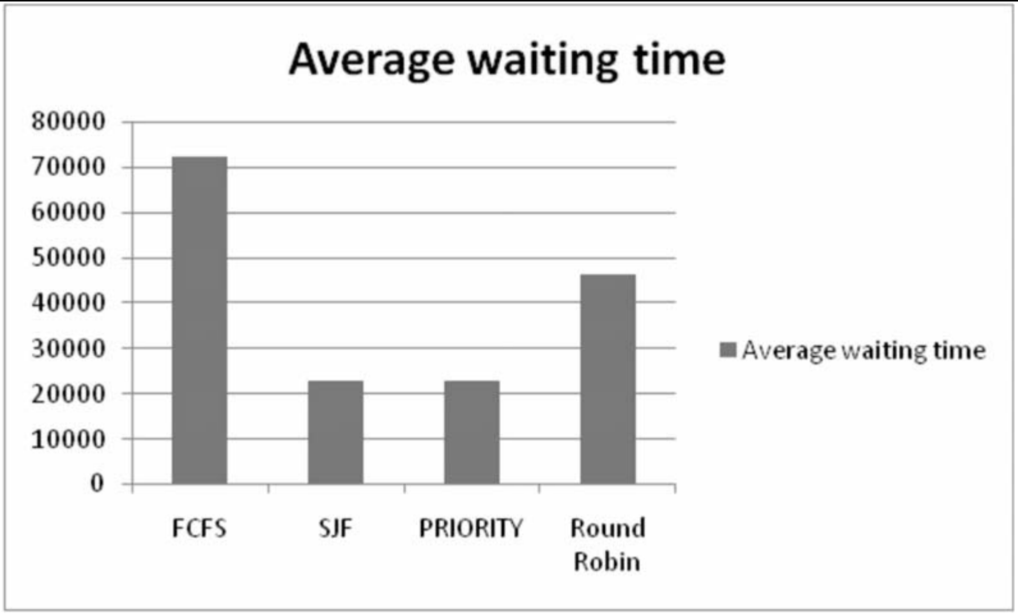
Graph 12:Average waiting Time for trapezoidal in random order



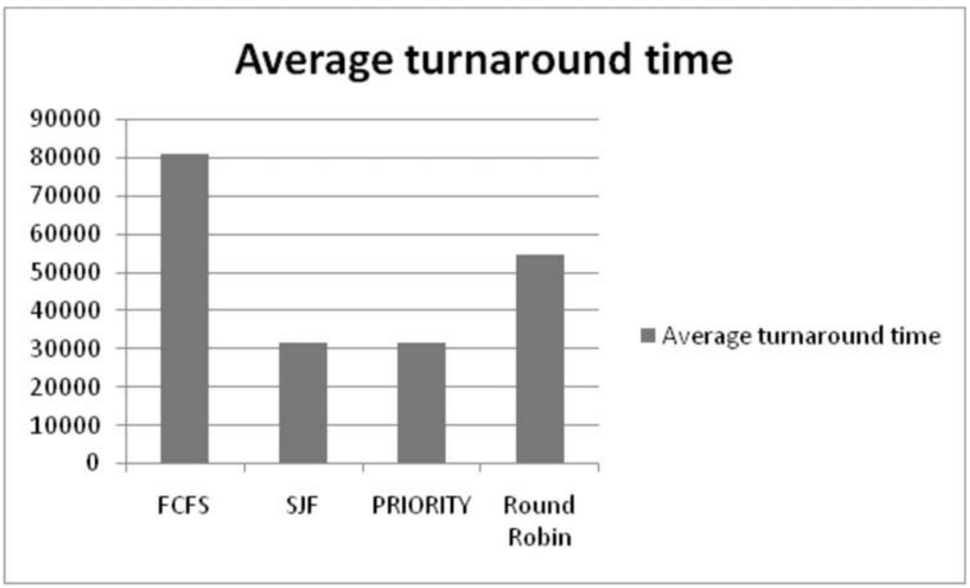
Graph 13:Average Turnaround Time for trapezoidal in random order

Table11:Comparison of Average waiting time and turn around time in trapezoidal fuzzy system
Total Processes=12(DESCENDING SORTED CPU BURST)

O/P Parameters	First Come First Serve	Shortest Job First	Priority	Round-Robin(TQ-50ns)
Average Waiting Time(ns)	72155.117188	22993.763672	22993.763672	46114.597656
Average Turnaround Time(ns)	80805.007812	31643.662109	31643.662109	54764.50000



Graph13:Average waiting Time for trapezoidal in descending order



Graph14:Average Turnaround Time for trapezoidal in descending order

5. Result Analysis:

First come First serve(FCFS),SJF,Priority,Round Robin have been calculated for the 12 processes. These scheduling algorithms have been implemented through graphs. The results obtained after comparative study have been presented in tables 6-11 taking variable time quanta for round robin algorithm.The analytical study has been made for both triangular and trapezoidal functions for the defuzzified value of burst time.We have shown the output of different scheduling policies through Bar Diagram.The fuzzy model design for comparative study of CPU Scheduling policies has also been depicted through Bar Diagram.

6. Conclusion and Further Scope

The analytical study for various policies through Bar graph shows that the average waiting time as well as average turnaround time of scheduling policies from FCFS, SJF, Priority and Round Robin gradually increases as depicted . It is because FCFS algorithm depends upon the order of the Burst Time of Processes lined up in a Queue. This further applies that if special hardware is attached to the CPU to sort the process in order of their Burst time whether in ascending order, descending order or random selection, the SJF policy gives almost a constant value of average waiting time and turnaround time in both triangular and trapezoidal cases which is minimum than other policies. It also maximize the CPU throughput. We conclude that SJF is best policy while Round Robin takes more time than other policies.

The work can be extended by taking multi processor on dividing the ready Queue into parts.Further work can be done by considering time quanta in Fuzzy logics.

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RELIABILITY AND PROFIT ANALYSIS OF A WATER PROCESS SYSTEM CONSIDERING NONE SWITCHING OF REDUNDANT UNITS AND PROPER/ IMPROPER REPAIRS OF MINOR FAULTS

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ABSTRACT :
A water process system in a Thermal Power Plant comprises of several redundant and non-redundant subsystems wherein the redundant subsystems are of two types, namely 1-out-of-2 (Type-I) and 2-out-of-3 (Type-II). On occurrence of a minor fault, the system goes to partial failure whereas on occurrence of a major fault the system goes to complete failure. Further on partial failure of the system, the available single repairman first inspects the subsystems to judge which subsystem has the minor fault and accordingly carries out the online repair of the infected subsystem, i.e. without halting the operation of the system, to avoid losses including loss of power generation etc. Even when a fault occurs in a redundant subsystem, switching of a redundant unit, being non-instantaneous and costly, is not done. However improper online repair of the infected subsystem sometimes results in to complete failure of the system. The present paper investigates a stochastic model developed for the system for the above practical situations. Various measures of system effectiveness for the model are obtained using Markov process and regenerative point technique. Using these measures, profit incurred to the system is also computed. The reliability and cost analyses of the system is presented through graphs for a particular case.
Keywords: ***Water process system, mean time to system failures, expected uptime with full/reduced capacity, Markov process, regenerative point technique.***

INTRODUCTION

In the present scenario, a variety of water process systems are being used in industries and households such as cooling water system, hot water system, steam generating system, water circulation system etc. The water process systems have important role in almost all industrial systems but these play crucial role especially in food and beverage industries, soda making plants, Thermal Power Plants, water purification plant, waste water treatment plant etc. In fact in these industries, reliability and cost of water process systems play a very significant role.

For the purpose of analyzing such systems, operation of a water process system at the Panipat Thermal Power Plant, Panipat was observed and real data regarding various faults, maintenances, inspections, repairs etc. of the system was collected. It was observed that the water process system has important role in the thermal power plant due to the dependency of whole process on circulation of water in different modes through various subsystems. Some of these subsystems are redundant like raw water pump, condensate exhaust pump and boiler feed pump etc. and others are non-redundant subsystems like gland steam cooler, low pressure heater, economizer and boiler drum etc. In other words, the water process system in the plant comprises of several redundant and non-redundant subsystems wherein the redundant subsystems are of two types, namely 1-out-of-2 (Type-I) and 2-out-of-3 (Type-II). These subsystems have different type of faults, some of these are minor faults such as vibration in motor of raw water pump, tripping in service water pump etc. and others are major faults such as casing leakage in main boiler feed pump (BFP), cartridge damaged in BFP etc. On occurrence of a minor fault, the system goes to partial failure whereas on occurrence of a major fault the system goes to complete failure. Further on partial failure of the system, the available single repairman first inspects the subsystems to judge which subsystem has the minor fault and accordingly carries out the online repair of the infected subsystem, i.e. without halting the operation of the system, to avoid losses including loss of power generation etc. Even when a fault occurs in a redundant subsystem, switching of a redundant unit, being non-instantaneous and costly, is not done. However improper online repair of the infected subsystem sometimes results in to complete failure of the system.

In the past, many researchers in the field of reliability modeling including Srinivasan and Gopalan (1973), Garg and Kumar (1977), Murari and Goyal (1983), Mahmoud(1989), Kumar et al (2001, 2009), Taneja et al (2004), Kumar and Bhatia (2011), Kumar and Batra (2012), Kumar and Rani (2013), Kumar and Kapoor (2014) Anita Taneja (2014) etc. discussed the reliability and cost-benefit analyses of different one and two-unit systems considering several aspects of the systems such as different types of faults, online/offline maintenances, repairs, inspections, rest/halt of system, degradation etc. but none of them analysed a water process system considering the above situations. To bridge this gap, the present paper presents reliability and profit analyses for the water process system through a stochastic model developed for the above mentioned situations. For the purpose, expressions for various measures of system effectiveness for the model such as mean time to system failure, expected uptime with full/reduced capacity, busy period of the repairman etc. are obtained using Markov processes and regenerative point techniques. The profit incurred to the system is also computed using these measures. In the last, for a particular case, reliability and cost analyses of the system is presented through graphs.

OTHER ASSUMPTIONS

1. The system is initially operative and various faults are self- announcing.
2. There is single repair facility that reaches the system in negligible time.
3. The priority for repair is given to the non-redundant subsystem on occurrence of faults in both the redundant and non-redundant subsystems.
4. The times to repair the different subsystems are different.
5. After each proper repair the system is as good as new.
6. The failure time distributions are taken exponential while other time distributions are considered general.
7. All the random variables are mutually independent.

STATES OF THE SYSTEM

O	:	Operative system.
O_i	:	Operative system under inspection.
O_{NRDr}	:	Operative non-redundant subsystem under repair.
O_{RDi}	:	Operative redundant subsystem under inspection.
$O_{RD\text{-}I}/O_{RD\text{-}Ir}$	:	Operative / operative under repair Type-I redundant subsystem.
$O_{RD\text{-}II}/O_{RD\text{-}IIr}$	:	Operative / operative under repair Type-II redundant subsystem.
F_r	:	Failed system under repair.

NOTATIONS

λ_1 / λ_2	:	Rate of major/minor faults.
x / y	:	Probability of fault in an non-redundant/redundant subsystem, y = 1- x.
a / b	:	Probability of fault in a Type-I redundant subsystem/Type-II redundant subsystem, b = 1- a.
$p_1/p_2/p_3$	:	Probability of doing proper repair of a minor fault in a non-redundant subsystem / Type-I redundant subsystem / Type-II redundant subsystem
$q_1/q_2/q_3$	:	Probability of doing improper repair of a minor fault in an non-redundant subsystem / Type-I redundant subsystem / Type-II redundant subsystem
$i_1(t)/I_1(t)$	:	P.d.f./c.d.f. of time to inspect the system.
$i_2(t)/I_2(t)$	:	P.d.f./c.d.f. of time to inspect the redundant subsystem.
$g(t)/G(t)$	:	P.d.f./c.d.f. of time to repair a major fault.
$g_1(t)/G_1(t)$	:	P.d.f./c.d.f. of time to repair a minor fault in the non-redundant subsystem.
$g_2(t)/G_2(t)$	:	P.d.f./c.d.f. of time to repair a minor fault in Type-I redundant subsystem.
$g_3(t)/G_3(t)$	:	P.d.f./c.d.f. of time to repair a minor fault in Type-II redundant subsystem.

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A transition diagram showing the various states of transition is shown as fig.1. The epochs of entry in to state 0, 1, 2, 3, 4, 5, 6 are regenerative point, i.e. all the states are regenerative state.

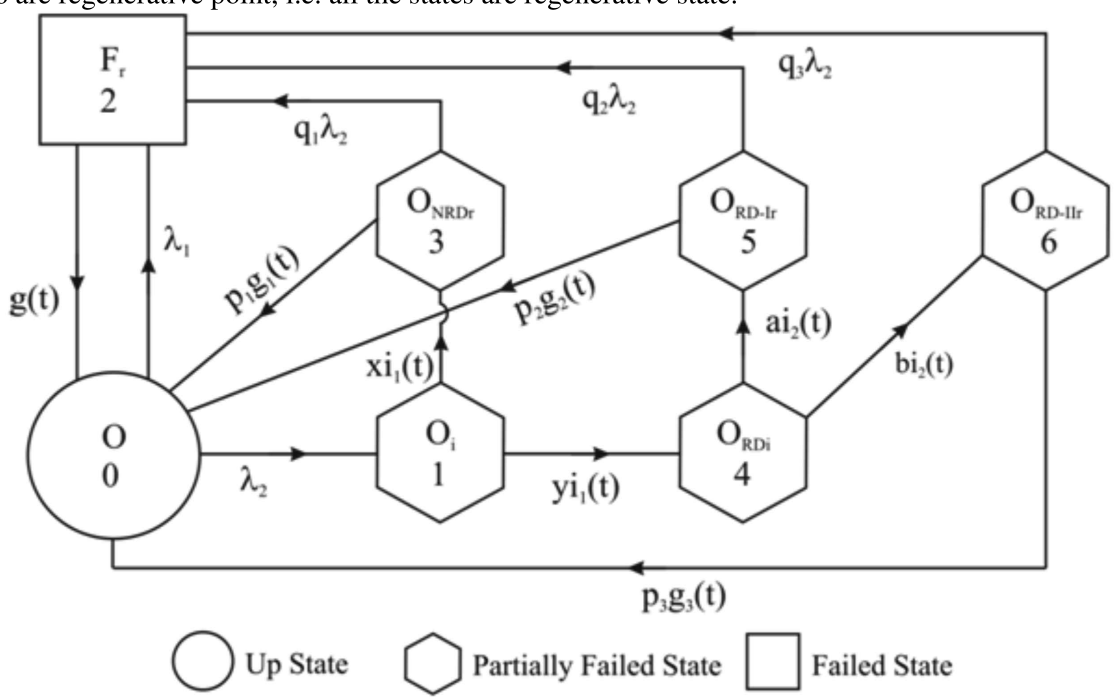

Fig.1: State Transition Diagram

The transition probabilities are:

$$dQ_{01}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}dt \qquad dQ_{02}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t}dt \qquad dQ_{13}(t) = xi_1(t)dt$$

$$dQ_{14}(t) = yi_1(t)dt \qquad dQ_{20}(t) = g(t)dt \qquad dQ_{30}(t) = p_1g_1(t)dt$$

$dQ_{32}(t)=q_1\lambda_2e^{-(\lambda_2)t}dt$ $\quad dQ_{45}(t)=ai_2(t)dt$ $\quad dQ_{46}(t)=bi_2(t)dt$

$dQ_{50}(t)=p_2g_2(t)dt$ $\quad dQ_{52}(t)=q_2\lambda_2e^{-(\lambda_2)t}dt$ $\quad dQ_{60}(t)=p_3g_3(t)dt$

$dQ_{62}(t)=q_3\lambda_2e^{-(\lambda_2)t}dt$

Taking L.S.T $Q_{ij}^{**}(s)$ and $p_{ij}=\lim\limits_{s\to 0}Q_{ij}^{**}(s)$, the non-zero elements $p_{ij,}$ are obtained as under:

$$p_{01}=\frac{\lambda_2}{\lambda_1+\lambda_2} \quad p_{02}=\frac{\lambda_1}{\lambda_1+\lambda_2} \quad p_{13}=xi_1^*(0) \quad p_{14}=yi_1^*(0)$$

$$p_{20}=g^*(0) \quad p_{30}=p_1g_1^*(0) \quad p_{32}=q_1 \quad p_{45}=ai_2^*(0)$$

$$p_{46}=bi_2^*(0) \quad p_{50}=p_2g_2^*(0) \quad p_{52}=q_2 \quad p_{60}=p_3g_3^*(0)$$

$$p_{62}=q_3$$

By these transition probabilities, it can be verified that

$$p_{01}+p_{02}=1, \quad p_{13}+p_{14}=1, \quad p_{30}+p_{32}=1, \quad p_{45}+p_{46}=1,$$

$$p_{50}+p_{52}=1, \quad p_{60}+p_{62}=1, \quad p_{20}=1$$

The unconditional mean time taken by the system to transit for any regenerative state ‘j’, when it is counted from epoch of entrance into that state ‘i’, is mathematically stated as

$$m_{ij}=\int_0^\infty t\,dQ_{ij}(t)=-q_{ij}^{*\prime}(0),$$

Thus we get

$$m_{01}+m_{02}=\mu_0 \quad m_{13}+m_{14}=\mu_1 \quad m_{20}=\mu_2 \quad m_{30}+m_{32}=\mu_3$$

$$m_{45}+m_{46}=\mu_4 \quad m_{50}+m_{52}=\mu_5 \quad m_{60}+m_{62}=\mu_6$$

The mean sojourn time (μ_i) in the regenerative state ‘i’ is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state ‘i’, then

$$\mu_0=\frac{1}{\lambda_1+\lambda_2} \quad \mu_1=-i_1^{*\prime}(0) \quad \mu_2=-g^{*\prime}(0) \quad \mu_3=\frac{q_1}{\lambda_2}-p_1g_1^*(0)$$

$$\mu_4=-i_2^{*\prime}(0) \quad \mu_5=\frac{q_2}{\lambda_2}-p_2g_2^*(0) \quad \mu_6=\frac{q_3}{\lambda_2}-p_3g_3^*(0)$$

MEASURES OF THE SYSTEM EFFECTIVENESS

Various measures of the system effectiveness in steady state obtained using the arguments of the theory of regenerative processes are as under:

Mean Time to System Failure (MTSF) **= N/D**

Expected Up-Time of the System with Full Capacity (AF_0) **= N_1/D_1**

Expected Up-Time of the System with Reduced Capacity (AR_0) **= N_2/D_1**

Busy Period of Repairman (Inspection time only) (B_i) **= N_3/D_1**

Busy Period of Repairman (Repair time only) (B_r) **= N_4/D_1**

where

$$N=\mu_0+p_{01}\left[\mu_1+p_{13}\mu_3+p_{14}\left(\mu_4+p_{45}\mu_5+p_{46}\mu_6\right)\right]$$

$$D=1-p_{01}\left[p_{13}p_{30}+p_{14}\left(p_{45}p_{50}+p_{46}p_{60}\right)\right]$$

$$N_1=\mu_0$$

$$D_1=\mu_0+p_{02}\mu_2+p_{01}\left[\mu_1+p_{13}\left(\mu_3+p_{32}\mu_2\right)+p_{14}\left\{\mu_4+p_{45}\left(\mu_5+p_{52}\mu_2\right)+p_{46}\left(\mu_6+p_{62}\mu_2\right)\right\}\right]$$

$$N_2 = p_{01}\left[\mu_1 + p_{13}\mu_3 + p_{14}\left(\mu_4 + p_{45}\mu_5 + p_{46}\mu_6\right)\right]$$

$$N_3 = p_{01}\left(\mu_1 + p_{14}\mu_4\right)$$

$$N_4 = p_{02}\mu_2 + p_{01}\left[p_{13}\left(p_{32}\mu_2 + \mu_3\right) + p_{14}\left\{p_{45}\left(p_{52}\mu_2 + \mu_5\right) + p_{46}\left(p_{62}\mu_2 + \mu_6\right)\right\}\right]$$

PROFIT ANALYSIS

The expected profit incurred of the system in the steady state is given by

$$P = C_0AF_0 + C_1AR_0 - C_2B_i - C_3B_r - C_4$$

where

C_0 = Revenue per unit uptime with full capacity of the system.

C_1 = Revenue per unit uptime with reduced capacity of the system.

C_2 = Cost per unit inspection of the subsystem

C_3 = Cost per unit repair of the subsystem

C_4 = Cost of installation of the system

GRAPHICAL INTERPRETATION AND CONCLUSIONS

For graphical analysis purpose, following particular case is considered:

$g(t) = \beta e^{-\beta t}$ $\quad g_1(t) = \beta_1 e^{-\beta_1 t}$ $\quad g_1(t) = \beta_1 e^{-\beta_1 t}$ $\quad g_3(t) = \beta_3 e^{-\beta_3 (t)}$

$i_1(t) = \alpha_1 e^{-\alpha_1 t}$ $\quad i_2(t) = \alpha_2 e^{-\alpha_2 t}$ $\quad p_{01} = \frac{\lambda_2}{\lambda_1 + \lambda_2}$ $\quad p_{02} = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

$p_{13} = x$ $\quad p_{14} = y$ $\quad p_{20} = 1$ $\quad p_{30} = p_1$

$p_{32} = q_1$ $\quad p_{45} = a$ $\quad p_{46} = b$ $\quad p_{50} = p_2$

$p_{52} = q_2$ $\quad p_{60} = p_3$ $\quad p_{62} = q_3$ $\quad \mu_0 = \frac{1}{\lambda_1 + \lambda_2}$

$\mu_1 = \frac{1}{\alpha_1}$ $\quad \mu_2 = \frac{1}{\beta}$ $\quad \mu_3 = \frac{q_1}{\lambda_2}\text{-}p_1$ $\quad \mu_4 = \frac{1}{\alpha_2}$

$\mu_5 = \frac{q_2}{\lambda_2}\text{-}p_2$ $\quad \mu_6 = \frac{q_3}{\lambda_2}\text{-}p_3$

Various graphs are plotted for MTSF, expected uptimes with full/reduced capacity and profit of the system taking different values of rates of different faults (λ_1, λ_2), repair rates (β, β_1, β_2, β_3), inspection rates (α_1, α_2) and various probabilities (a, x, p_1, p_2, p_3).

Following is concluded from the various plotted graphs:

Fig. 2 gives the graph between MTSF (T_0) and rate of minor faults (λ_2) for different values of rate of major faults (λ_1). It is concluded that the MTSF decreases with increase in the values of the rates of major/minor faults.

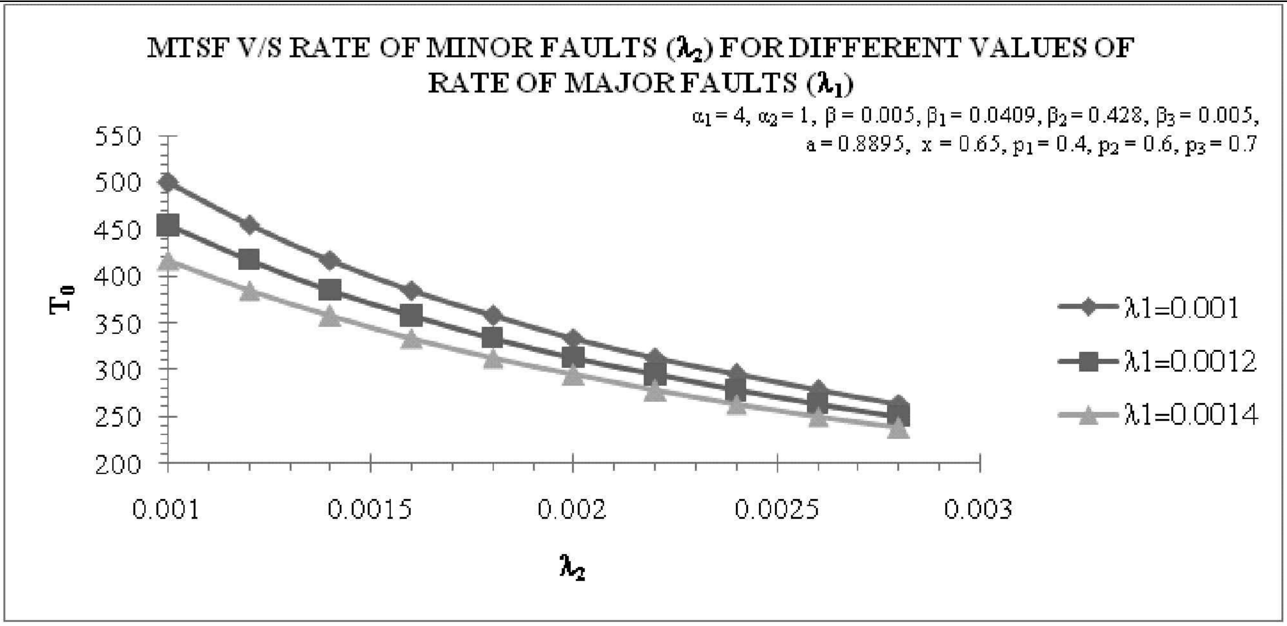

Fig.2

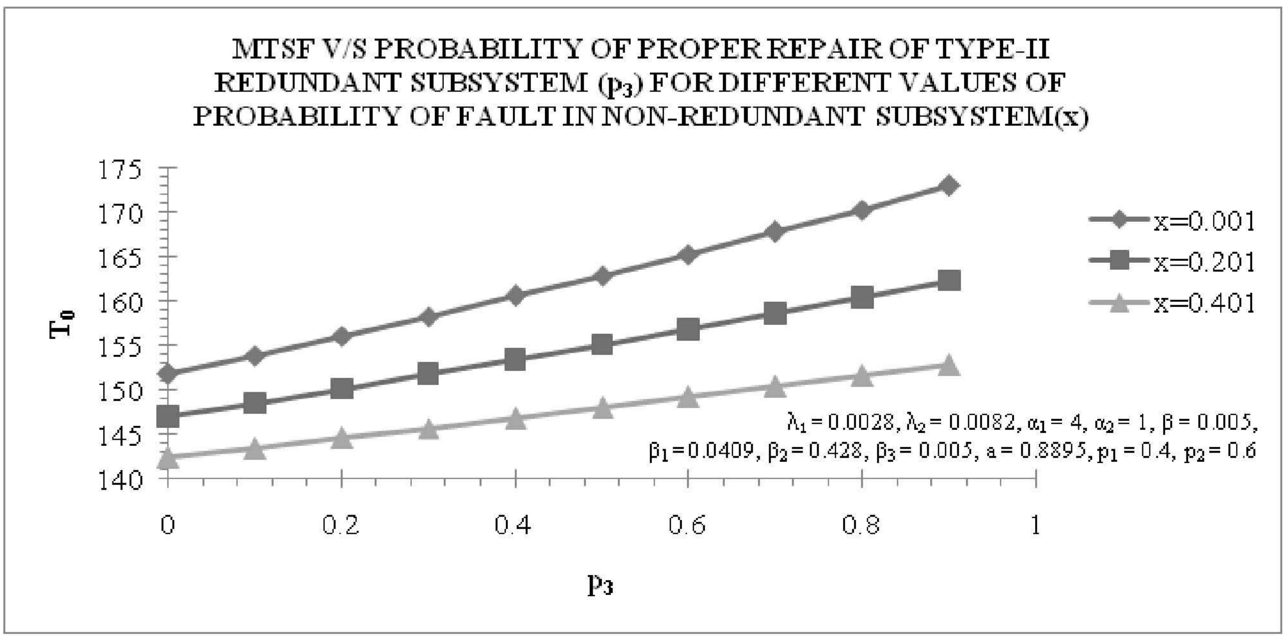

Fig.3

The curves in fig. 3 show the behaviour of MTSF (T_0) with respect to probability of proper repair of a Type-II redundant subsystem (p_3) for different values of probability of fault in non-redundant subsystem (x). The graph reveals that the MTSF increases with increase in the values of the probability of proper repair of a Type-II redundant subsystem and MTSF decreases with increase in the values of the probability of fault in non-redundant subsystem.

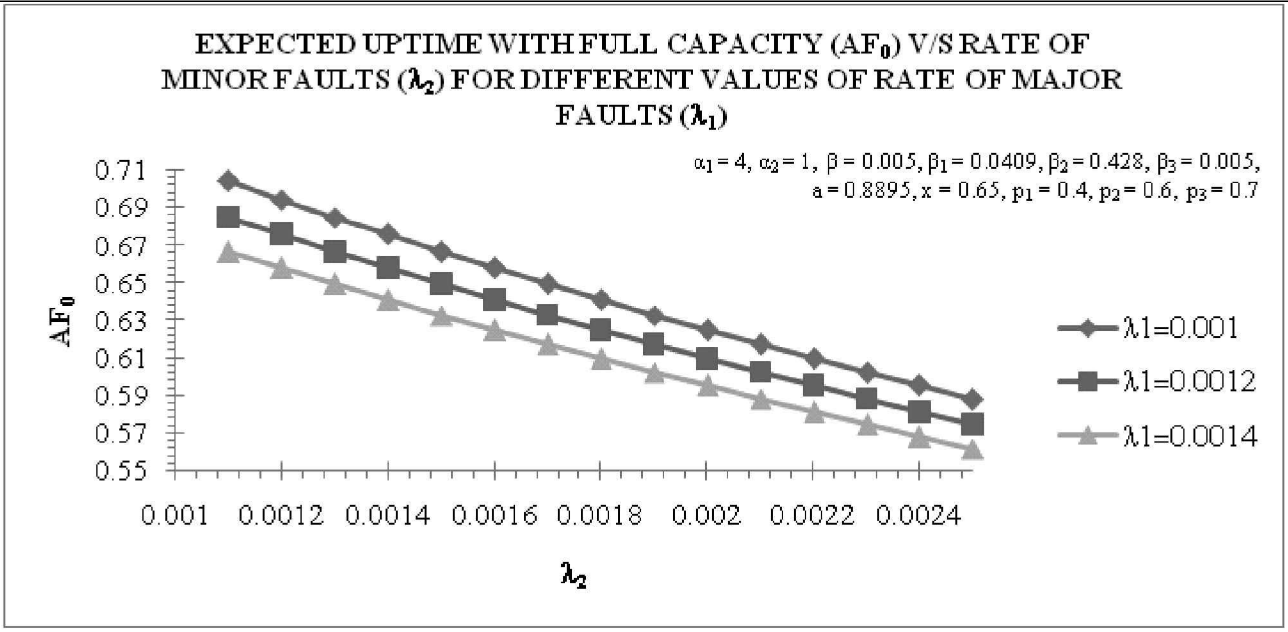

Fig.4

Fig. 4 presents the graph between expected uptime with full capacity of the system (AF_0) and rate of minor faults (λ_2) for different values of rate of major faults (λ_1). It can be concluded that the expected uptime with full capacity of the system decreases with increase in the values of the rates of minor/ major faults.

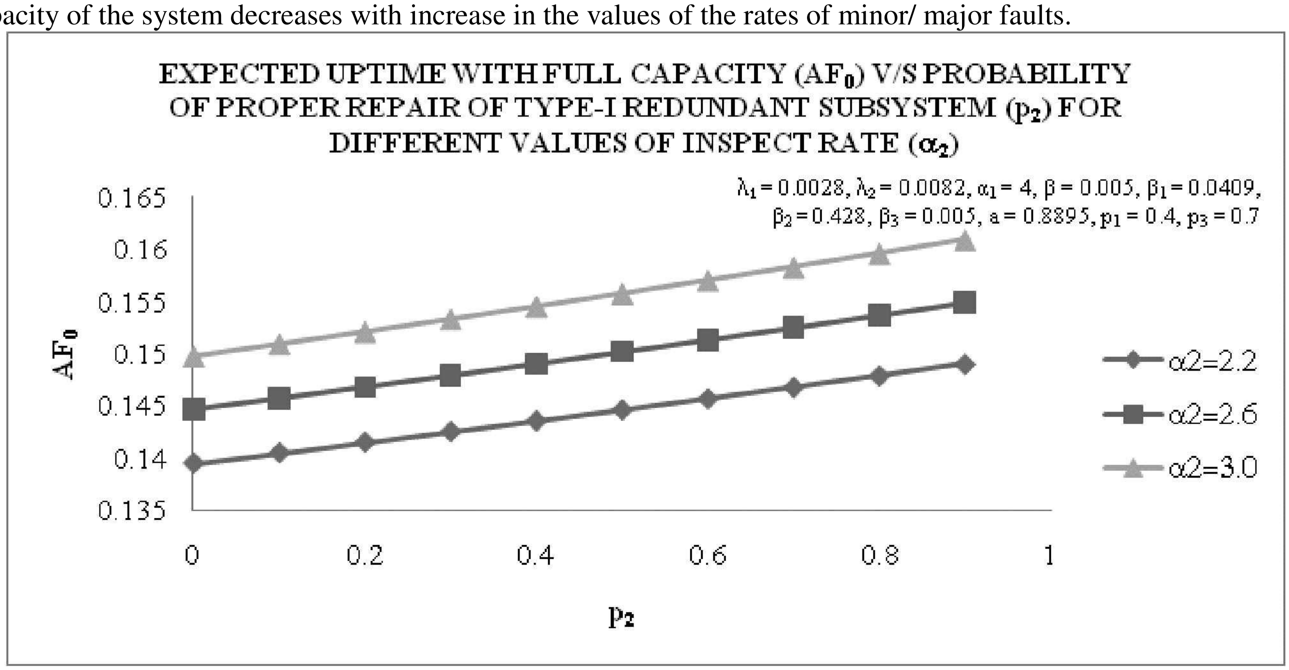

Fig.5

The curves in fig. 5 present the behaviour of expected uptime with full capacity of the system (AF_0) with respect to probability of proper repair of a Type-I redundant subsystem (p_2) for different values of inspection rate (α_2). It is concluded that the expected uptime with full capacity of the system increases with increase in the values of the probability of proper repair of a Type-I redundant subsystem and has higher values for higher values of inspection rate.

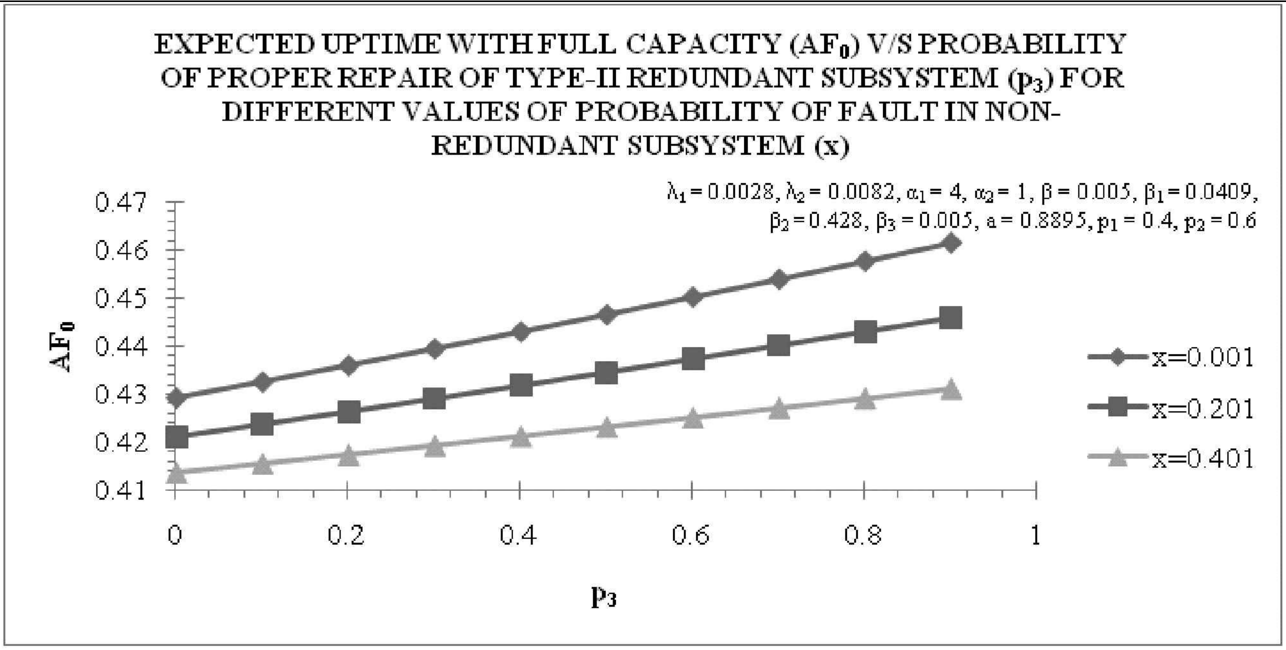

Fig.6

The graph in fig. 6 gives patterns of expected uptime with full capacity of the system (AF_0) with respect to probability of proper repair of a Type-II redundant subsystem (p_3) for different values of probability of fault in non-redundant subsystem (x). From the graph it can be concluded that the expected uptime with full capacity of the system increases with increase in the values of the probability of proper repair of a Type-II redundant subsystem and has lower values for higher values of probability of fault in non-redundant subsystem.

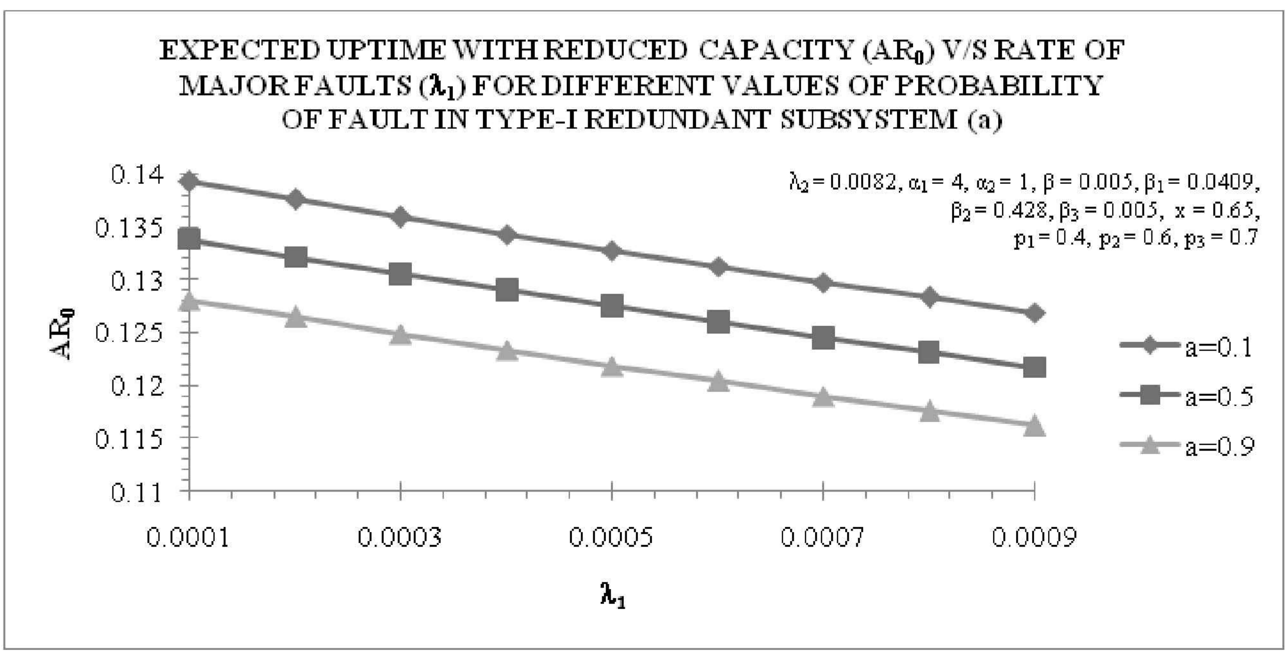

Fig.7

Fig. 7 presents the graph between expected uptime with reduced capacity of the system (AR_0) and rate of major faults (λ_1) for different values of probability of fault in Type-I redundant subsystem (a). The graph reveals that the expected uptime with reduced capacity of the system decreases with increase in the values of rate of major faults and has lower values for higher values of probability of fault in Type-I redundant subsystem.

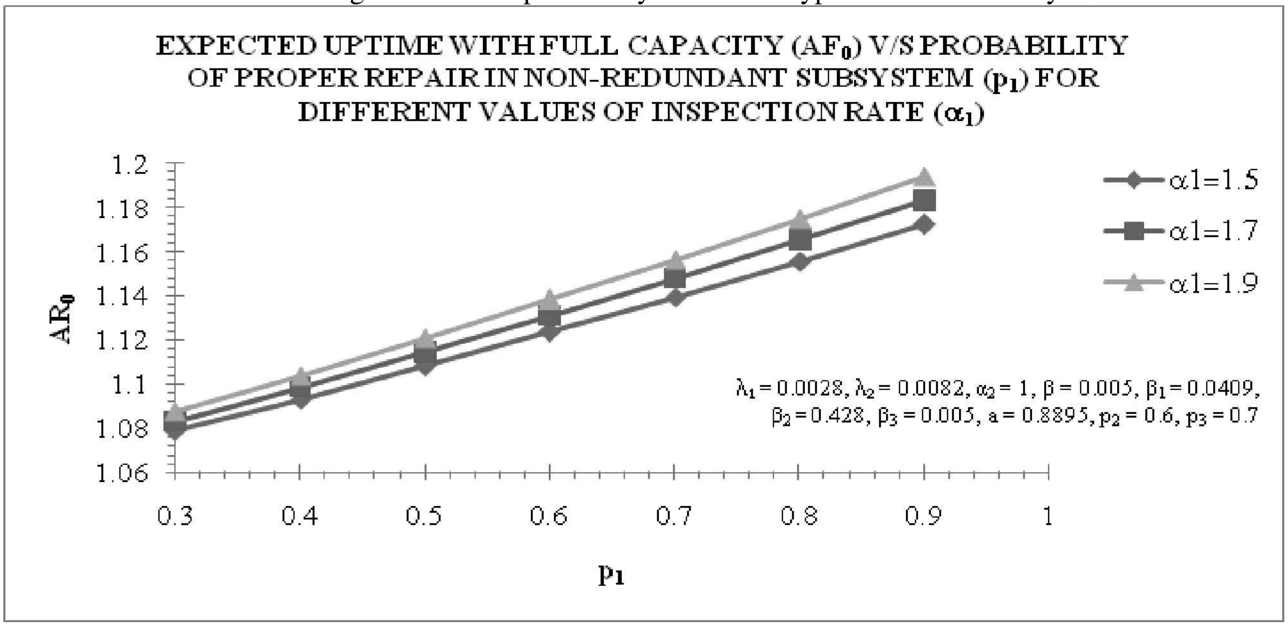

Fig.8

The curves in fig. 8 reveal the behaviour of expected uptime with reduced capacity of the system (AR_0) with respect to probability of proper repair of a non-redundant subsystem (p_1) for different values of inspection rate (α_1). It can be concluded from the graph that the expected uptime with reduced capacity of the system increases with increase in the values of the probability of proper repair of an non-redundant subsystem and has higher values for higher values of the inspection rate.

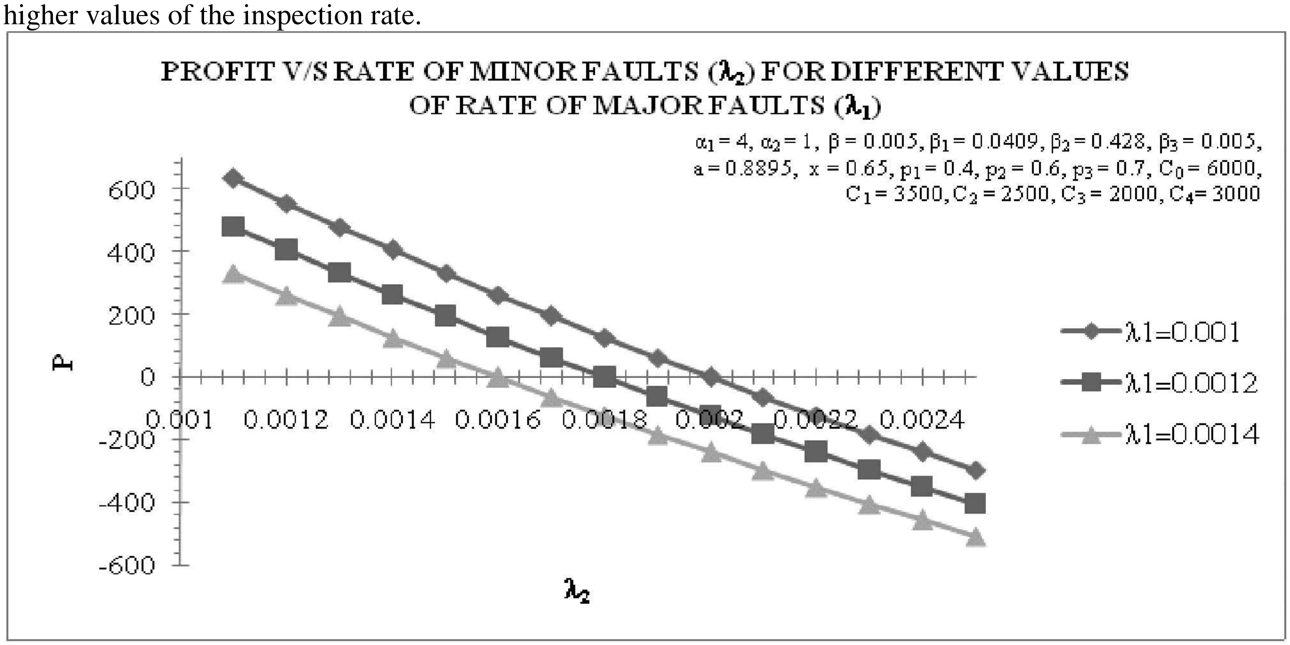

Fig.9

Fig. 9 gives the patterns of the profit incurred from the system (P) with respect to the rate of minor faults (λ_2) for the different values of rate of major faults (λ_1). It is concluded that the profit decreases with the increase in the values of the rate of minor faults and has lower values for higher values of the rate of major faults when other parameters remain fixed. From the fig. 9, it can also be observed that for $\lambda_1 = 0.001$, the profit is positive or zero or negative according as λ_2 is < or = or > 0.0016. Thus, in this case, the system is profitable whenever $\lambda_2 < 0.0016$. Similarly, for $\lambda_1 = 0.0012$ and $\lambda_1 = 0.0014$, the system is profitable whenever $\lambda_2 < 0.0018$ and 0.002 respectively.

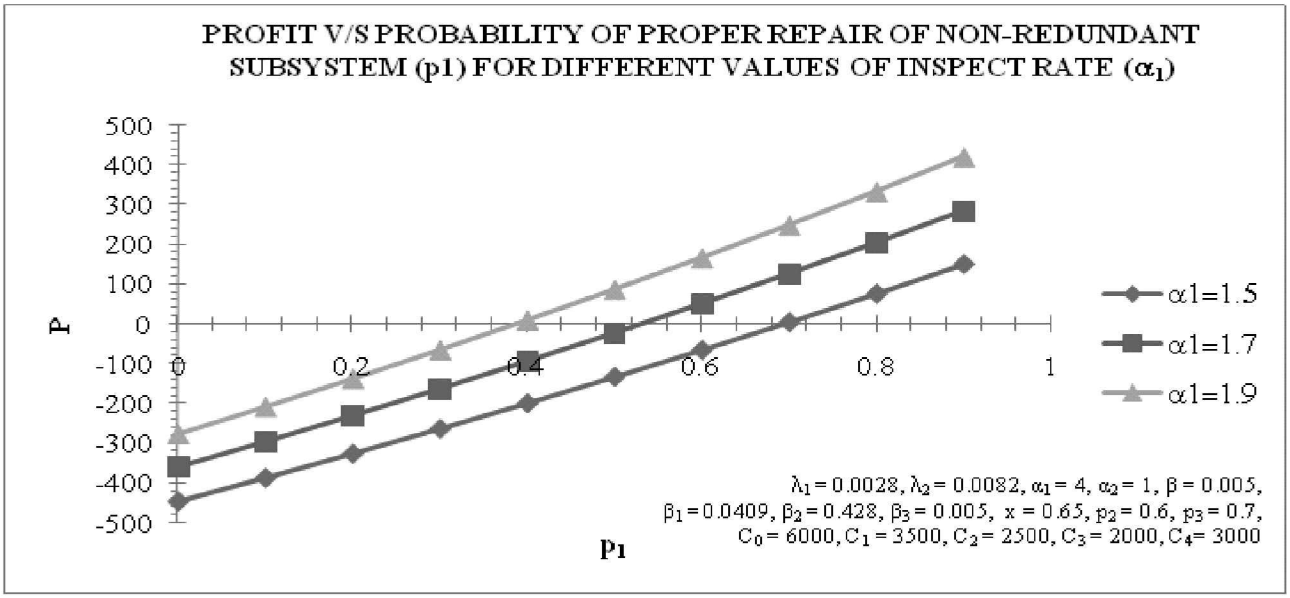

Fig.10

The curves in the fig. 10 show the behaviour of profit of the system (P) with respect to the probability of proper repair of a non-redundant subsystem (p_1) for different values of inspect rate (α_1). It is evident from the graph that the profit increases with the increase in the values of probability of proper repair of a non-redundant subsystem and has lower values for higher values of inspection rate when other parameters remain fixed. From the fig. 10, it can also be observed that for $\alpha_1 = 1.5$, the profit is negative or zero or positive according as $p_1$is < or = or > 0.694030. Thus, in this case, the system is profitable whenever $p_1 > 0.694030$. Similarly, for $\alpha_1 = 1.7$ and $\alpha_1 = 1.9$, the system is profitable whenever $p_1 > 0.532147$ and 0.385979 respectively.

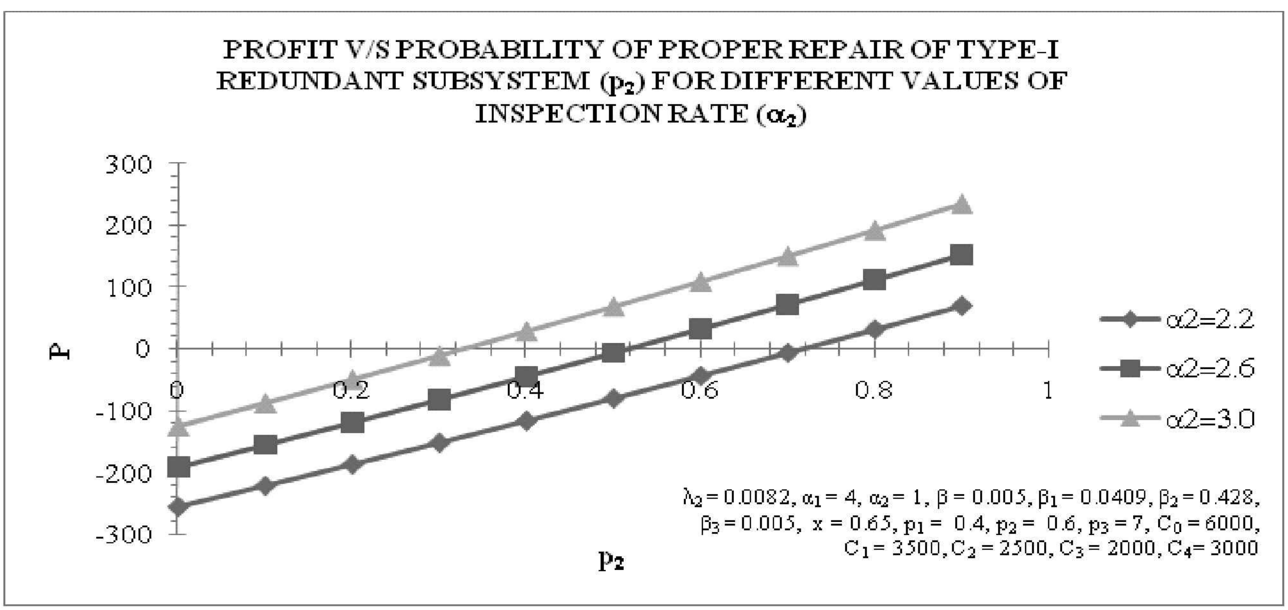

Fig.11

Fig. 11 highlights the behaviour of profit incurred from the system (P) with respect to probability of proper repair of a Type-I redundant subsystem (p_2) for different values of inspection rate (α_2). It can be concluded from the graph that the profit increases with the increase in the values of probability of proper repair of a Type-I redundant subsystem and has lower values for higher values of inspection rate when other parameters remain fixed. From the fig.11, it can also be observed that for α_2 = 2.2, the profit is negative or zero or positive according as p is < or = or > 0.717649. Thus, in this case, the system is profitable whenever p_2 > 0.717649. Similarly, for α_2 = 2.6 and α_2 = 3.0, the system is profitable whenever p_2 > 0.517551 and 0.328838 respectively.

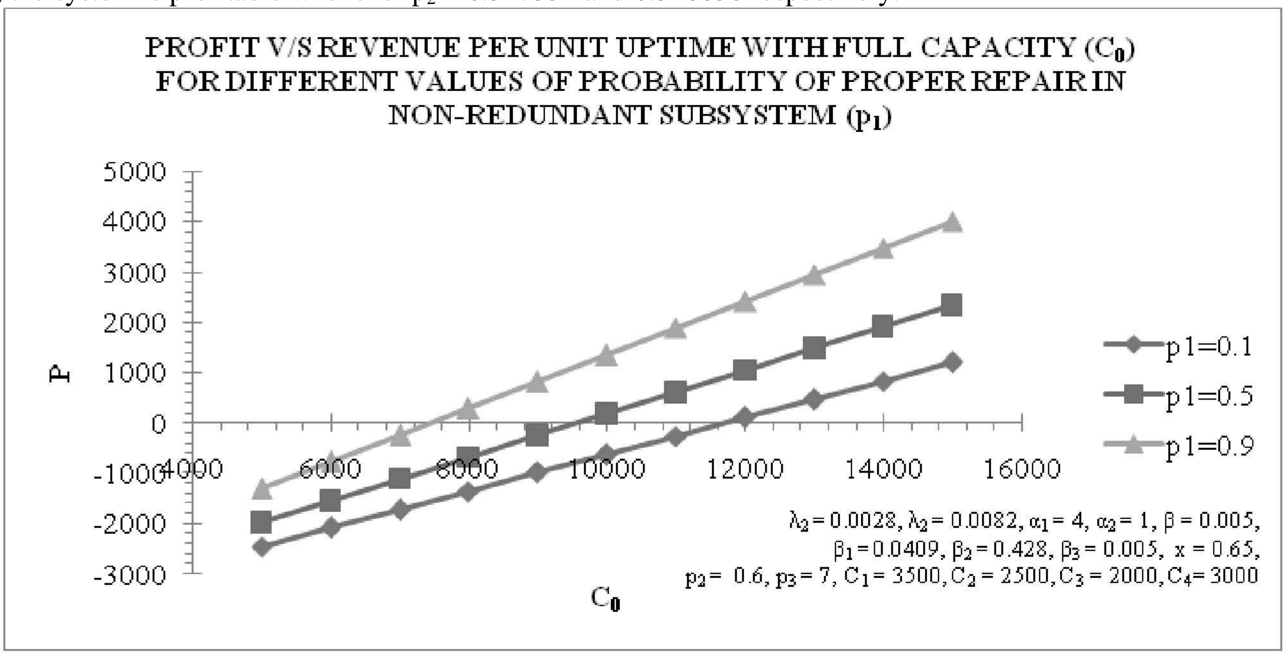

Fig.12

The curves in the fig.12 show the behaviour of profit with respect to revenue per unit uptime with full capacity of the system (C_0) for different values of probability of proper repair of an non-redundant subsystem (p_1). It is concluded from the graph that the profit increases with the increase in the values of revenue per unit up time with full capacity and has higher values for higher values of probability of proper repair of a non-redundant subsystem when other parameters remain fixed. From the fig.12 it can also be observed that for p_1 = 0.1, the profit is negative or zero or positive according as C_0 is < or = or > 11716.28. Thus in this case, the system is profitable whenever C_0 > 11716.28. Similarly, for p_1 = 0.5 and p_1 = 0.9, the system is profitable whenever p_2 > 9581.08 and 7445.88 respectively.

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MATHEMATICAL MODELING OF FEEDBACK BITANDEM QUEUE NETWORK WITH LINKAGE TO COMMON CHANNEL

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ABSTRACT :
The present paper makes an attempt to model a network of queues in which a common service channel is linked with each of two biserial channels along with a feedback from common channel i.e., arrival and service pattern both follow poisson distribution. All the activity in the system is performed under stochastic environment. The differential difference equations have been framed for the model and steady state behavior of the system has been studied.
The system performance characteristics have been determined by using generating function technique, laws of calculus, statistical formulae of marginal distribution etc. Numerical illustration has been given to demonstrate the result. The model finds its application in industries, banking system, administrative setup, computer network, super markets and many other business service systems.
Keywords: - ***Steady state behavior, Bitandem queues, Feedback, Marginal distribution, Poisson law***

1. Introduction:

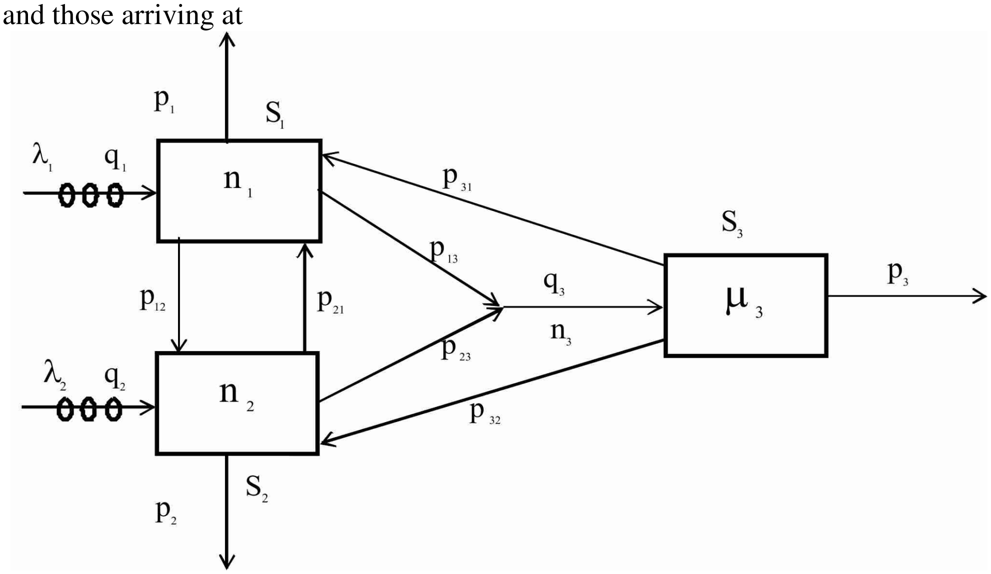
A queue network can be termed as a collection of service centers organized in such a way that the customers may proceed from one facility to other in order to fulfill their demands. Feedback concept relate to those queues in which a customer served once when his service becomes unsuccessful and are served again and again till it become successful. Many real life situations can be modeled as a feedback biserial queue network.
A vast amount of research in the field of serial and biserial queue network has been conducted by researchers during the past several decades. Jackson (1957) first showed the solution of steady state queue network system is of product form.Koiengsberg (1958) investigated the buffer storage problem as a system of cyclic queues. Finch(1959) extended the work of cyclic queues with feedback. Maggu (1970) introduced the concept of bitandem in queuing theory. Later on the work was extended by Singh T.P etal (1996, 2005). Singh T.P and Vinod (2005) studied the transient behavior of a queue network with parallel biseries queue linked with a common channel. Singh T.P and Kusum(2010) discussed feedback queue model under different parameters. Further Singh T.P and Arti (2012) discussed more generalized feedback serial queue network with different angles and augmentation.
This work is an extended generalized work as it combines both the concepts bitandem and feedback together. There is a network of queues in which the common channel is linked in a series with two distinct service channels which are biserially linked to each other.There is a feedback service from the common channel to each of channel. The feedback arises due to unsuccessful service to make successful one from the initial stage. All the activities in the system are performed under stochastic environment. It means the arrival follow Poisson distribution at each channel and the service times are distributed exponentially at each channel. The various system characteristics have been explicitly found using generating function technique, laws of calculus, L' Hospital's rule and statistical formulae. Numerical illustration has been given to demonstrate the result.

1.1 Practical Situation of the Model

A lot of practical situations of the model arise in industries, administrative set up, banking system, computer networks, office management, etc. At district level, various officers such as Tehsildars, SDM has each linkage to district commissioner (DC)for smooth running of administration and government policy. The people first put their problems before Tehsildar or SDM who solve at their own level or pass to each other ordirectly (a bitendem formation) sent to DC depending upon the nature ofgrievances/problems. The DC office if findaccurate, serves at its own level or in case of any shortcoming, feedback to again these officers for more clarifications. Similar the cases holds in production concern where the defective items are sent back to initial stage for making it correct. Recycling is common in manufacturing system where quality control inspections are performed after certain stages and components which do not meets quality standard are sent back for reprocessing. Similarly, telecommunication network may process message through randomly selected sequence of nodes with the probability that some message will require rerouting on occasion through the initial stage. The practical situations can also be observed in maintenance and repair facility system, production & assembly lines system and transportation system etc.

2. The Model:

The network of queue system is comprised of three service channels S_1 , S_2 & S_3 where S_3 is commonly linked with each of two servers S_1 & S_2 in bitandem. The customer demanding services arrive in Poisson stream at S_1 & S_2 with respective mean arrival rates λ_1 and λ_2 . Queues q_1 , q_2 & q_3 are formed in front of server S_1 , S_2 & S_3 , if they are busy. Customers coming at the rate λ_1 either go to the network $S_1 \to S_3$ directly or $S_1 \to S_2 \to S_3$ in biserial fashion and those arriving at



the rate λ_2 either go to $S_2 \to S_3$ directly or $S_2 \to S_1 \to S_3$ (in biserial manner). A customer at S_1 may either depart with probability p_1 or join S_2 or S_3 with probabilities p_{12} and p_{13} such that $p_1 + p_{12} + p_{13} = 1$ .Similarly, a customer after service at S_2 may either depart with probability p_2 or may join S_1 or S_3 with probabilities p_{21} and p_{23} such that $p_2 + p_{21} + p_{23} = 1$. Further, after successful service, the customers depart from server S_3 with probability p_3 and unsuccessful service may feedback from the common channel S_3 again to S_1 or S_2 with probabilities p_{31} and p_{32} such that $p_3 + p_{31} + p_{32} = 1$, till the service becomes successful.

3. Mathematical Description of the Model:-

On the basis of probability consideration, the differential difference equation in the steady state can be expresses as:

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{n_1,n_2,n_3} = \lambda_1 P_{n_1-1,n_2,n_3} + \lambda_2 P_{n_1,n_2-1,n_3} + \mu_1 p_1 P_{n_1+1,n_2,n_3} + \mu_1 p_{12} P_{n_1+1,n_2-1,n_3}$$
$$+\mu_1 p_{13} P_{n_1+1,n_2,n_3-1} + \mu_2 p_2 P_{n_1,n_2+1,n_3} + \mu_2 p_{21} P_{n_1-1,n_2+1,n_3} + \mu_2 p_{23} P_{n_1,n_2+1,n_3-1} + \mu_3 p_{31} P_{n_1-1,n_2,n_3+1}$$
$$+\mu_3 p_{32} P_{n_1,n_2-1,n_3+1} + \mu_3 p_3 P_{n_1,n_2,n_3+1} \qquad ; n_1, n_2, n_3 > 0 \qquad (1)$$

For $n_1 = 0 \quad ,n_2, n_3 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{0,n_2,n_3} = \lambda_2 P_{0,n_2-1,n_3} + \mu_1 p_1 P_{1,n_2,n_3} + \mu_1 p_{12} P_{1,n_2-1,n_3} + \mu_1 p_{13} P_{1,n_2,n_3-1}$$
$$+\mu_2 p_2 P_{0,n_2+1,n_3} + \mu_2 p_{23} P_{0,n_2+1,n_3-1} + \mu_3 p_{32} P_{0,n_2-1,n_3+1} + \mu_3 p_3 P_{0,n_2,n_3+1} \qquad (2)$$

For $n_2 = 0 \quad ,n_1, n_3 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{n_1,0,n_3} = \lambda_1 P_{n_1-1,0,n_3} + \mu_1 p_1 P_{n_1+1,0,n_3} + \mu_1 p_{13} P_{n_1+1,0,n_3-1} + \mu_2 p_2 P_{n_1,1,n_3}$$
$$+\mu_2 p_{21} P_{n_1+1,1,n_3} + \mu_2 p_{23} P_{n_1,1,n_3-1} + \mu_3 p_{31} P_{n_1-1,0,n_3+1} + \mu_3 p_3 P_{n_1,n_2,n_3+1} \qquad (3)$$

For $n_3 = 0 \quad ,n_1, n_2 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{n_1,n_2,0} = \lambda_1 P_{n_1-1,n_2,0} + \lambda_2 P_{n_1,n_2-1,0} + \mu_1 p_1 P_{n_1+1,n_2,0} + \mu_1 p_{12} P_{n_1+1,n_2-1,0}$$
$$+\mu_2 p_2 P_{n_1,n_2+1,0} + \mu_2 p_{21} P_{n_1+1,n_2+1,0} + \mu_3 p_{31} P_{n_1-1,n_2,1} + \mu_3 p_{32} P_{n_1,n_2-1,1} + \mu_3 p_3 P_{n_1,n_2,1} \qquad (4)$$

For $n_1 = 0, n_2 = 0 \quad ,n_3 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{0,0,n_3} = \mu_1 p_1 P_{1,0,n_3} + \mu_1 p_{13} P_{1,0,n_3-1} + \mu_2 p_2 P_{0,1,n_3} + \mu_2 p_{23} P_{0,1,n_3-1}$$
$$+\mu_3 p_3 P_{0,0,n_3+1} \qquad (5)$$

For $n_1 = 0, n_3 = 0 \quad ,n_2 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{n_1,n_2,n_3} = \lambda_2 P_{0,n_2-1,0} + \mu_1 p_1 P_{1,n_2,0} + \mu_1 p_{12} P_{1,n_2-1,0} + \mu_2 p_2 P_{0,n_2+1,0}$$
$$+\mu_3 p_{32} P_{0,n_2-1,1} + \mu_3 p_3 P_{0,n_2,1} \qquad (6)$$

For $n_2 = 0, n_3 = 0 \quad ,n_1 > 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{n_1,n_2,n_3} = \lambda_1 P_{n_1-1,0,0} + \mu_1 p_1 P_{n_1+1,0,0} + \mu_2 p_2 P_{n_1,1,0} + \mu_2 p_{21} P_{n_1-1,1,0}$$
$$+\mu_3 p_{31} P_{n_1-1,0,1} + \mu_3 p_3 P_{n_1,0,1} \qquad (7)$$

For $n_1 = 0, n_2 = 0, n_3 = 0$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)P_{0,0,0} = \mu_1 p_1 P_{1,0,0} + \mu_2 p_2 P_{0,1,0} + \mu_3 p_3 P_{0,0,1} \qquad (8)$$

with initial condition

$$P_{n_1,n_2,n_3}(0)=\begin{cases}1 & (n_1,n_2,n_3=0)\\ 0 & \text{otherwise}\end{cases} \tag{9}$$

4. Solution Process:-

For Steady state solution of equations from (1) to (8), we apply Generating function technique as :

$$F(x,y,z)=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}P_{n_1,n_2,n_3}x^{n_1}y^{n_2}z^{n_3} \tag{10}$$

Also

$$F_{n_2,n_3}(x)=\sum_{n_1=0}^{\infty}P_{n_1,n_2,n_3}x^{n_1} \tag{11}$$

$$F_{n_3}(x,y)=\sum_{n_2=0}^{\infty}F_{n_2,n_3}(x)\,y^{n_2} \tag{12}$$

$$F(x,y,z)=\sum_{n_3=0}^{\infty}F_{n_3}(x,y)\,z^{n_3} \tag{13}$$

Multiplying (1), (3), (4) and (7) by x^{n_1} and summing over n_1 from 0 to ∞, using (2), (5), (6) and (8) with the help of (11) we get(14) and other subsidiary equations

$$\begin{aligned}&(\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3)F_{n_2,n_3}(x)-\mu_1P_{0,n_2,n_3}=\lambda_1xF_{n_2,n_3}(x)+\lambda_2F_{n_2-1,n_3}(x)\\&+\frac{\mu_1p_1}{x}F_{n_2,n_3}(x)-\frac{\mu_1p_1}{x}P_{0,n_2,n_3}+\frac{\mu_1p_{12}}{x}F_{n_2-1,n_3}(x)-\frac{\mu_1p_{12}}{x}P_{0,n_2-1,n_3}+\frac{\mu_1p_{13}}{x}F_{n_2,n_3-1}(x)\\&-\frac{\mu_1p_{13}}{x}P_{0,n_2,n_3-1}+\mu_2p_{23}F_{n_2+1,n_3-1}(x)+\mu_3p_{31}xF_{n_2,n_3+1}(x)+\mu_3p_{32}F_{n_2-1,n_3+1}(x)\\&+\mu_2p_2F_{n_2+1,n_3}(x)+\mu_2xp_{21}F_{n_2+1,n_3}(x)+\mu_3p_3F_{n_2,n_3+1}(x)\qquad;\quad n_1=0,n_2,n_3>0\end{aligned} \tag{14}$$

Now multiplying (14) by y^{n_2} and summing over n_2 from 0 to ∞, using subsidiary equation with the help of (12) we get

$$\begin{aligned}&(\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3)F_{n_3}(x,y)-\mu_2F_{0,n_3}(x)-\mu_1F_{n_3}(y)\\&=\lambda_1xF_{n_3}(x,y)+\lambda_2yF_{n_3}(x,y)+\frac{\mu_1p_1}{x}F_{n_3}(x,y)-\frac{\mu_1p_1}{x}F_{n_3}(y)+\frac{\mu_1p_{12}}{x}yF_{n_3}(x,y)\\&-\frac{\mu_1p_{12}}{x}yF_{n_3}(y)+\frac{\mu_1p_{13}}{x}F_{n_3-1}(x,y)-\frac{\mu_1p_{13}}{x}F_{n_3-1}(y)+\frac{\mu_2p_{23}}{y}F_{n_3-1}(x,y)-\frac{\mu_2p_{23}}{y}F_{0,n_3-1}(x)\\&+\mu_3p_{31}xF_{n_3+1}(x,y)+\mu_3p_{32}yF_{n_3+1}(x,y)+\frac{\mu_2p_2}{y}F_{n_3}(x,y)-\frac{\mu_2p_2}{y}F_{0,n_3}(x)\\&+\frac{\mu_2xp_{21}}{y}F_{n_3}(x,y)-\frac{\mu_2xp_{21}}{y}F_{0,n_3}(x)+\mu_3p_3F_{n_3+1}(x,y)\qquad;n_1,n_2=0,n_3>0\end{aligned} \tag{15}$$

Multiplying (15) by z^{n_3} and summing over n_3 from 0 to ∞, using subsidiary equation with the help of (13) we get

$$
\begin{aligned}
&(\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3)F(x,y,z)-\mu_3F(x,y)-\mu_2F(x,z)-\mu_1F(y,z)\\
&=\lambda_1xF(x,y,z)+\lambda_2yF(x,y,z)+\frac{\mu_1p_1}{x}F(x,y,z)-\frac{\mu_1p_1}{x}F(y,z)+\frac{\mu_1p_{12}}{x}yF(x,y,z)\\
&-\frac{\mu_1p_{12}}{x}yF(y,z)+\frac{\mu_1p_{13}}{x}zF(x,y,z)-\frac{\mu_1p_{13}}{x}zF(y,z)+\frac{\mu_2p_2}{y}F(x,y,z)\\
&+\frac{\mu_2p_{23}z}{y}F(x,y,z)-\frac{\mu_2p_2}{y}F(x,z)+\frac{\mu_2p_{21}x}{y}F(x,y,z)-\frac{\mu_2p_{2,1}x}{y}F(x,z)\\
&+\frac{\mu_3p_{32}y}{z}F(x,y,z)-\frac{\mu_2p_{23}z}{y}F(x,z)+\frac{\mu_3p_{31}x}{z}F(x,y,z)-\frac{\mu_3p_{31}x}{z}F(x,y)\\
&-\frac{\mu_3p_{32}y}{z}F(x,y)+\frac{\mu_3p_3}{z}F(x,y,z)-\frac{\mu_3p_3}{z}F(x,y)\quad;\quad n_1,n_2,n_3=0 \qquad (16)
\end{aligned}
$$

On simplifying equation (16),we get

$$
\begin{aligned}
&\left[\begin{array}{l}\lambda_1(1-x)+\lambda_2(1-y)+\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}-\frac{p_{13}z}{x}\right)+\\ \mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)+\mu_3\left(1-\frac{p_3}{z}-\frac{p_{31}x}{z}-\frac{p_{32}y}{z}\right)\end{array}\right]F(x,y,z)=\\
&\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}-\frac{p_{13}z}{x}\right)F(y,z)\\
&+\mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)F(x,z)+\mu_3\left(1-\frac{p_3}{z}-\frac{p_{31}x}{z}-\frac{p_{32}y}{z}\right)F(x,y)
\end{aligned}
$$

$$
F(x,y,z)=\frac{\begin{array}{l}\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}-\frac{p_{13}z}{x}\right)F(y,z)+\mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)F(x,z)\\ +\mu_3\left(1-\frac{p_3}{z}-\frac{p_{31}x}{z}-\frac{p_{32}y}{z}\right)F(x,y)\end{array}}{\left[\begin{array}{l}\lambda_1(1-x)+\lambda_2(1-y)+\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}-\frac{p_{13}z}{x}\right)+\\ \mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)+\mu_3\left(1-\frac{p_3}{z}-\frac{p_{31}x}{z}-\frac{p_{32}y}{z}\right)\end{array}\right]} \qquad (17)
$$

We define $F(y,z)=F_0,\quad F(x,z)=F_1,\quad F(x,y)=F_2$

Using L' Hospital rule, we get

1. When $x,y=1,\quad z\to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$-\mu_1p_{13}F_0-\mu_2p_{23}F_1+\mu_3F_2=-\mu_1p_{13}-\mu_2p_{23}+\mu_3 \qquad (18)$$

2. When $x, z = 1, \quad y \to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$-\mu_1 p_{12} F_0 + \mu_2 F_1 - \mu_3 p_{32} F_2 = -\lambda_2 - \mu_1 p_{12} + \mu_2 - \mu_3 p_{32} \tag{19}$$

3. When $y, z = 1, \quad x \to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$\mu_1 F_0 - \mu_2 p_{21} F_1 - \mu_3 p_{31} F_2 = -\lambda_1 + \mu_1 - \mu_2 p_{21} - \mu_3 p_{31} \tag{20}$$

On solving, we get

$$F_0 = 1 - \frac{\lambda_1 (1 - p_{32} p_{23}) + \lambda_2 (p_{21} + p_{31} p_{23})}{\mu_1 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{21}$$

$$F_1 = 1 - \frac{\lambda_1 (p_{12} + p_{32} p_{13}) + \lambda_2 (1 - p_{31} p_{13})}{\mu_1 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{22}$$

$$F_2 = 1 - \frac{p_{23} (\lambda_1 p_{12} + \lambda_2) + p_{13} (\lambda_2 p_{21} + \lambda_1)}{\mu_1 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{23}$$

$$P_{n_1, n_2, n_3} = \rho_1^{n_1} \rho_2^{n_2} \rho_3^{n_3} (1 - \rho_1)(1 - \rho_2)(1 - \rho_3)$$

Where
$$\rho_1 = \frac{\lambda_1 (1 - p_{32} p_{23}) + \lambda_2 (p_{21} + p_{31} p_{23})}{\mu_1 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{24}$$

$$\rho_2 = \frac{\lambda_1 (p_{12} + p_{32} p_{13}) + \lambda_2 (1 - p_{31} p_{13})}{\mu_2 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{25}$$

$$\rho_3 = \frac{p_{23} (\lambda_1 p_{12} + \lambda_2) + p_{13} (\lambda_2 p_{21} + \lambda_1)}{\mu_3 (1 - p_{32} p_{23} - p_{31} p_{13} - p_{12} p_{21} - p_{12} p_{31} p_{23} - p_{32} p_{13} p_{21})} \tag{26}$$

The solution of steady state exist if $\rho_1, \rho_2, \rho_3 < 1$

5. System Characteristics:-

(a) Average Number of Customers:-

$$L = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} (n_1 + n_2 + n_3) P_{n_1, n_2, n_3}$$

$$L = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} n_1 P_{n_1, n_2, n_3} + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} n_2 P_{n_1, n_2, n_3} + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} n_3 P_{n_1, n_2, n_3}$$

$$= \left[(1 - \rho_1)(1 - \rho_2)(1 - \rho_3) \right] \begin{bmatrix} \left(\sum_{n_2=0}^{\infty} \rho_2^{n_2} \sum_{n_3=0}^{\infty} \rho_3^{n_3} \sum_{n_1=0}^{\infty} n_1 \rho_1^{n_1} \right) + \\ \left(\sum_{n_2=0}^{\infty} \rho_1^{n_1} \sum_{n_3=0}^{\infty} \rho_3^{n_3} \sum_{n_1=0}^{\infty} n_2 \rho_2^{n_2} \right) \\ + \left(\sum_{n_2=0}^{\infty} \rho_1^{n_1} \sum_{n_3=0}^{\infty} \rho_2^{n_2} \sum_{n_1=0}^{\infty} n_3 \rho_3^{n_3} \right) \end{bmatrix}$$

$$= \left[(1-\rho_1)\sum_{n_1=0}^{\infty} n_1\rho_1^{n_1} + (1-\rho_2)\sum_{n_1=0}^{\infty} n_2\rho_2^{n_2} + (1-\rho_3)\sum_{n_1=0}^{\infty} n_3\rho_3^{n_3} \right]$$

On Solving, we get

$$L = (1-\rho_1)^{-1}\rho_1 + (1-\rho_2)^{-1}\rho_2 + (1-\rho_3)^{-1}\rho_3$$

On putting the value of ρ_1, ρ_2 and ρ_3 from equation (24), (25) and (26), we get

$$L = \left(\frac{\lambda_1(1-p_{32}p_{23}) + \lambda_2(p_{21}+p_{31}p_{23})}{\mu_1(1-p_{32}p_{23}-p_{31}p_{13}-p_{12}p_{21}-p_{12}p_{31}p_{23}-p_{32}p_{13}p_{21}) - \lambda_1(1-p_{32}p_{23}) + \lambda_2(p_{21}+p_{31}p_{23})} \right)$$
$$+ \left(\frac{\lambda_1(p_{12}+p_{32}p_{13}) + \lambda_2(1-p_{31}p_{13})}{\mu_1(1-p_{32}p_{23}-p_{31}p_{13}-p_{12}p_{21}-p_{12}p_{31}p_{23}-p_{32}p_{13}p_{21}) - \lambda_1(p_{12}+p_{32}p_{13}) + \lambda_2(1-p_{31}p_{13})} \right)$$
$$+ \left(\frac{p_{23}(\lambda_1 p_{12}+\lambda_2) + p_{13}(\lambda_2 p_{21}+\lambda_1)}{\mu_1(1-p_{32}p_{23}-p_{31}p_{13}-p_{12}p_{21}-p_{12}p_{31}p_{23}-p_{32}p_{13}p_{21}) - p_{23}(\lambda_1 p_{12}+\lambda_2) + p_{13}(\lambda_2 p_{21}+\lambda_1)} \right)$$

(b). Variance:-

$$\sigma^2 = \frac{\rho_1}{(1-\rho_1)^2}.\frac{\rho_2}{(1-\rho_2)^2}.\frac{\rho_3}{(1-\rho_3)^2}$$

(c). Average Waiting Time:-

$$\text{Average wating time} = \frac{\text{Queue Length}}{\lambda} = \frac{L}{\lambda} \text{ where } \lambda = \lambda_1 + \lambda_2$$

6. Numerical Illustration:-

S.No	Number of Customers	Mean Service Rate	Mean Arrival Rate	Probabilities
1	$n_1 = 4$	$\mu_1 = 5$	$\lambda_1 = 2$	$p_1 = 0.4$, $p_{12} = 0.4$, $p_{13} = 0.2$
2	$n_2 = 5$	$\mu_2 = 6$	$\lambda_2 = 3$	$p_2 = 0.3$, $p_{21} = 0.3$, $p_{23} = 0.4$
3	$n_3 = 6$	$\mu_3 = 7$		$p_3 = 0.5$, $p_{31} = 0.3$, $p_{32} = 0.2$

$$\rho_1 = \frac{\lambda_1(1 - p_{32}p_{23}) + \lambda_2(p_{21} + p_{31}p_{23})}{\mu_1(1 - p_{32}p_{23} - p_{31}p_{13} - p_{12}p_{21} - p_{12}p_{31}p_{23} - p_{32}p_{13}p_{21})} = 0.9117$$

$$\rho_2 = \frac{\lambda_1(p_{12} + p_{32}p_{13}) + \lambda_2(1 - p_{31}p_{13})}{\mu_2(1 - p_{32}p_{23} - p_{31}p_{13} - p_{12}p_{21} - p_{12}p_{31}p_{23} - p_{32}p_{13}p_{21})} = 0.090$$

$$\rho_3 = \frac{p_{23}(\lambda_1 p_{12} + \lambda_2) + p_{13}(\lambda_2 p_{21} + \lambda_1)}{\mu_3(1 - p_{32}p_{23} - p_{31}p_{13} - p_{12}p_{21} - p_{12}p_{31}p_{23} - p_{32}p_{13}p_{21})} = 0.4411$$

$$L = (1 - \rho_1)^{-1}\rho_1 + (1 - \rho_2)^{-1}\rho_2 + (1 - \rho_3)^{-1}\rho_3 = 20.019$$

$$\text{Average wating time} = \frac{\text{Queue Length}}{\lambda} = \frac{L}{\lambda} = 4.0038$$

6.1 Analysis of the Results:-

From above numerical illustration, it is clear that about 91% utilization is made at first service channel while 9% of service has been utilized at second service channel. The third service channel has approximately 44% of utilization of service. Since very less service utilization has been made at level of second server. The second server may be assigned some extra work in vacation period.

7. Conclusion:

With the help of above queue characteristics, the analyst can find optimal number of channels to maintain the service rate. The analyst can also redesign the waiting facility by getting the information regarding the possible size of queue to plan for waiting queues. A computer program can also be framed to compute the joint probabilities and various queue characteristics.

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PERFORMANCE ANALYSIS OF A DESALINATION PLANT AS A SINGLE UNIT WITH MANDATORY SHUTDOWN DURING WINTER

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ABSTRACT :
The aim of this paper is to present a reliability and availability analysis of a desalination plant as a single unit. The plant operates round the clock for water production and the complete plant is under shut down state for 30 days during winter season for annual maintenance. The water supply during the shutdown period is maintained through ground water and storage system. Any failure or annual maintenance brings the plant to a complete halt and the plant goes under forced outage state. Tripping statesare also noted which requires only servicing. For the present analysis, seven years failure data has been extracted from operations and maintenance reports of a desalination plant in Oman. Measures of the plant effectiveness have been obtained using semi-Markov processes and regenerative point techniques

Keywords : ***Desalination plant, failures, repairs, replacement, mandatory shutdown, Semi – Markov, regenerative processes***

NOTATIONS

β_1	Rate of the unitmoving from winter to summer
β_2	Rate of the unit moving from winter to summer
λ_2	Rate of failure of any component of the plant due to repair
λ_3	Rate of failure of any component of the plant due to replacement
λ_4	Rate of failure of any component of the plant due to tripping
α_2	Repair rate of the unit
α_3	Replacement rate of the unit
α_4	Rate of recovery after tripping
γ_1	Rate of Mandatory shut down
©	Symbol for Laplace Convolution
Ⓢ	Symbol for Stieltje's convolution
*	Symbol for Laplace Transforms
**	Symbol for Laplace Stieltje's transforms
$\varphi_i(t)$	c.d.f. of first passage time from a regenerative state i to a failed state j
$p_{ij}(t)$, $Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in (0, t]
$g_2(t)$, $G_2(t)$	p.d.f. and c.d.f. of repair rate
$g_3(t)$, $G_3(t)$	p.d.f. and c.d.f. of replacement rate
$g_4(t)$, $G_4(t)$	p.d.f. and c.d.f. of recovery rate after tripping

1. INTRODUCTION

Desalination is a water treatment process that removes the salt from sea water or brackish water. It is the only option in arid regions, since the rainfall is marginal. In many desalination plants, multi stage flash desalination process is normally used for water purification which is very expensive and involves sophisticated systems. Since, desalination plants are designed to fulfill the requirement of water supply for a larger sector in arid regions, they are normally kept in continuous production mode especially during summer except for emergency/forced/planned outages. It is therefore, very important that the efficiency and reliability of such a complex system is maintained in order to avoid big loses. Many researchers have spent a great deal of efforts in analyzing industrial systems to achieve the reliability results that are useful in understanding the system behavior. Munoli&Suranagi [1] predicted the reliability indices in fatal and non-fatal shock model, Singh &Satyavati [2] analyzed a screening system in paper industry, Mathew et al. [3] analyzed an identical two-unit parallel CC plant system operative with full installed capacity, Padmavathi et al. [4] carried out an analysis for desalination plant with online repair and emergency shutdowns. Recently, some more case studies have been reported by Rizwan et al. [5] & Padma et al. [6], [7] for a desalination plant with seven evaporators under various failure and repair situations. Thus, the methodology for system analysis under various failure and repair assumptions has been widely presented in the literature and the novelty of this work lies in its case study. The numerical results are extremely helpful in understanding the significance of these failures/maintenances on plant reliability and availability.

Thus, the paper is an attempt to present a case analysis of the desalination plant as a single unit and no standby support system is available unlike the analysis shown in [6]& [7] where seven units / evaporators at a time have been considered. The situation of tripping is also included in the present analysis as noted in the data. This is equally important to note the variations on the reliability indices when there is no standby system available.Failure data for seven years have been collected from the operations and maintenance record of the plant in Oman. Component failure, maintenance, and plant shutdown rates, and various maintenance costs involved are estimated from the data. The desalination plant in this case operates round the clock as a single unit no standby evaporator is available upon failure of the same, and rather the complete plant goes into forced outage state in this case. The complete plant needs to be shut down for one month during the end of winter season for annual maintenance because of the low consumption of water for annual maintenance; the water supply during this period is maintained through ground water and storage system. The evaporator fails due to the failures which are repairable and due to the failures where only replacement is possible to restore the services.

Using the data, following values of rates are estimated:
Estimated rate of failure of any component of the unitdue to repairs (λ_2)=0.00002457per hour
Estimated rate of failure of any component of the unit due to replacements (λ_3) = 0.00003002per hour
Estimated rate of tripping (λ_4) = 0.00003118 per hour
Estimated value of repair rate(α_2) = 0.08420299per hour
Estimated value of replacement rate (α_3) = 0.12916976 per hour
Estimated rate of recovery after tripping (α_4) = 0.16746411 per hour
Estimated rate of shutting down (γ_1) = 0.00012438 per hour
Estimated rate of the unit moving from winter to summer (β_1) =0.0002315 per hour
Estimated rate of the unit moving from summer to winter (β_2) = 0.0002315 per hour

The plant is analyzed using semi-Markov processes and regenerative point techniques. The mean time to failure of the plant, the plant availability, expected busy period for repairs, replacements and tripping areestimated numerically.

2. MODEL DESCRIPTION AND ASSUMPTIONS

- Various states of the plant are: 0(operative state during summer); 1(operative state during winter); 2(failed state under repair during summer); 3(failed state under replacement during summer); 4(unit is down due to tripping during summer); 5(mandatory shut down at the end of winter season); 6(failed state under repair during winter); 7(failed state under replacement during winter); 8(unit is down due to tripping during winter).

- If a unit is failed in one season, it gets repaired in that season only.
- All failure times are assumed to have exponential distribution with failure rate (λ) whereas other times have general distributions.
- Mandatory shutdown during winter season after 11 months.

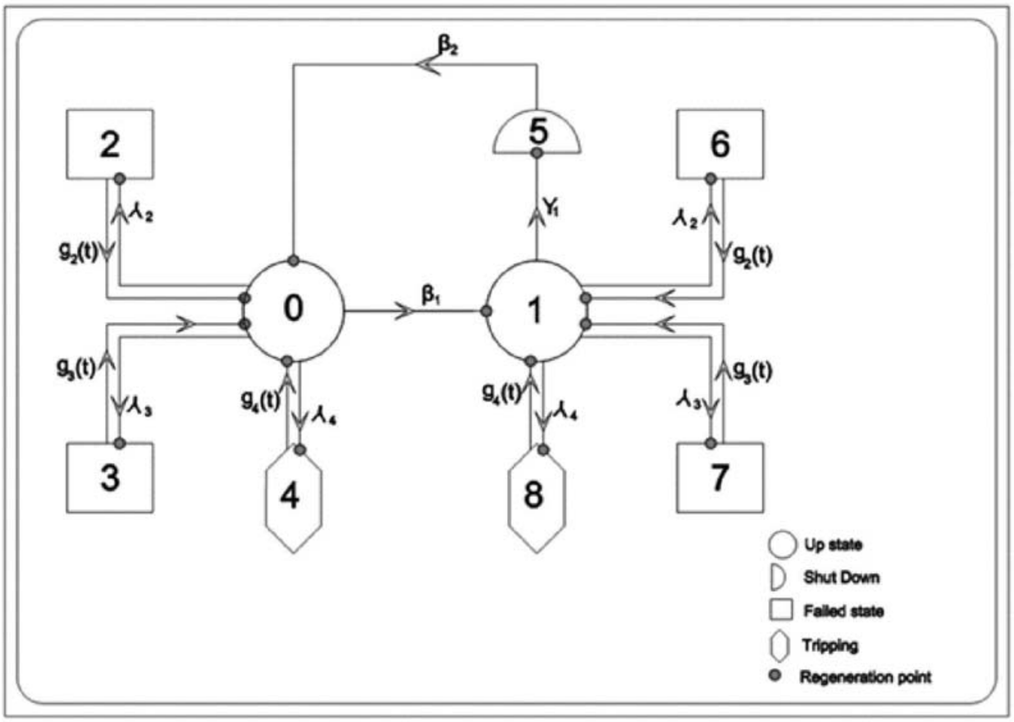

Fig. 1: State Transition Diagram

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram showing the possible states of transition of the plant is shown in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7 and 8 are regeneration points and hence these states are regenerative states. The transition probabilities are given by:

$dQ_{01} = \beta_1 e^{-(\lambda_2+\lambda_3+\lambda_4+\beta_1)t}\,dt,\; dQ_{02} = \lambda_2 e^{-(\lambda_2+\lambda_3+\lambda_4+\beta_1)t}\,dt,\; dQ_{03} = \lambda_3 e^{-(\lambda_2+\lambda_3+\lambda_4+\beta_1)t}\,dt,$

$dQ_{04} = \lambda_4 e^{-(\lambda_2+\lambda_3+\lambda_4+\beta_1)t}\,dt,\; dQ_{15} = \gamma_1 e^{-(\lambda_2+\lambda_3+\lambda_4+\gamma_1)t}\,dt,\; dQ_{16} = \lambda_2 e^{-(\lambda_2+\lambda_3+\lambda_4+\gamma_1)t}\,dt$

$dQ_{17} = \lambda_3 e^{-(\lambda_2+\lambda_3+\lambda_4+\gamma_1)t}\,dt,\; dQ_{18} = \lambda_4 e^{-(\lambda_2+\lambda_3+\lambda_4+\gamma_1)t}\,dt,\; dQ_{51} = \beta_2 e^{-\beta_2 t}\,dt$

$$dQ_{20} = g_2(t)dt, dQ_{30} = g_3(t)dt, dQ_{40} = g_4(t)dt$$

$dQ_{61} = g_2(t)dt, dQ_{71} = g_3(t)dt, dQ_{81} = g_4(t)dt$ (1- 15)

The non-zero element p_{ij} can be obtained by,

$$p_{ij} = \lim_{s\to 0}\int_0^\infty q_{ij}\,(t)dt$$

The transition probabilities p_{ij} are given below:

$$p_{01} + p_{02} + p_{03} + p_{04} = 1$$

$$p_{15} + p_{16} + p_{17} + p_{18} = 1$$

$p_{20} = p_{30} = p_{40} = p_{50} = p_{61} = p_{71} = p_{81} = 1$ (16-18)

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state 'i', then:

$$\mu_i = E(T) = P(T > t)$$

$$\mu_0 = \frac{1}{(\lambda_2 + \lambda_3 + \lambda_4 + \beta_1)}$$

$$\mu_1 = \frac{1}{(\lambda_2 + \lambda_3 + \lambda_4 + \gamma_1)}$$

$$\mu_2 = \mu_6 = \frac{1}{\alpha_2}, \mu_3 = \mu_7 = \frac{1}{\alpha_3}, \mu_4 = \mu_8 = \frac{1}{\alpha_4}, \mu_5 = \frac{1}{\beta_2} \quad (19\text{-}24)$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entry into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0), \quad \sum_j m_{ij} = \mu_i$$

$$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$$
$$m_{15} + m_{16} + m_{17} + m_{18} = \mu_1$$
$$m_{20} = \mu_2 \,;\, m_{30} = \mu_3 \,;\, m_{40} = \mu_4 \,;\, m_{50} = \mu_5 \,;\, m_{61} = \mu_6 \,; m_{71} = \mu_7 \,; m_{81} = \mu_8 \quad (25\text{-}33)$$

4. THE MATHEMATICAL ANALYSIS

4.1 Mean time to plant failure

Regarding the failed states 2, 3, 6 & 7 are absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ is obtained:

$$\varphi_0(t) = Q_{01}(t) Ⓢ \varphi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t) Ⓢ \varphi_4(t)$$
$$\varphi_1(t) = Q_{15}(t) Ⓢ \varphi_5(t) + Q_{16}(t) + Q_{17}(t) + Q_{18}(t) Ⓢ \varphi_8(t)$$
$$\varphi_4(t) = Q_{40}(t) Ⓢ \varphi_0(t)$$
$$\varphi_5(t) = Q_{50}(t) Ⓢ \varphi_0(t)$$
$$\varphi_8(t) = Q_{81}(t) Ⓢ \varphi_1(t) \quad (34\text{-}38)$$

Solving the above equation for $\phi_o^{**}(s)$ by taking Laplace Stieltje's transforms, the mean time to plant failurewhen the unit started at the beginning of state 0, is

$$MTPF = \lim_{s\to 0} \frac{1-\emptyset_0^{**}(s)}{s} = \lim_{s\to 0} \frac{1-\frac{N(s)}{D(s)}}{s} \quad (39)$$

Where,

$$N(s) = Q_{02}^{**}(s) + Q_{03}^{**}(s) + Q_{01}^{**}(s)Q_{16}^{**}(s) + Q_{01}^{**}(s)Q_{17}^{**}(s) - Q_{02}^{**}(s)Q_{18}^{**}(s)Q_{81}^{**}(s) - Q_{03}^{**}(s)Q_{18}^{**}(s)Q_{81}^{**}(s)$$
$$D(s) = 1 - Q_{18}^{**}(s)Q_{81}^{**}(s) - Q_{04}^{**}(s)Q_{40}^{**}(s) - Q_{01}^{**}(s)Q_{15}^{**}(s)Q_{50}^{**}(s) + Q_{04}^{**}(s)Q_{40}^{**}(s)Q_{18}^{**}(s)Q_{81}^{**}(s) \quad (40\text{-}41)$$

4.2 Availability analysis of the plant

Using the probabilistic arguments and defining $A_i(t)$ as the probability of unit entering into upstate at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $A_i(t)$:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) + q_{03}(t)©A_3(t) + q_{04}(t)©A_4(t)$$
$$A_1(t) = M_1(t) + q_{15}(t)©A_5(t) + q_{16}(t)©A_6(t) + q_{17}(t)©A_7(t) + q_{18}(t)©A_8(t)$$
$$A_2(t) = q_{20}(t)©A_0(t)$$
$$A_3(t) = q_{30}(t)©A_0(t)$$
$$A_4(t) = M_4(t) + q_{40}(t)©A_0(t)$$
$$A_5(t) = q_{50}(t)©A_0(t)$$
$$A_6(t) = q_{61}(t)©A_1(t)$$
$$A_7(t) = q_{71}(t)©A_1(t)$$
$A_8(t) = M_8(t) + q_{81}(t)©A_1(t)$(42-50)

Where

$M_0(t) = e^{-(\lambda_2+\lambda_3+\lambda_4+\beta_1)t}$; $M_1(t) = e^{-(\lambda_2+\lambda_3+\lambda_4+\gamma_1)t}$; $M_4(t)=M_8(t)=\overline{G_4}(t)$

On taking Laplace transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability is given by,

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_1(s)}{D_1(s)} \quad (51)$$

where

$$N_1(s) = M_0^*(s) - M_0^*(s)q_{16}^*(s)q_{61}^*(s) - M_0^*(s)q_{17}^*(s)q_{71}^*(s) - q_{18}^*(s)q_{81}^*(s)M_0^*(s) + q_{01}^*(s)M_1^*(s) + M_4{}^*(s)q_{04}^*(s) - M_4{}^*(s)q_{04}^*(s)q_{16}^*(s)q_{61}^*(s) - M_4{}^*(s)q_{04}^*(s)q_{17}^*(s)q_{71}^*(s) - M_4{}^*(s)q_{04}^*(s)q_{18}^*(s)q_{81}^*(s) + M_8{}^*(s)q_{18}^*(s)q_{81}^*(s)$$

$$D_1(s) = 1 - q_{16}^*(s)q_{61}^*(s) - q_{17}^*(s)q_{71}^*(s) - q_{18}^*(s)q_{81}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s) - q_{04}^*(s)q_{40}^*(s) - q_{01}^*(s)q_{15}^*(s)q_{50}^*(s) + q_{02}^*(s)q_{16}^*(s)q_{61}^*(s)q_{20}^*(s) + q_{02}^*(s)q_{17}^*(s)q_{71}^*(s)q_{20}^*(s) + q_{02}^*(s)q_{18}^*(s)q_{81}^*(s)q_{20}^*(s) + q_{03}^*(s)q_{30}^*(s)q_{16}^*(s)q_{61}^*(s) + q_{03}^*(s)q_{30}^*(s)q_{17}^*(s)q_{71}^*(s) + q_{03}^*(s)q_{30}^*(s)q_{18}^*(s)q_{81}^*(s) + q_{04}^*(s)q_{40}^*(s)q_{16}^*(s)q_{61}^*(s) + q_{04}^*(s)q_{40}^*(s)q_{17}^*(s)q_{71}^*(s) + q_{04}^*(s)q_{40}^*(s)q_{18}^*(s)q_{81}^*(s)$$

(52-53)

4.3 Busy period analysis of repairman for repair

Using the probabilistic arguments and defining $B_i^R(t)$ as the probability of unit is busy for repair at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^R(t)$:

$$B_0{}^R(t) = q_{01}(t)©B_1{}^R(t) + q_{02}(t)©B_2{}^R(t) + q_{03}(t)©B_3{}^R(t) + q_{04}(t)©B_4{}^R(t)$$
$$B_1{}^R(t) = q_{15}(t)©B_5{}^R(t) + q_{16}(t)©B_6{}^R(t) + q_{17}(t)©B_7{}^R(t) + q_{18}(t)©B_8{}^R(t)$$
$$B_2{}^R(t) = W_2(t) + q_{20}(t)©B_0{}^R(t)$$
$$B_3{}^R(t) = q_{30}(t)©B_0{}^R(t)$$
$$B_4{}^R(t) = q_{40}(t)©B_0{}^R(t)$$
$$B_5{}^R(t) = q_{50}(t)©B_0{}^R(t)$$
$$B_6{}^R(t) = W_6(t) + q_{61}(t)©B_1{}^R(t)$$
$$B_7{}^R(t) = q_{71}(t)©B_1{}^R(t)$$
$$B_8{}^R(t) = q_{81}(t)©B_1{}^R(t) \quad (54\text{-}62)$$

Where,

$$W_2(t) = e^{-\alpha_2 t};\ W_6(t) = e^{-\alpha_2 t}$$

Taking Laplace Transforms of the above equations and solving them for $B_0^{R*}(s)$, the following is obtained:

$$B_0^R = \lim_{s\to 0} sB_0^{R*}(s) = \lim_{s\to 0} s\frac{N_2(s)}{D_1(s)}$$

$$N_2(s) = q_{02}^*(s)W_2^*(s) - q_{02}^*(s)W_2^*(s)q_{16}^*(s)q_{61}^*(s) - q_{02}^*(s)W_2^*(s)q_{17}^*(s)q_{71}^*(s) - q_{02}^*(s)W_2^*(s)q_{18}^*(s)q_{81}^*(s) + q_{01}^*(s)q_{16}^*(s)W_6^*(s)$$

4.4 Busy period analysis of repairman for replacements

Using the probabilistic arguments and defining $B_i^{RP}(t)$ as the probability of unit is busy for replacement at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^{RP}(t)$:

$$B_0{}^{RP}(t) = q_{01}(t)©B_1{}^{RP}(t) + q_{02}(t)©B_2{}^{RP}(t) + q_{03}(t)©B_3{}^{RP}(t) + q_{04}(t)©B_4{}^{RP}(t)$$

$$B_1{}^{RP}(t) = q_{15}(t)©B_5{}^{RP}(t) + q_{16}(t)©B_6{}^{RP}(t) + q_{17}(t)©B_7{}^{RP}(t) + q_{18}(t)©B_8{}^{RP}(t)$$

$$B_2{}^{RP}(t) = q_{20}(t)©B_0{}^{RP}(t)$$

$$B_3{}^{RP}(t) = W_3(t) + q_{30}(t)©B_0{}^{RP}(t)$$

$$B_4{}^{RP}(t) = q_{40}(t)©B_0{}^{RP}(t)$$

$$B_5{}^{RP}(t) = q_{50}(t)©B_0{}^{RP}(t)$$

$$B_6{}^{RP}(t) = q_{61}(t)©B_1{}^{RP}(t)$$

$$B_7{}^{RP}(t) = W_7(t) + q_{71}(t)©B_1{}^{RP}(t)$$

$$B_8{}^{RP}(t) = q_{81}(t)©B_1{}^{RP}(t) \qquad (63\text{-}71)$$

Where,

$$W_3(t) = e^{-\alpha_3 t};\ W_7(t) = e^{-\alpha_3 t}$$

Taking Laplace Transforms of the above equations and solving them for $B_0^{RP*}(s)$, the following is obtained:

$$B_0^{RP} = \lim_{s\to 0} sB_0^{RP*}(s) = \lim_{s\to 0} s\frac{N_3(s)}{D_1(s)}$$

$$N_3(s) = q_{03}^*(s)W_3^*(s) - q_{03}^*(s)W_3^*(s)q_{16}^*(s)q_{61}^*(s) - q_{03}^*(s)W_3^*(s)q_{17}^*(s)q_{71}^*(s) - q_{03}^*(s)W_3^*(s)q_{18}^*(s)q_{81}^*(s) + q_{01}^*(s)q_{17}^*(s)W_7^*(s)$$

4.5 Busy period analysis of repairman for tripping

Using the probabilistic arguments and defining $B_i^T(t)$ as the probability of unit is busy for repair at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^T(t)$:

$$B_0{}^{R}(t) = q_{01}(t)©B_1{}^{T}(t) + q_{02}(t)©B_2{}^{T}(t) + q_{03}(t)©B_3{}^{T}(t) + q_{04}(t)©B_4{}^{T}(t)$$

$$B_1{}^{T}(t) = q_{15}(t)©B_5{}^{T}(t) + q_{16}(t)©B_6{}^{T}(t) + q_{17}(t)©B_7{}^{T}(t) + q_{18}(t)©B_8{}^{T}(t)$$

$$B_2{}^{T}(t) = W_2(t) + q_{20}(t)©B_0{}^{T}(t)$$

$$B_3{}^{T}(t) = q_{30}(t)©B_0{}^{T}(t)$$

$$B_4{}^{T}(t) = W_4(t) + q_{40}(t)©B_0{}^{T}(t)$$

$$B_5{}^{T}(t) = q_{50}(t)©B_0{}^{T}(t)$$

$$B_6{}^T(t) = q_{61}(t) © B_1{}^T(t)$$

$$B_7{}^T(t) = q_{71}(t) © B_1{}^T(t)$$

$$B_8{}^T(t) = W_8(t) + q_{81}(t) © B_1{}^T(t) \qquad (72\text{-}80)$$

Where,

$$W_4(t) = e^{-\alpha_4 t};\ W_8(t) = e^{-\alpha_4 t}$$

Taking Laplace Transforms of the above equations and solving them for $B_0^{R*}(s)$, the following is obtained:

$$B_0^T = \lim_{s\to 0} sB_0^{T*}(s) = \lim_{s\to 0} s\frac{N_4(s)}{D_1(s)}$$

$$N_4(s) = q_{04}^*(s)W_4^*(s) - q_{04}^*(s)W_4^*(s)q_{16}^*(s)q_{61}^*(s) - q_{04}^*(s)W_4^*(s)q_{17}^*(s)q_{71}^*(s) - q_{04}^*(s)W_4^*(s)q_{18}^*(s)q_{81}^*(s) + q_{01}^*(s)q_{18}^*(s)W_8^*(s)$$

5. Particular Case

For the particular case, it is assumed that the failure rates are exponentially distributed whereas the other rates are general,

$g_2(t) = \alpha_2 e^{-\alpha_2 t}, g_3(t) = \alpha_3 e^{-\alpha_3 t}, g_4(t) = \alpha_4 e^{-\alpha_4 t}$

$$p_{01} = \frac{\beta_1}{(\lambda_2 + \lambda_3 + \lambda_4 + \beta_1)},\ p_{02} = \frac{\lambda_2}{(\lambda_2 + \lambda_3 + \lambda_4 + \beta_1)},\ p_{03} = \frac{\lambda_3}{(\lambda_2 + \lambda_3 + \lambda_4 + \beta_1)}$$

$$p_{04} = \frac{\lambda_4}{(\lambda_2 + \lambda_3 + \lambda_4 + \beta_1)},\ p_{15} = \frac{\gamma_1}{(\lambda_2 + \lambda_3 + \lambda_4 + \gamma_1)},\ p_{16} = \frac{\lambda_2}{(\lambda_2 + \lambda_3 + \lambda_4 + \gamma_1)}$$

$$p_{17} = \frac{\lambda_3}{(\lambda_2 + \lambda_3 + \lambda_4 + \gamma_1)},\ p_{18} = \frac{\lambda_4}{(\lambda_2 + \lambda_3 + \lambda_4 + \gamma_1)}$$

$$p_{20} = p_{30} = p_{40} = p_{50} = p_{61} = p_{71} = p_{81} = 1$$

$\mu_0 = 3151.894,\ \mu_1 = 4758.558,\ \mu_2 = 11.876,\ \mu_3 = 7.742,\ \mu_4 = 5.971$

$\mu_5 = 4319.654,\ \mu_6 = 0011.876, \mu_7 = 07.742,\ \mu_8 = 5.971$

Using the data as summarized in section 1 and expressions for MTSF, availability and expected busy period as obtained in section 4, the following values of plant effectiveness are estimated:

- Mean time to plant failure = 23873 hours = 256 days
- Availability (A_0) = 0.7406
- Expected Busy period for repairs (B_0^R) = 0.00083
- Expected Busy period for replacements (B_0^{RP}) = 0.00017
- Expected Busy period during tripping (B_0^T) = 0.00014

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PRELIMINARY ASSESSMENT OF IRNSS SATELLITE CLOCK BEHAVIOR

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ABSTRACT :
Time and frequency of an atomic clock plays an essential role in satellite navigation systems: since the distance can be measured from a time, any error on the measure of time leads to an error on the user's position. Hence, an accurate and reliable prediction of satellite clock offsets is the necessary condition for the satellite based navigation system. The IRNSS Satellites are equipped with three space qualified high quality atomic clocks with Rubidium atomic frequency standards (RAFS) and two Atomic Clock Monitoring Units (ACMU). One RAFS is configured as primary frequency standard which will be used for navigation signal generation purpose and the second RAFS is configured as stand by to primary. The third RAFS is maintained as cold redundant. After the launch, when satellite reaches the desired orbit followed by routine activation and stabilization of all payloads, the primary and secondary RAFS were switched ON. Subsequently the onboard clocks are synchronized with the IRNSS reference time and appropriate frequency correction offsets to active ACMU is given through telecommand. The behavior of these RAFS is being continuously monitored through simultaneous reception of data from IRNSS Range and Integrity Monitoring Stations. The clock solution is obtained in post-processing mode. The other alternative means of evaluating the relative performance of active onboard clocks is done through monitoring telemetry phase meter data.
The paper presents the behavior of available IRNSS satellites in terms of satellite clock offsets and its drift with respect to IRNSS reference time, which are estimated by ground based Navigation Software and uplinked to the respective satellites as one of the primary navigation parameter. The results of on-board clocks show satisfactory performance with respect to the mission specifications.
Keywords : ***IRNSS; Satellite Clock; Atomic clock;***

I. Introduction

IRNSS: Indian Regional Navigation Satellite System

Indian Space Research Organization (ISRO) is in process of deploying an independent regional satellite based navigation system of India compatible with other existing GNSS services. ISRO has taken its first step towards the realization of the same by successful launch of its first ever navigation satellite IRNSS-1A on 1st July 2013. IRNSS-1A is placed in GSO orbit with longitude crossing of 55^0 with RAAN 139^0 and inclination of 27^0. The second satellite in the series, IRNSS-1B was launched on 4th April 2014, which is placed in GSO orbit with longitude crossing of 55^0 with RAAN 312^0 and inclination of 31^0. The third navigation satellite of IRNSS constellation, IRNSS-1C was launched on 16th October 2014 and is placed in GSO orbit with longitude crossing of 83.5^0 with RAAN 272^0 and inclination of 5^0. Until the end of year 2014, total three navigation satellites of IRNSS constellation are operational. IRNSS is designed to provide navigation services with position accuracy better than 20m for dual frequency users over India and the region extending about 1500 Km around India which is area covered by Latitude -30^0 to $+50^0$ and Longitude 30^0 to 130^0. IRNSS system consists of Space segment, Ground segment and User segment. The IRNSS architecture is shown in **Error! Reference source not found.**.

The Space segment of IRNSS constellation will consists of seven satellites. The geometry of constellation provides visibility of all the satellites in the Indian region and ensures a Geometric Dilution of Precision (GDOP) of better than 3.0 in the primary service area. The ground trace of IRNSS satellites forms loops of figure of eight with longitude crossing as mentioned above, to provide an optimum coverage in Indian region and its neighbouring countries.

The Ground segment of IRNSS consists of IRNSS Navigation Control Centre (INC), IRNSS CDMA Ranging Stations (IRCDR), IRNSS Timing Facility (IRNWT), Satellite LASER Ranging Stations (SLR), IRNSS Spacecraft Control Facility (IRSCF) and IRNSS Range and Integrity Monitoring Stations (IRIMS). Currently, most of the IRIMS are operational and providing the one-way pseudo range measurements for all three IRNSS satellites. The Bangalore IRIMS station is the Master reference station which is directly fed with IRNSS reference time.

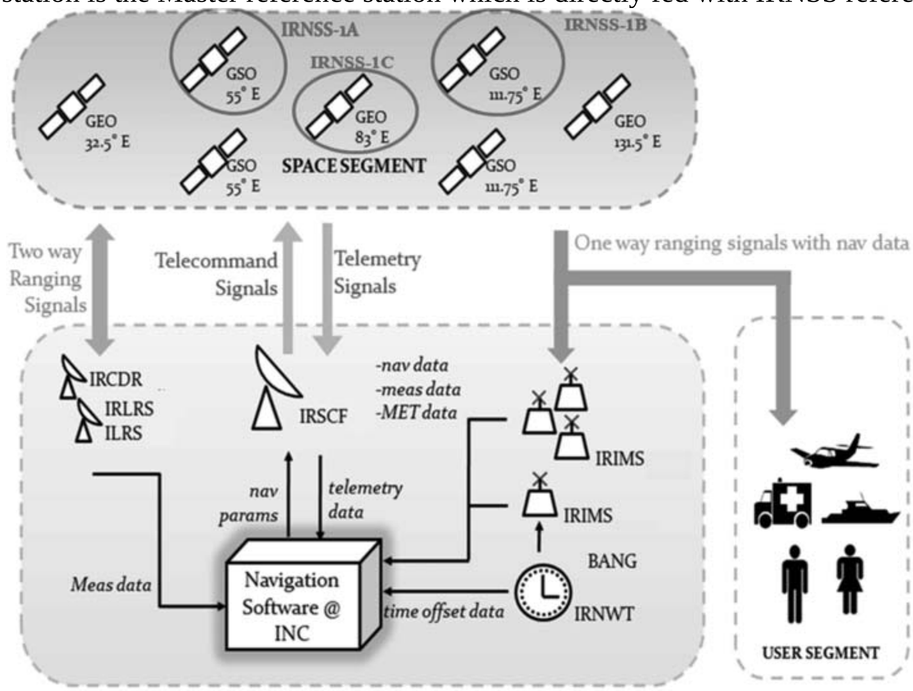

Figure 1 : IRNSS Architecture

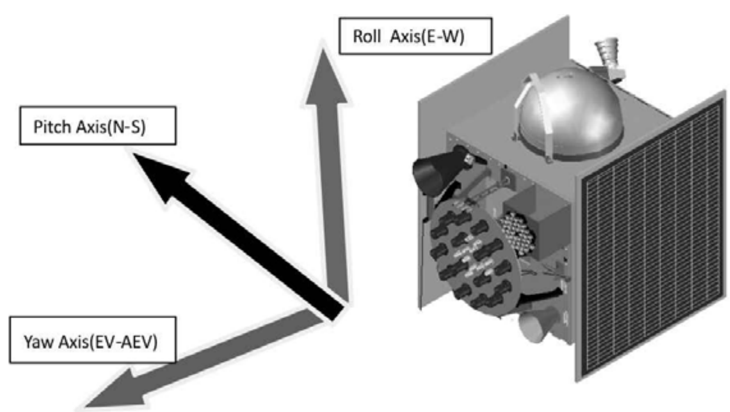

Figure 2 : Axis definition of IRNSS spacecraft

The IRNSS user segment consists of Standard Position Service (SPS) and Restricted Service (RS) users. The navigation payload will have down links in L5 and S bands of the frequency spectrum. IRNSS will provide basically two types of services SPS and RS. These services will be provided on two signals with frequencies in L5 band and S-band. Thus, user receiver can operate either in single and or dual frequency operation mode.

The Telemetry and Tele-command subsystem provide links for up-link and down-link functions in C band. The ground segment infrastructure provide all the necessary facilities and tools for the required functionalities, covering the data acquisition , archiving, the operations of the major processing facilities, the management and wide dispatching of the results to internal and external users. All IRNSS satellites has passed all In-Orbit tests flawlessly and meets all spacecraft and navigation payload specifications. The IRNSS-1A was set healthy for public use within 2 months after its launch whereas IRNSS-1B and IRNSS-1C were set healthy for public use after almost one month from their launch. It is expected to have fully functional IRNSS services with all seven satellites in orbit by end of year 2015.

All three IRNSS satellites are identical in their hardware and functional specifications. The figure 2 shows the axis definition of IRNSS satellites. The +ve Yaw direction is towards earth viewing, +ve Roll is towards East direction and Pitch direction completes the right-hand system. The atomic clocks are placed along the panel with normal in +ve Roll direction. The satellites are maintained in Yaw-Roll plane and attitude is maintained such that the atomic clock never sees the sun.

In the present paper, the behavior of IRNSS Satellite clocks in terms of estimated clock offsets and its drift with respect to IRNWT using one way measurement is presented. The section II presents the brief description about Rubidium atomic frequency standards given by Astrium. Section III presents the one-way pseudorange measurement system of IRNSS. Section IV presents the performance methodology adopted for studying the IRNSS onboard clock behavior. Section V presents the results of the IRNSS onboard clock performance. Finally conclusion is drawn in last section.

II. Rubidium Atomic Frequency Standards

Atomic clock signals allow the most precise and accurate measurements and their excellent performances are very important in complex systems such as telecommunication networks, global navigation satellite systems, and tests of fundamental physics, which may fulfill their demanding requirements only if the clock signal ensures its top-level characteristics. A common assumption when analyzing atomic clock data is that clock noise is stationary or that at least the increments in the frequency values are stationary, and the data can be examined with stability analysis tools such as the Allan variance. In practice, atomic clocks can show a non-stationary behavior due to aging, the external environment effects, or the mismatch of the internal components frequencies.

The onboard clocks of IRNSS satellites are based on the space qualified high performance of 10.00 MHz frequency standard based on the atomic reference given by the spectral absorption line of the Rb87 isotope. The specifications of the onboard RAFS for IRNSS mission are as follows:

1. Frequency stability: $5*10^{-12}$ @ 1 s
   $1.5*10^{-12}$ @ 10 s
   $5*10^{-13}$ @ 100 s
   $1.5*10^{-13}$ @ 1000 s
2. Frequency accuracy $\pm 10^{-9}$
3. Flicker floor: $< 5*10^{-14}$ (drift removed)
4. Thermal sensitivity: $< \pm 1*10^{-13}$ / °C
5. Magnetic sensitivity: $< \pm 1*10^{-13}$ / Gauss
6. Output Frequency: 10 MHz
7. Power: 30 W
8. Operating Temperature: -5 °C to +10 °C
9. Reliability: < 1,200 FIT
10. Magnetic field: < 5 Gauss
11. Radiation tolerance Suitable for LEO,MEO and GEO

Figure 3: RAFS Clock

The fundamental frequency of the IRNSS system is selected as 10.23MHz. All clocks on the IRNSS satellites and Ground reference receivers operate on this frequency. If all the IRNSS satellites are working simply on the fundamental frequency f_0(10.23MHz), then a frequency f will be observed at our reference point, and f is not the same as f_0 due to relativistic effects. For IRNSS the relativistic frequency offset is approximately 5.5155 mHz. This offset due to the relativistic effects has been implemented in the satellite clock frequency settings, and therefore users do not need to consider this effect.

Every satellite in IRNSS constellation will carry three atomic clocks with Rubidium atomic frequency standards and two Atomic Clock Monitoring Units (ACMU). The RAFS along with ACMU for IRNSS is procured by Astrium. One RAFS will be configured as primary clock which will be used for ranging purpose and other two being redundant. Both Primary and one of the secondary redundant clocks is ON at any given time. If required, it is possible to switch the hot redundant RAFS to primary RAFS through the ACMU in a seamless manner without abrupt phase change. The Phase meter equipped in ACMU is essential for the computation of phase difference between two active RAFS. The phase meter determines the phase difference between the primary RAFS and the secondary RAFS kept in hot redundancy. It is necessary to obtain stability information about the redundant clock before switching it as a primary clock. The relative stability of primary and redundant clocks can be studied using the phase meter data and further this can be compared with the behavior of primary clocks observed through measurements to find the probable behavior of redundant clock.

III. IRNSS-measurement System

In order to monitor and process the signals transmitted by IRNSS satellites, to assess the ability of Positioning, Navigation and Timing of IRNSS system, ISRO Navigation Center (INC) has been established in 2012. The prime source of measurement data for IRNSS mission is the one-way pseudorange observables provided by IRIMS reference stations located within and outside the territory of Indian land mass. Most of the IRIMS stations are operational and supporting the mission and remaining other IRIMS stations will be soon commissioned and will help in improving the IRNSS performances. The measurements are with respect to the onboard primary RAFS. The multi-channel IRIMS G-III reference receivers from Novatel are configured to generate the pseudorange and phase observables. The smoothened Code measurements were used for the clock characterization of all the available and configured satellites. All computations are done with respect to the IRNSS Network time (IRNWT). The reference clock, IRNWT is the paper clock obtained through the ensembling of Active Hydrogen Masers and Cesium atomic clocks. The IRNSS system reference time, IRNWT and IRIMS stations are maintained by ISRO Tracking, Telemetry and Commanding Center, Bangalore India. All IRNSS Satellite onboard clocks and IRIMS reference station clocks are synchronized with IRNWT. The IRNSS reference clock, IRNWT is continuously monitored and compared with respect to external reference time scales as the Universal Coordinated Time (UTC) realized by the BIPM.

The onboard satellite clocks behavior is studied using the Orbit and Clock Estimation algorithm which processes the smoothened one-way pseudorange measurements from all operational IRIMS stations. The Orbit and Clock Estimation process solves for satellite state vectors, onboard clock parameters, and the reference station parameters. This algorithm uses dedicated algorithms to deal with different effects (ionosphere, troposphere, relativity, phase center offsets corrections, tides, site displacements, ocean loading etc.).

Any miss-modeling of the involved deterministic effects will propagate together with other dominant perturbances and noise, which can result in abnormal effects on user position. The apparent clock behavior estimated as phase offset with respect to IRNWT will not coincide with the real physical onboard clock behavior since it includes stochastic and deterministic residuals errors introduced by the measurement System. And hence it is very important to separate the errors and provide the correct orbit and satellite clock information to users.

IV. Performance assessment method

The Navigation software at INC is fed with the data collected from all active IRIMS stations. The performance of the IRNSS onboard clocks is based on processing of dual-frequency iono-free carrier smoothened observables obtained from active IRIMS stations. The Orbit and Clock estimation process is a batch least-squares algorithm that processes the iono-free carrier smoothened measurements. The code measurements are smoothed with phase measurements using a hatch-filter. The Orbit and Clock estimation algorithm of Navigation software estimates the orbit, satellite clock parameters and the IRIMS stations clock parameters. One of the outputs of this process is, estimation of the offset between the onboard clock and the IRNSS reference timescale IRNWT, as seen through the whole measurement system. The satellite clock offsets are estimated using polynomial model. These estimated clocks parameters forms one of the primary navigation messages to be uplinked to the respective satellite which is further broadcasted to the user community. It is important to note that this estimated clock ("apparent clock") is different from the actual onboard clock ("true clock") as it is affected by the noise of the measurement system. There are various possible causes for such system noise (e.g., variation of hardware delays, inadequate orbital modeling, receiver related noise etc.). In following section, the results of satellite clock behavior in terms of estimated clock offsets and its drift with respect to the IRNWT for IRNSS-1A, IRNSS-1B and IRNSS-1C are discussed. Further the frequency stability is obtained in terms of overlapped Allan deviation.

results

This section presents up-to date results and observations of the IRNSS onboard clock based on the one-way measurement system and techniques described in the previous section. The data available for the analysis are the estimated phase offsets between the onboard clock and IRNWT. The availability of data is obviously subject to ground network availability as well as spacecraft transmission.

A) IRNSS-1A Satellite Clock behavior:

The first navigation satellite of IRNSS, IRNSS-1A was successfully launched on 1st July 2013. After final orbit injection and stabilization of all payloads, the RAFS were switched ON by end of July 2013 for operation. The RAFS-1 was chosen as primary clock for the ranging operation and RAFS-3 made as hot redundant clock. The fine Z-count setting i.e. setting of onboard clock for IRNSS-1A was done on 6th August, 2013. On 2nd September 2013, the RAFS-1 was given factory frequency offset correction to reduce the drift of onboard primary clock.

The figure 4 shows the variation of onboard satellite clock offsets and its drift from September, 2013 to Feb, 2014. Initially the clock is not stabilized and it is observed that it is having high drift rate. Due to its high drift rate (a2 term), the satellite clock offsets reached near to its maximum possible bit allocation memory(1ms i.e. ~300Km equivalent range) allocated for it in navigation sub frame on 25th Feb 2014 as shown in figure 5.

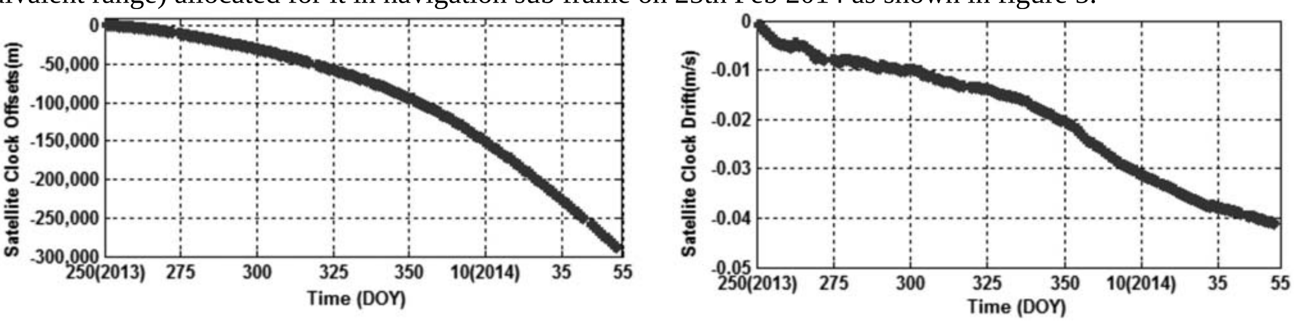

Figure 4 : IRNSS-1A Satellite Clock Offsets and drift before RAFS Switch over

The primary RAFS-1 clock behavior has been studied using the Orbit and Clock estimation algorithm using one-way pseudo measurements. The relative clock behavior of active onboard clocks is studied using phase meter data between primary RAFS-1 and secondary RAFS-3. From this study, it is been observed that the secondary clock RAFS-3 is having less drift rate and the offsets can continue to be within the specified bit allocation memory for longer duration, if Z-count and frequency correction was set appropriately. The drift direction for RAFS-3 is predicted to be positive from the combined study of phase meter analysis and through Orbit and clock estimation.

So the decision for switching over of primary clock is taken on 6th Mar2014 (DOY65) and RAFS-3 was made as primary clock and RAFS-1 was made as hot redundant clock. The Figure 5 : RAFS Switchover event for IRNS-1A

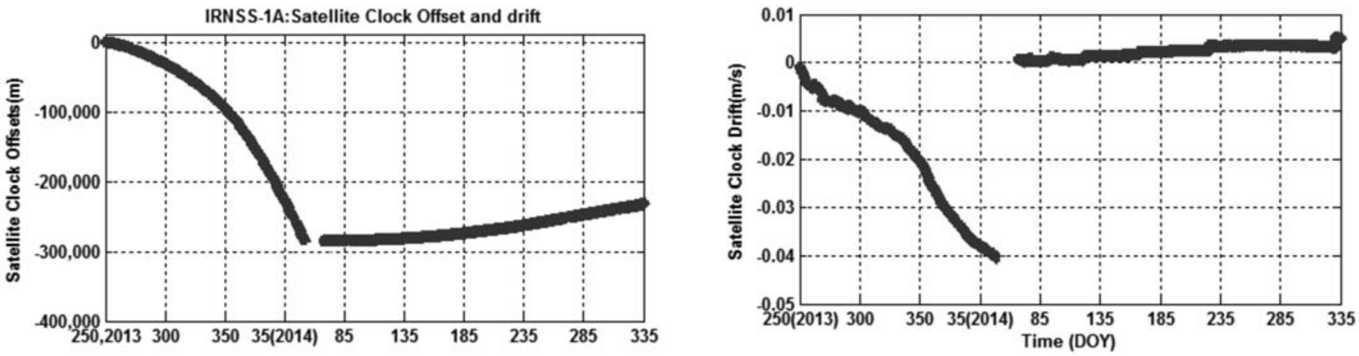

Figure 5 : shows the RAFS switch over event.

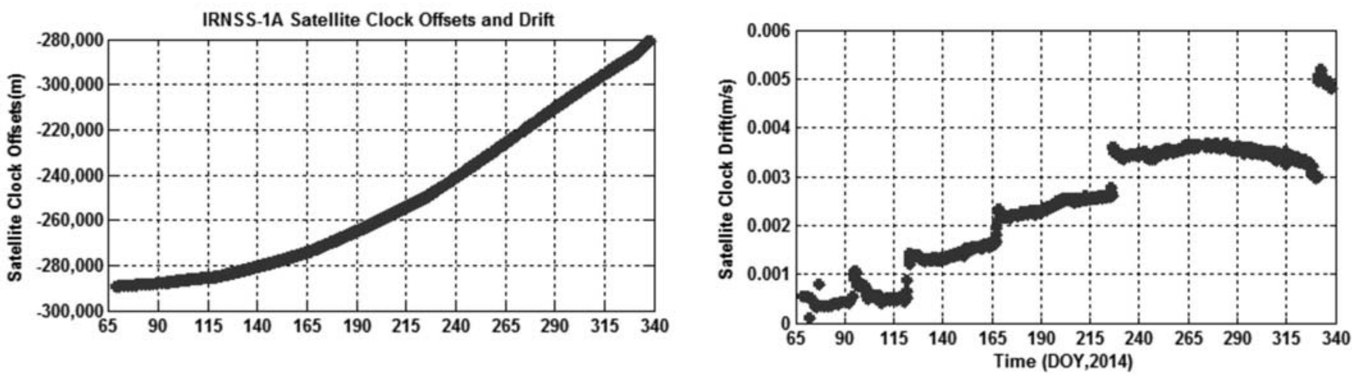

Figure 6: IRNSS-1A Clock behaviour after RAFS switchover

As per the study, the Z-count for the RAFS-3 is set near to -1ms (~ -300Km equivalent range) and it is observed that the new primary clock is drifting with positive slope. So from 6th March 2014 onwards, RAFS-3 is serving as primary clock and observations are with respect to this clock. The Figure 6 shows the variation of satellite clock behavior of RAFS-3. It can be further observed that, initially the number of satellite clock frequency jumps occurrences was more and with time the number of jumps has reduced significantly. The occurrence of clock jumps is detected in both one-way measurement data and the phase meter data.

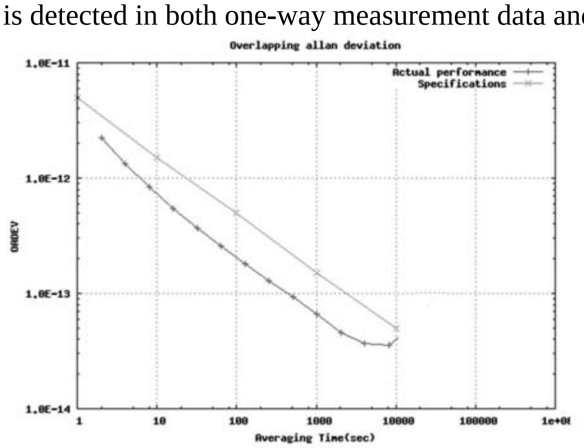

Figure 7: Frequency Stability of IRNSS-1A satellite clock

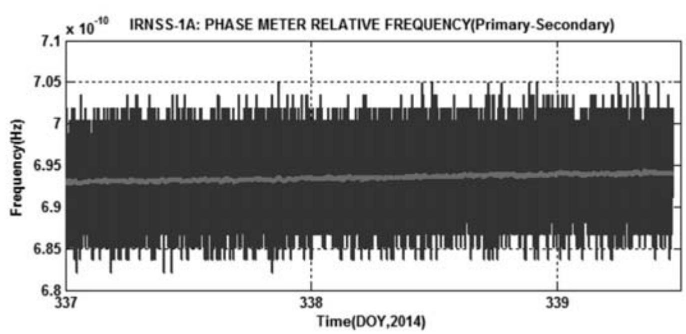

Figure 8: Phase meter relative frequency

The shows the relative frequency variation of onboard secondary clock with respect to primary clock. The shows the frequency stability plot for primary RAFS-3 using the Clock estimated through navigation software. The frequency stability is obtained in terms of overlapped Allan deviation. It can be observed that, IRNSS-1A onboard clock is meeting the specifications up to 10000sec. Further the current frequency accuracy for IRNSS-1A onboard clock is observed to be $5.0014*10^{-12}$, which is also within the specification.

B) IRNSS-1B Satellite Clock behavior:

The second satellite in the IRNSS series, IRNSS-1B was launched on 4th April 2014. After final orbit injection and stabilization of all payloads, the RAFS were switched ON on 2nd May 2014 for the operation. The RAFS-3 was chosen as primary clock for the ranging operation and RAFS-1 was made as hot redundant. The fine Z-count setting for IRNSS-1B was carried out on 2nd May 2014. The figure 9 shows the variations of Satellite clock offsets and drift from DOY125 to DOY 300 of year 2014.

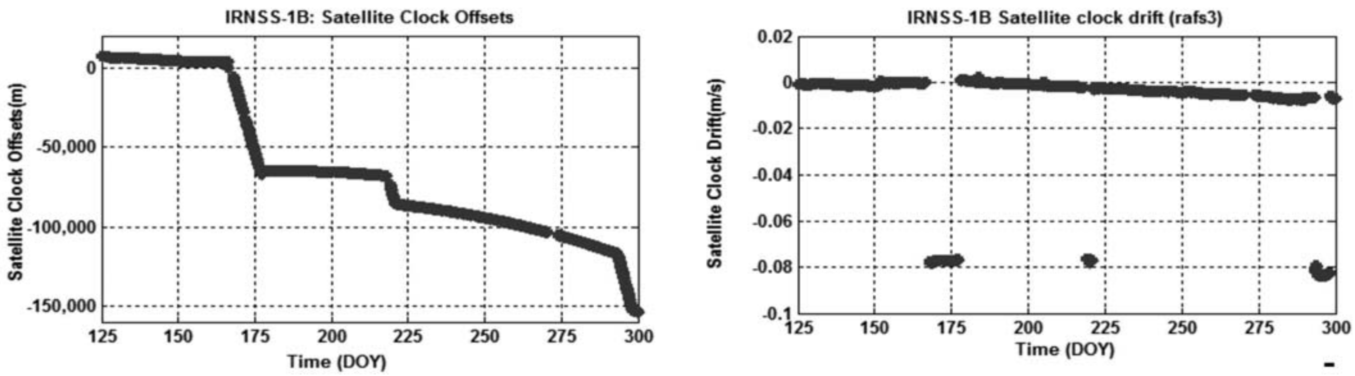

Figure 9 : IRNSS-1B, Satellite clock offsets and drift

It is observed that, the RAFS-3 performance is not satisfactory as there were many small to very big frequency jumps in order of kilometers (per day) were observed. The figure 10 shows the occurrence of big frequency jump events which are retained for longer durations (in days) and again come back to previous frequency. In addition to this, many small and large size frequency jumps were also observed which retained for very short durations (less than 5 min) and again came back to its previous value. In such cases, the range measurement data will be flagged bad and becomes unusable for further processing of orbit and clock estimation. About more than 50 such small and large frequency jumps were observed for onboard primary clock from DOY122 to DOY302 of year 2014. The result of satellite clock estimated by navigation software is compared with the phase meter frequency data between onboard primary and redundant clocks. The occurrence of frequency jumps in phase meter data is mapped to the frequency jump observations using Orbit and clock estimation. It is observed that there is no frequency jump observed for redundant clock. Combining results of clock behavior from measurements and phase meter data, it is observed that the behavior of secondary clock is satisfactory. Also it is observed that the secondary clock is drifting with negative slope. Depending upon the above study, the decision for primary clock switch over has been taken on 30thOct 2014.

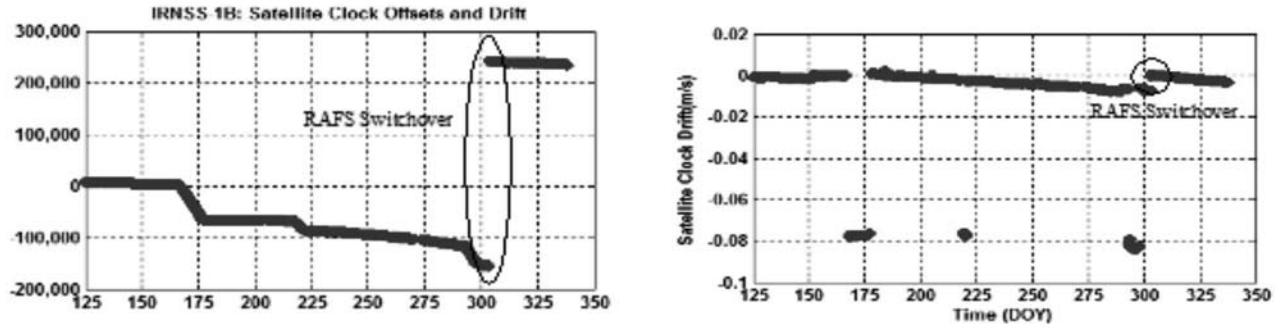

Figure 10 : RAFS switchover event for IRNSS-1B

On 30th Oct 2014, the RAFS switch over has been performed to make RAFS-1 as primary and RAFS-3 as secondary clock. The   shows the switchover event for IRNSS-1B satellite. Since the drift direction for RAFS-1 is computed to be negative, so after switchover Z-count setting was done by setting the onboard time near +0.8ms (equivalent to ~ +240Km) offset from IRNWT and appropriate frequency offset correction was given.

The figure 11 shows the behavior of IRNSS-1B satellite clock offsets and its drift of RAFS-1 after switchover. One small frequency jump has been observed on onboard clock after RAFS switchover.

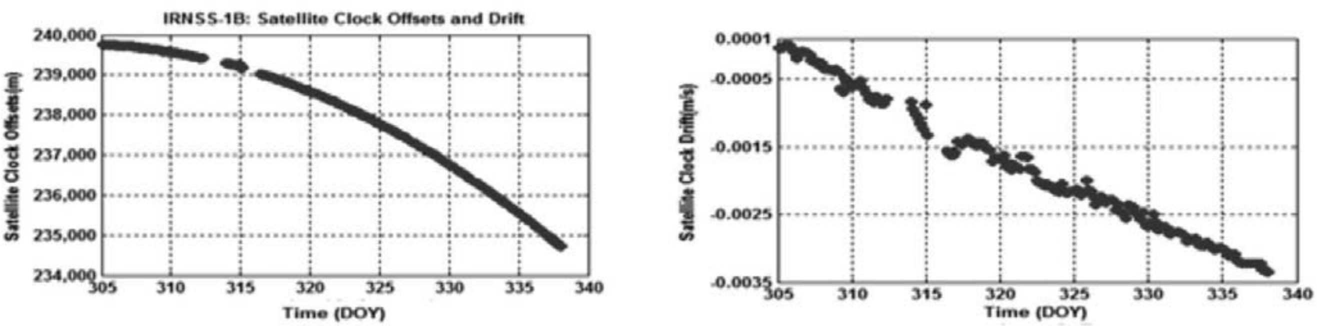

Figure 11 : IRNSS-1B, Satellite Clock behaviour after RAFS switchover

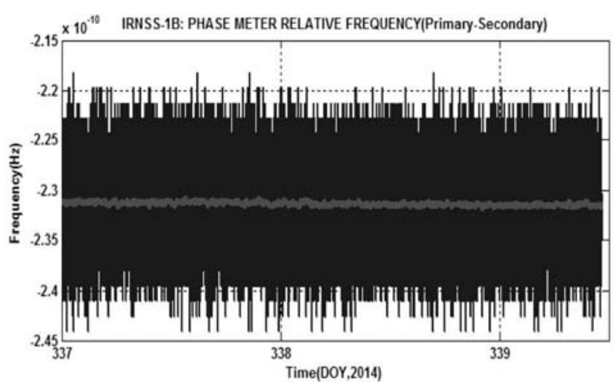

Figure 12 : Phase meter relative frequency data

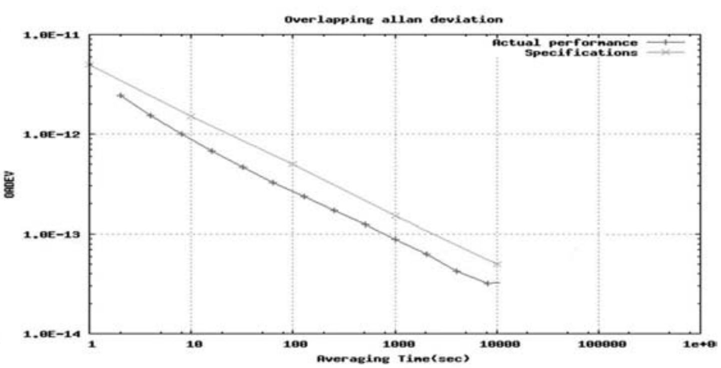

Figure 13: Frequency stability of IRNSS-1B Satellite clock

The figure 12 shows the relative frequency variation of onboard secondary clock with respect to primary clock for IRNSS-1B. The figure 13 shows the frequency stability plot for primary RAFS-1 using the Clock estimated through navigation software. It can be observed that, IRNSS-1B onboard clock is meeting the specifications up to 10000sec. Further the current frequency accuracy for IRNSS-1B onboard clock is observed to be 1.0009*10-11, which is also within the specification.

C) IRNSS-1C Satellite Clock behavior:

The third satellite of IRNSS constellation, IRNSS-1C was launched on 16th Oct 2014. After final orbit injection and stabilization of all payloads, the RAFS were switched ON on 7th Nov 2014 for the operation. The RAFS-3 was chosen as primary clock for the ranging operation and RAFS-1 was made as hot redundant. After performing coarse level Z-count setting, it was observed that the clock drift is having positive direction. Hence the fine Z-count setting with initial offset of about -0.5ms (equivalent to ~ -150Km) was performed. The following fig. 14 shows the behavior of onboard primary satellite clock from DOY316 to DOY338 of year 2014. So far one small clock frequency jump has been observed for IRNSS-1C clock. The curvature in the clock offsets plot is due to slight over correction of frequency offset for onboard primary clock.

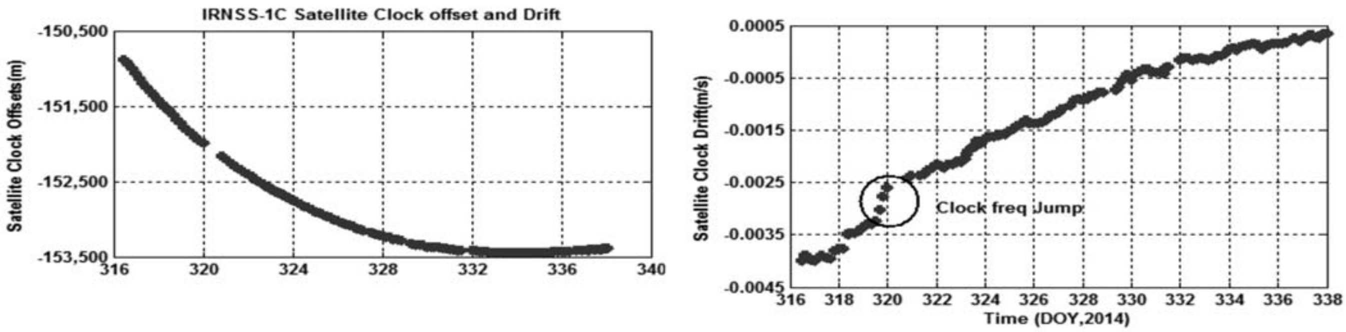

Figure 14 : IRNSS-1C, Satellite clock offsets and drift

The figure 15 shows the relative frequency variation of onboard secondary clock with respect to primary clock for IRNSS-1C.

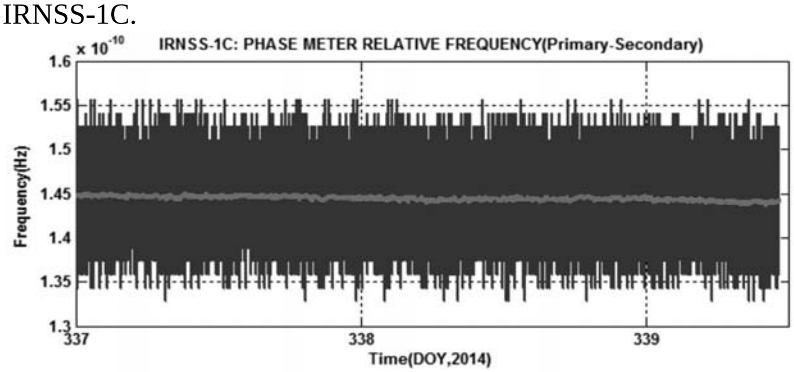

Figure 15: Phase meter relative frequency

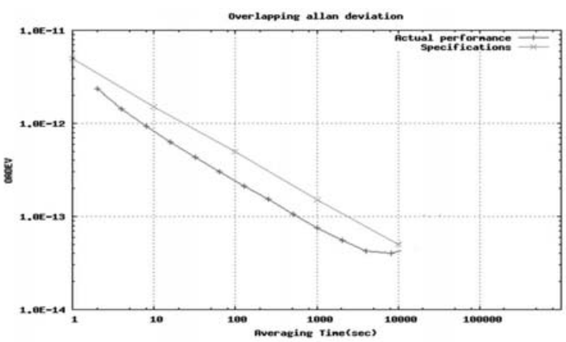

Figure 16: Frequency stability for IRNSS-1C satellite clock

The figure 16 shows the frequency stability plot for primary RAFS-3 using the Clock estimated through navigation software. It can be observed that, IRNSS-1C onboard clock is meeting the specifications up to 10000sec. Further the current frequency accuracy for IRNSS-1C onboard clock is observed to be $1.667*10^{-12}$, which is within the specification.

Conclusion and Summary

In this paper the performance of IRNSS onboard clock is presented. Almost 18months of IRNSS-1A and 7 months of IRNSS-1B in orbit has provided a good amount of valuable data for studying the behavior of satellite clock. It has been observed that the onboard satellite clocks are performing satisfactorily and are meeting the mission specifications. Apart from this, the performance of the IRNSS onboard clock is validated with the frequency stability provided by vendor. It has been found that the performance of onboard clocks meeting the requirement of Rubidium frequency standard.. Only one month data is available for the IRNSS-1C, and the behavior of its satellite clock is under study. On IRNSS-1A and IRNSS-1B, it can be concluded that RAFS are performing well and their frequency stability is meeting the requirements most of the time as observed from measurements. The better clock predictability at the timescales of several hours has been observed for all three satellites. The IRNSS satellite clock assessment study is very beneficial for developing the new strategies for handling the satellite clock anomalies for the current and next development phase of IRNSS project.

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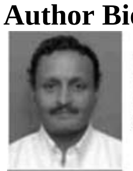

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