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RELIABILITY ANALYSIS OF A POWER GENERATING SYSTEM THROUGH GAS AND STEAM TURBINES WITH SCHEDULED INSPECTION

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ABSTRACT
A reliability model for a power generating system through gas and steam turbines is developed wherein scheduled inspection is done at regular intervals of time for maintenance. Initially, both the units i.e. the gas turbine as well as the steam turbine are operating. On failure of the gas turbine , system goes to down state , whereas on failure of the steam turbine, the system may be kept in the up state with only gas turbine working or put to down state according as the buyer of the power so generated is ready to pay higher amount or not . Three types of scheduled inspection, that is, minor, path and major inspection are done in this order at regular intervals of time for maintenance. System is analysed by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made and interesting conclusions are drawn.

INTRODUCTION
Single unit, two-unit or multi-unit systems have been widely studied in the field of reliability by various researchers including Parashar and Taneja (2007), Goyal et al. (2010) and Padmavathi et al. (2012). Contributors for the analysis of reliability models for systems with two similar units include Tuteja et al. (1991), Duhan et al. (2004), Rizwan et. al. (2010) and Mathew et al. (2011). Systems with two dissimilar units have also been analyzed by numerous researchers including Su Baohe (1997), Tuteja et. al. (2001) and Taneja et. al. (2011). In most of the studies on two dissimilar units, one unit was taken as operative and other as standby. Both the dissimilar units have also been taken as operative simultaneously in some of the studies. Tuteja et. al. (2001) discussed a two unit system where it has been considered that both the units may be operative at a time and the operation of main unit depends on the sub-unit, e.g., computer system as main unit and electricity as sub-unit. Two units for the systems discussed by them were totally dissimilar i.e. their nature was different. However, there may be practical situations where the two units are dissimilar but the nature of the work done by them is same; and failure in either of the units affects the working of the other. Such a situation was observed by the authors when they visited some gas turbine plants. Reliability modelling for such systems has not been done so far in the field of reliability and our aim is to bridge in such a gap.

The present paper develops a reliability model for a power generative system having gas and steam turbines. Initially, both the gas turbine as well as the steam turbine are operative. On failure of the gas turbine, system goes to down state as the steam turbine cannot work in this case; whereas on failure of the steam turbine, it may be kept in upstate with only gas turbine working or put to down state according as the buyer of the power so generated is ready to pay higher amount or not. The concept of scheduled inspection is also incorporated as the same was also observed while gathering information from gas turbine plant during the visit of the authors. The scheduled inspection is done at regular intervals of time for maintenance and is of three types — Minor, Path and Major Inspection.

System is analyzed by making use of semi-Markov processes and regenerative point techniques. Studies through graphs is also made and interesting conclusions are drawn.

OTHER ASSUMPTIONS FOR THE MODEL

(1) Failure, requirement of inspection and inspection times are assumed to follow exponential distribution whereas the repair times have arbitrary distributions.

(2) After every repair, unit becomes as good as new.

(3) All the random variables are independent.

(4) System fails completely on the failure of both the units.

(5) System works at reduced capacity when only the gas turbine is operative and such type of working is called single cycle.

(6) System is put to downstate during inspections and also when steam turbine is failed with no buyer of power which is generated in single cycle.

NOTATIONS

O_{gt} : Gas turbine operative

O_{gt1} : Gas turbine operative after 1st inspection / Scheduled inspection

O_{gt2} : Gas turbine operative after 2nd inspection / Scheduled inspection

O_{st} : Steam turbine operative

O_{st1} : Steam turbine operative after 1st inspection / Scheduled inspection

O_{st2} : Steam turbine operative after 2nd inspection / Scheduled inspection

U_{rgt} : Gas turbine under repair

U_{rst} : Steam turbine under repair

U_{Rst} : Repair of steam turbine continuing from previous state

d_{gt} : Gas turbine put to down mode

d_{st} : Steam turbine put to down mode

W_{rgt} : Gas turbine waiting for repair

W_{rst} : Steam turbine waiting for repair

$Insp_1$: First type of inspection (Minor inspection)

$Insp_2$: Second type of inspection (Path inspection)

$Insp_3$: Third type of inspection (Major inspection)

λ : Failure rate of gas turbine

α : Failure rate of steam turbine

p : Probability that there is dire demand of electricity and the customer is ready to pay higher amounts.

q : 1-p i.e the probability that the customer is not ready to pay the amount higher than the normal rates.

$g_1(t)$, $G_1(t)$: pdf and cdf of repair time of gas turbine

$g_2(t)$, $G_2(t)$: pdf and cdf of repair time of steam turbine

β_1 : Rate of requirement of scheduled inspection/ Maintenance

γ_1 : Rate of doing minor inspection or maintenance

γ_2 : Rate of doing path inspection or maintenance

γ_3 : Rate of doing major inspection or maintenance

Transition Probabilities and Mean Sojourn Times

The transition diagram showing the various states of the system is shown as in **Fig. 1.1**. The epochs of entry into states 0, 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14 and 15 are regeneration points and thus 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14 and 15 are regenerative states . States 4, 10, 16 are failed states. States 0, 6, 12 are up states. States 2, 8, 14 are single cycle up states, when only gas turbine is operative. States 5, 11, 17 are down states due to inspection and 1, 3, 7, 9, 13, 15 are also down state due to putting the operable unit to down mode when gas turbine is failed.

The transition probabilities are :-

$q_{01}(t) = \lambda e^{-(\lambda t\alpha+\beta 1)}$

$q_{02}(t) = p\alpha e^{-(\alpha+\lambda+\beta 1)t}$

$q_{03}(t) = q\alpha e^{-(\alpha+\lambda+\beta 1)t}$, $q_{05}(t) = \beta_1 e^{-(\alpha+\lambda+\beta 1)t}$

$q_{10}(t) = g_1(t)$, $q_{20}(t) = e^{-\lambda t} g_1(t)$

$q_{24}(t) = \lambda e^{-\lambda t}\, \overline{G}_2(t)$, $q_{21}^{(4)}(t) = [\lambda e^{-\lambda t} © 1] g_2(t) = (1-e^{-\lambda t})\, g_2(t)$

$q_{30}(t) = g_2(t)$, $q_{56}(t) = \gamma_1 e^{-\gamma 1t}$, $q_{67}(t) = \lambda e^{-(\lambda+\alpha+\beta 1)t}$

$q_{68}(t) = p\alpha e^{-(\alpha+\lambda+\beta 1)t}$, $q_{69}(t) = q\alpha e^{-(\alpha+\lambda+\beta 1)t}$, $q_{6,11}(t) = \beta_1 e^{-(\alpha+\lambda+\beta 1)t}$, $q_{76}(t) = g_1(t)$

$q_{86}(t) = e^{-\lambda t} g_2(t)$, $q_{8,10}(t) = \lambda e^{-\lambda t}\, \overline{G}_2(t)$

$q_{87}^{(10)}(t) = (\lambda e^{-\lambda t} ©) g_2(t) = (1 - e^{\lambda t}) g_2(t), \qquad q_{96} = g_2(t)$

$q_{11,12}(t) = \gamma e^{-\gamma 2t}$, $q_{12,13}(t) = \lambda e^{-(\alpha+\lambda+\beta 1)t}$

$q_{12,14}(t) = p\alpha e^{-(\alpha+\lambda+\beta 1)t}$, $q_{12.15}(t) = q\alpha e^{-(\alpha+\lambda+\beta 1)t}$

$q_{12,17} = \beta_1 e^{-(\alpha+\lambda+\beta 1)t}$, $q_{13,12}(t) = g_1(t)$

$q_{14,12}(t) = e^{-\lambda t} g_2(t)$, $q_{14,16}(t) = \lambda e^{-\lambda t}\, \overline{G}_2(t)$

$q_{14,15}^{16}(t) = (1 - e^{-\lambda t})\, g_2(t)$, $q_{15,12}(t) = g_2(t)$

$q_{17,0}(t) = \gamma_3 e^{-\gamma 3t}$

The non-zero elements $p_{ij} = \lim\limits_{s \to 0} q_{ij}(s)$ can be obtained using the above transition probabilities.

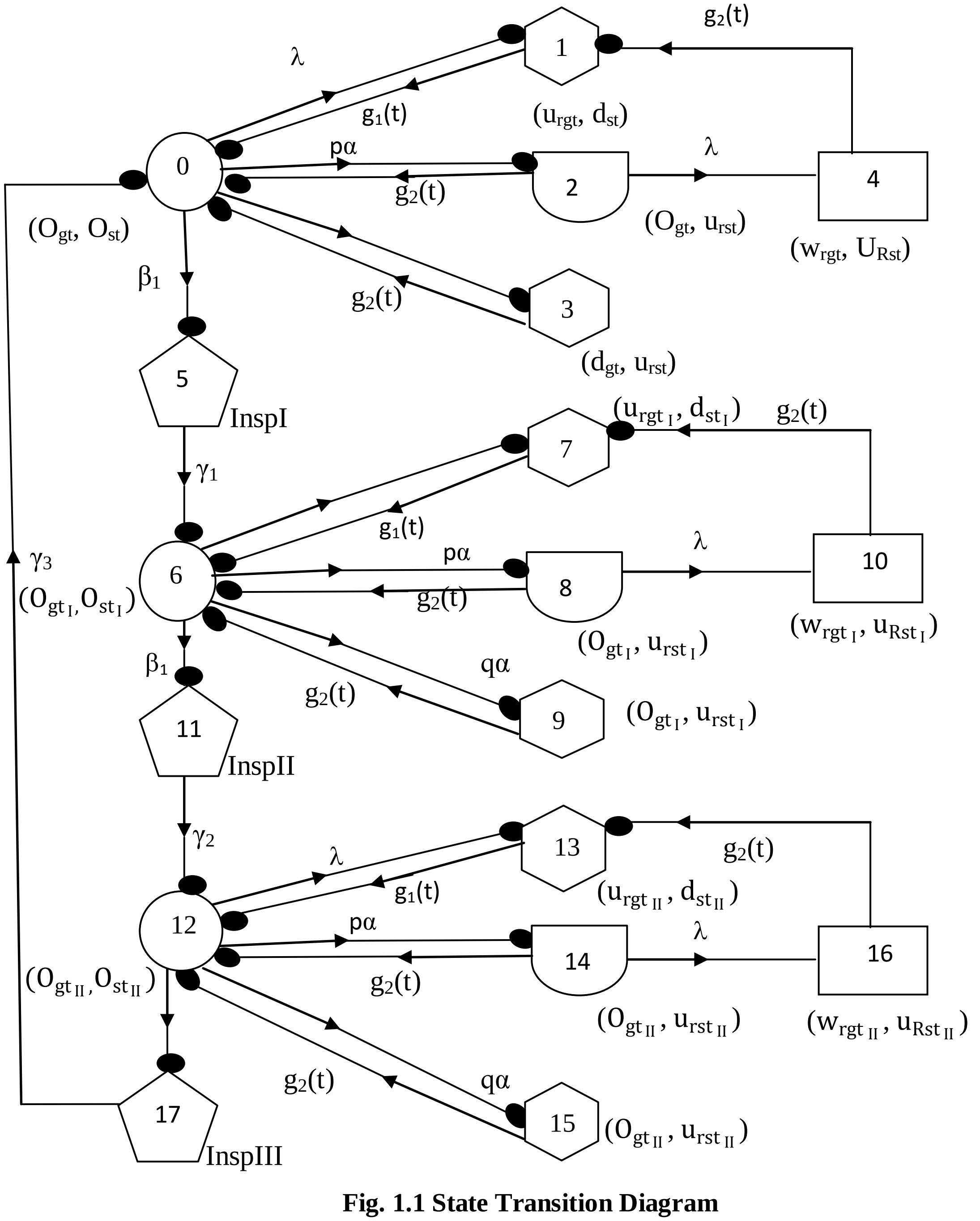
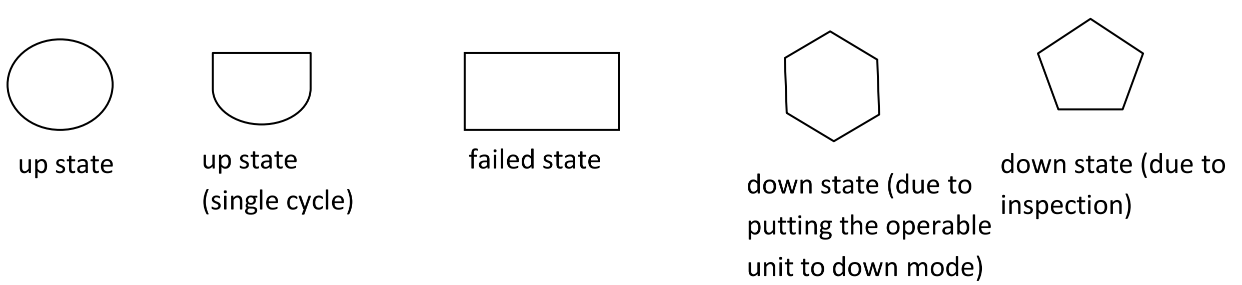


Fig. 1.1 State Transition Diagram



Mean Sojourn time (μ_i) in state i, i.e., the expected time of stay in state i is

$$\mu_0 = \int_0^\infty e^{-(\lambda+\alpha+\beta_1)}dt = \frac{1}{\lambda+\alpha+\beta_1}, \qquad \mu_1 = \int_0^\infty t g_1(t)dt$$

$$\mu_2 = \int_0^\infty e^{-\lambda t}\overline{G}_1(t)dt = \frac{1-g_2^*(\lambda)}{\lambda}, \qquad \mu_3 = \int_0^\infty t g_2(t)dt$$

$$\mu_5 = \int_0^\infty e^{-\gamma_1 t}dt = \frac{1}{\gamma_1}, \qquad \mu_6 = \mu_0, \qquad \mu_7 = \mu_1, \qquad \mu_8 = \mu_2$$

$$\mu_9 = \mu_3, \qquad \mu_{11} = \int_0^\infty e^{-\gamma_2 t}dt = \frac{1}{\gamma_2}, \qquad \mu_{12} = \mu_0, \qquad \mu_{13} = \mu_1$$

$$\mu_{14} = \mu_2, \qquad \mu_{15} = \mu_3, \qquad \mu_{17} = \int_0^\infty e^{-\gamma_3 t}dt = \frac{1}{\gamma_3}$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states as absorbing states. By probabilistic arguments, we obtain the following recursive relation for $\phi_i(t)$

$$\phi_0(t) = Q_{01}(t) ⓢ \phi_1(t) + Q_{02}(t) ⓢ \phi_2(t) + Q_{03}(t) ⓢ \phi_3(t) + Q_{05}(t) ⓢ \phi_5(t)$$

$$\phi_1(t) = Q_{10}(t) ⓢ \phi_0(t)$$

$$\phi_2(t) = Q_{20}(t) ⓢ \phi_0(t) + Q_{2,4}(t)$$

$$\phi_3(t) = Q_{30}(t) ⓢ \phi_6(t)$$

$$\phi_6(t) = Q_{67}(t) ⓢ \phi_7(t) + Q_{68}(t) ⓢ \phi_8(t) + Q_{69}(t) ⓢ \phi_9(t) + Q_{6,11}(t) ⓢ \phi_{11}(t)$$

$$\phi_7(t) = Q_{76}(t) ⓢ \phi_6(t)$$

$$\phi_8(t) = Q_{86}(t) ⓢ \phi_6(t) + Q_{8,10}(t)$$

$$\phi_9(t) = Q_{96}(t) ⓢ \phi_6(t)$$

$$\phi_{11}(t) = Q_{11,12}(t) ⓢ \phi_{12}(t)$$

$$\phi_{12}(t) = Q_{12,13}(t) ⓢ \phi_{13}(t) + Q_{12,14}(t) ⓢ \phi_{14}(t) + Q_{12,15}(t) ⓢ \phi_{15}(t) + Q_{12,17}(t) ⓢ \phi_{17}(t)$$

$$\phi_{13}(t) = Q_{13,12}(t) ⓢ \phi_{12}(t)$$

$$\phi_{14}(t) = Q_{14,12}(t) ⓢ \phi_{12}(t) + Q_{14,16}$$

$$\phi_{15}(t) = Q_{15,12}(t) ⓢ \phi_{12}(t)$$

$$\phi_{17}(t) = Q_{17,0}(t) ⓢ \phi_0(t)$$

Taking Laplace Stieltjes Transform (L.S.T) of these relations and solving them for $\phi_0^{**}(s)$, the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = (p_{6,11} + p_{6,8}p_{8,10})(p_{12,17} + p_{12,14}p_{14,16})(\mu_0) + (p_{6,11} + p_{68}p_{8,10})(\mu_0) + (p_{6,11} + p_{68}p_{8,10})$$
$$(p_{01} + p_{12,14}p_{14,16})[p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3] + p_{05}[p_{6,11}(p_{12,17} + p_{12,14}p_{14,16}) + p_{68}p_{8,10}$$
$$(p_{12,17} + p_{12,14}p_{14,16})]\mu_5 + p_{05}[p_{12,17} + p_{12,14}p_{14,16}][(\mu_0 + p_{67}\mu_1 + p_{68}\mu_2 + \mu_3 + p_{6,11}\mu_{11})]$$
$$+ p_{05}p_{6,11}(\mu_0 + p_{12,13}\mu_1 + p_{12,14}\mu_2 + p_{12,15}\mu_3 + p_{12,17}\mu_{17})]$$
$$D = (p_{12,17} + p_{12,14}p_{14,16})[p_{02}p_{24}p_{6,11} + p_{02}p_{24}p_{68}p_{8,10} + p_{05}p_{68}p_{8,10}) + p_{05}p_{6,11}p_{12,14}p_{14,16}$$

Availability At Full Capacity

Using the arguments of the theory of regenerative process, the availability $A_i(t)$ is seen to be satisfy the following recursive relations.

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t)© A_2(t) + q_{03}(t) © A_3(t) + q_{05}(t)© A_5(t)$
$A_1(t) = q_{10}(t) © A_0(t)$
$A_2(t) = q_{20}(t) © A_0(t) + q_{21}^{(4)}(t) © A_1(t)$
$A_3(t) = q_{30}(t) © A_0(t)$
$A_5(t) = q_{56}(t) © A_6(t)$
$A_6(t) = M_6(t) + q_{67}(t) © A_7(t) + q_{68}(t)© A_8(t) + q_{69}(t) © A_9(t) + q_{6,11}(t) Ⓢ A_{11}(t)$
$A_7(t) = q_{76}(t) © A_6(t)$
$A_8(t) = q_{86}(t) © A_6(t) + q_{87}^{(10)}(t) © A_7(t)$
$A_9(t) = q_{96}(t)© A_6(t)$
$A_{11}(t) = q_{11,12}(t)© A_{12}(t)$
$A_{12}(t) = M_{12}(t) + q_{12,13}(t)©A_{13}(t) + q_{12,14}(t)© A_{14}(t)$
$+ q_{12,15}(t) © A_{15}(t) + q_{12,17}(t)© A_{17}(t)$
$A_{13}(t) = q_{13,12}(t)© A_{12}(t)$
$A_{14}(t) = q_{14,12}(t)© A_{12}(t) + q_{14,13}^{(16)}(t)© A_{13}(t)$
$A_{15}(t) = q_{15,12}(t)© A_{12}(t)$
$A_{17}(t) = q_{17,0}(t)© A_0(t)$

where

$M_0(t) = M_6(t) = M_{12}(t) = e^{-(\lambda+\alpha+\beta_1)t}$

Taking Laplace Transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0[(p_{05} + p_{6,11})p_{12,17} + p_{05}p_{6,11}]$$
$$D_1 = p_{6,11}p_{12,17}[\mu_0 + (p_{01} + p_{02}p_{21}^{(4)})\mu_1 + (p_{02} + p_{03})\mu_3] + p_{05}p_{12,17}[\mu_0 + (p_{67} + p_{68}p_{87}^{(10)})\mu_1$$
$$+(p_{68} + p_{69})\mu_3] + p_5p_{6,11}[\mu_0 + (p_{12,13} + p_{12,14}p_{14,13}^{(16)})\mu_1 + (p_{12,14} + p_{12,15})\mu_3]$$
$$+p_{05}p_{6,11}p_{12,17}(\mu_5 + \mu_{11} + \mu_{17})$$

Availability in Single Cycle

Using the arguments of the theory of regenerative processes, the availability in single cycle $A_i^s(t)$ is seen to satisfy the following recursive relations

$A_0^s(t) = q_{01}(t) © A_1^s(t) + q_{02}(t)© A_2^s(t) + q_{03}(t) © A_3^s(t) + q_{05}(t)© A_5^s(t)$
$A_1^s(t) = q_{10}(t) © A_0^s(t)$
$A_2^s(t) = M_2(t) + q_{20}(t) © A_0^s(t) + q_{21}^{(4)}(t) © A_1^s(t)$

$A_3^s(t) = q_{30}(t) © A_0^s(t)$
$A_5^s(t) = q_{56}(t) © A_6^s(t)$
$A_6^s(t) = (t) © A_7^s(t) + q_{68}(t) © A_8^s(t) + q_{69}(t) © A_9^s(t) + q_{6,11}(t) Ⓢ A_{11}^s(t)$
$A_7^s(t) = q_{76}(t) © A_6^s(t)$
$A_8^s(t) = M_8(t) + q_{86}(t) © A_6^s(t) + q_{87}^{(10)}(t) © A_7^s(t)$
$A_9^s(t) = q_{96}(t) © A_6^s(t)$
$A_{11}^s(t) = q_{11,12}(t) © A_{12}^s(t)$
$A_{12}^s(t) = q_{12,13}(t) © A_{13}^s(t) + q_{12,14}(t) © A_{14}^s(t) + q_{12,15}(t) © A_{15}^s(t) + q_{12,17}(t) © A_{17}^s(t)$
$A_{13}^s(t) = q_{13,12}(t) © A_{12}^s(t)$
$A_{14}^s(t) = M_{14}(t) + q_{14,12}(t) © A_{12}^s(t) + q_{14,13}^{(16)}(t) © A_{13}^s(t)$
$A_{15}^s(t) = q_{15,12}(t) © A_{12}^s(t)$
$A_{17}^s(t) = q_{17,0}(t) © A_0^s(t)$
where $M_2(t) = M_8(t) = M_{14}(t) = e^{-\lambda t}\overline{G_2}(t)$

Taking Laplace Transforms of the above equations and solving them for $A_0^{s*}(s)$, the steady state availability in single cycle is given by :-

$$A_0^s = \lim_{s\to 0} sA_0^{s*}(s) = \frac{N_2}{D_1}$$

where $N_2 = p_{12,17}[p_{02}p_{6,11} + p_{05}p_{68}]\mu_2 + p_{05}p_{6,11}p_{12,14}\mu_2$ and D_1 is already specified.

Similarly,

the **Expected Down Time** in steady state is

$$DT_0 = \lim_{s\to 0} sDT_0^*(s) = \frac{N_3}{D_1}$$

the **Expected Time for Minor Inspection** is

$$MI_0 = \lim_{s\to 0} sMI_0^*(s) = \frac{N_4}{D_1},$$

the **Expected Time for Path Inspection** is

$$PI_0 = \lim_{s\to 0} sPI_0^*(s) = \frac{N_5}{D_1},$$

the **Expected Time for Major Inspection** is

$$MJ_0 = \lim_{s\to 0} sMJ_0^*(s) = \frac{N_6}{D_1},$$

the **Expected Fraction of Time** for which the Repairman is busy for Repair is

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_7}{D_1}.$$

where

$$N_3 = p_{12,17}[\{p_{05}\mu_5 + p_{03}\mu_3 + (p_{01} + p_{02}p_{21}^{(4)})\mu_1\}p_{6,11} + \{p_{6,11} + p_{69}\mu_3 + (p_{67} + p_{68}p_{87}^{(10)})\mu_1\}p_{05}]$$
$$+[p_{12,17}\mu_{17} + p_{12}\mu_3 + (p_{12,13} + p_{12,14}p_{14,13}^{(16)})\mu_1 p_{05}p_{6,11}$$

$N_4 = \mu_5 p_{05}p_{6,11}p_{12,17}$, $N_5 = p_{05}p_{6,11}p_{12,17}\mu_{11}$, $N_6 = p_{05}p_{6,11}p_{12,17}\mu_{17}$

$$N_7 = [(p_{03} + p_{02})\mu_3(p_{01} + p_{02}p_{21}^{(4)}\mu_1]p_{6,11}p_{12,17} + [(p_{69} + p_{68})\mu_3 + (p_{67} + p_{68}p_{87}^{(10)})\mu_1]p_{05}p_{12,17}$$
$$+[(p_{12,15} + p_{12,14})\mu_3 + (p_{12,13} + p_{14,13}^{(16)}p_{12,14})\mu_1]p_{05}p_{6,11}$$

and D_1 is already specified.

Cost-Benefit Analysis

Expected profit incurred to the system is the excess of revenue over cost and in steady state is given by

Profit $(P) = C_0A_0 + C_1A_0^s - C_2DT_0 - C_3MI_0 - C_4PI_0 - C_5MJ_0 - C_6B_0$

C_0 = Revenue per unit uptime with full capacity.

C_1 = Revenue per unit uptime in single cycle
C_2 = Loss per unit time for which the system is in down state (other than failed state)
C_3 = Cost per unit time for which the system is under minor inspection.
C_4 = Cost per unit time for which the system is undergone for path inspection.
C_5 = Cost per unit time for which the major inspection goes on.
C_6 = Cost per unit time for engaging the repairman for doing repair.

CONCLUSION

A system comprising one gas turbine and one steam turbine has been analysed. Various measures of system effectiveness have been obtained. Using the equations obtained in this paper for the MTSF and the profit, the user of such systems can obtain the expressions for his/her system putting the numerical values of various rates/costs experienced by him/her. Graphs can be plotted to find the cut-off points for various rates/costs/revenue i.e. the values beyond or prior to which the system is profitable or not. These will be helpful in taking important decisions with regard to reliability and profitability of the system.

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