Study of Different Stages of EpidemicOutbreak

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Abstract:The population of the world is around 7 billion out of which millions of people are suffering through some or other disease. The man can get infected by manyways such as through air pollution, contaminated food & water, biting of insects or by getting in connected with infected persons. But large population gets infected by the epidemic fallout. If any virus spread in a country or a state then that virus is carried through the infected person to other country or a state, this way the epidemic will spread out.The modeling of infectious diseases is a tool which has been used to study the mechanisms by which diseases spread, to predict the future course of an outbreak and to evaluate strategies to control an epidemic. Here, an attempt is done to known the different stages of epidemic fallout so that precaution can be taken to control it. The epidemic model is been designed for different stages of epidemic, showing the number of infected and susceptible persons.

Keywords: Epidemic, Fuzzy logic, infected, Mathematical modeling, susceptible

I. INTRODUCTION

Epidemics have ever been a great concern of human kind and we are stillmoved by the dramatic descriptions that arrive to us from the past.The conditions which govern the outbreak of epidemics include infected food supplies such as contaminated drinking water and the migration of populations of certain animals, such as rats or mosquitoes, which can act as disease vectors.In the whole world, thousands of people died of epidemic.This can be controlled by proper medication or by taking precaution. When this epidemic spread, three types of person involved in it, first one susceptible, second one infected and last one is recovered person. A mathematical modeling is carried out for these kinds of people. Now, taking some assumptions a model is prepared using linear Ordinary differential equation. Here, the equation is frame innonlinearode and it is converted into linear form. By solving these equationswe will get number of susceptible and infected persons. Epidemic model have different stages.Firstly the person get susceptible to the virus and got infected by it and at the last it got recovered (hospitalization or death). Sometimes after getting recovered,the person can become susceptible if he doesn't get proper medication. On the basis of these situations the mathematical model is done.

II. A SIMPLE EPIDEMIC MODEL

Let S(t) & I(t) be the number of susceptible and infected person at time t. Let initially there are n susceptible and one infected person.

$$S(t) + I(t) = n + 1 \qquad (1)$$

$$S(0) = n\,, I(0) = 1$$

If the number of infected person grows at a rate proportional to the product of susceptible and infected and number of susceptible persons decreases at the same rate i.e. infection rate 'β', then we have

$$\frac{dS}{dt} = -\beta SI \qquad (2)$$

$$\frac{dI}{dt} = \beta SI \qquad (3)$$

From equation (2) and (3),

$$\frac{dS}{dt} + \frac{dI}{dt} = 0$$

Using (1) in equation (2) , we get

$$\frac{dS}{dt} = -\beta S(n + 1 - S)$$

$$\frac{dS}{dt} + \beta(n + 1)S = \beta S^2 \qquad (4)$$

which is non-linear differential equation.

Solving equation (4) using Bernoulli's equation, converting into linear differential equation,

Let $U = S^{-1}$ (5)

Equation (4) becomes

$$\frac{dU}{dt} - \beta(n + 1)U = -\beta \qquad (6)$$

The general solution will be

$$Ue^{-\beta(n+1)t} = \frac{e^{-\beta(n+1)t}}{n + 1} + C$$

From equation (5), we have

$$S^{-1}e^{-\beta(n+1)t} = \frac{e^{-\beta(n+1)t}}{n+1} + C \qquad (7)$$

Now from equation (1), $S(0) = n$, we have

$$\frac{1}{n} = \frac{1}{n + 1} + C$$

$$\Rightarrow C = \frac{1}{n(n + 1)}$$

From equation (7)

$$S^{-1} = \frac{n + e^{-\beta(n+1)t}}{n(n + 1)}$$

$$\therefore\ S(t) = \frac{n(n+1)}{n+e^{-\beta(n+1)t}} \qquad (8)$$

From equation (1),

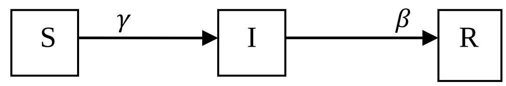
$$I(t) = n + 1 - S(t)$$

$$\therefore I(t) = \frac{(n+1)e^{\beta(n+1)t}}{n+e^{\beta(n+1)t}} \qquad (9)$$

Equation (8) and (9) shows the number of susceptible and infected persons respectively.[1]

III. SIR MODEL (SUSCEPTIBLE INFECTED REMOVED MODEL)

Here, the person get susceptible then infected and in the end got removed either by death or hospitalization/recoveryat a rate proportional to number of infective. Here, 'γ' is the rate of recovery.



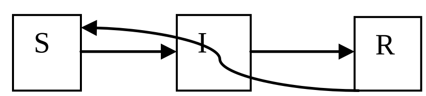
The governing equation can be written as

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

IV. SIS MODEL (SUSCEPTIBLE INFECTED SUSCEPTIBLE MODEL)



If the person loses its immunity i.e. a susceptible person can become infected at a rate proportional toβI and an infected person can recovered and can again become susceptible at a rate proportional to γI.

$$\frac{dS}{dt} = -\beta SI + \gamma I \qquad (10)$$

$$\frac{dI}{dt} = \beta SI - \gamma I \qquad (11)$$

Substituting from equation (1) in equation (11), we get

$$\frac{dI}{dt} = \beta I((n + 1) - I) - \gamma I$$

$$\Rightarrow \frac{dI}{dt} - \beta I\big((n + 1) - \gamma\big)I = \beta I^2 \qquad (12)$$

which is a non-linear differential equation. Using Bernoulli's equation converting into linear differential equation, we have

Let $U = I^{-1}$

equation (12) becomes

$$\frac{du}{dt} + (\beta(n+1) - \gamma)U = \beta$$

The general solution of above linear differential equation is

$$I^{-1} e^{(\beta(n+1)-\gamma)t} = \frac{\beta e^{(\beta(n+1)-\gamma)t}}{\beta(n+1)-\gamma} + C \quad (13)$$

From equation (1),$I(0) = 1$,applying initial condition in equation (13), we get

$$C = 1 - \frac{\beta}{\beta(n+1) - \gamma}$$

Substituting the value of C in equation (13) and deriving the value of $I(t)$ as

$$I(t) = \frac{\beta(n+1)-\gamma}{\beta + (\beta(n+1)-\gamma-\beta)e^{-(\beta(n+1)-\gamma)t}} \quad (14)$$

V. SIS MODEL (SUSCEPTIBLE INFECTED SUSCEPTIBLE MODEL) WITH CARRIERS

Most of the time epidemic is spread due to animals or insects that are known as carriers. These carriers are already been affected by germs (or scientifically said virus, protozoa, bacteria, etc.). In this model only carriers spread the diseases and their number decreases exponentially with time as these are identified and removed. 'C' - carriers

$$\frac{dS}{dt} = -\beta S(t)C(t) + \gamma I(t) \quad (15)$$

$$\frac{dI}{dt} = \beta S(t)C(t) - \gamma I(t) \quad (16)$$

We have, $S(t) + I(t) = N$ (17)

Now, as the carriers' decreases we have,

$$\frac{dc}{dt} = -\alpha C$$

$$\rightarrow \frac{dc}{C} = -\alpha dt$$

$$\rightarrow C = C_0 e^{-\alpha t} \quad (18)$$

The above expression shows that the carriers are decreasing exponentially with respect to time.

Now, using the value of C from equation (17), equation (16) becomes,

$$\frac{dI}{dt} = \beta C_0 N e^{-\alpha t} - (\beta C_0 e^{-\alpha t} + \gamma) I(t)$$

which is a linear differential equation. It can be solved as follows

$$\frac{dI}{dt} + (\beta C_0 e^{-\alpha t} + \gamma)\, I = \beta C_0 N e^{-\alpha t}$$

Integrating Factor (I.F.) = $e^{-\beta C_0 e^{-\alpha t} + \gamma t}$

The solution is

$$I(t) = \frac{\beta C_0 N e^{-\alpha t}}{-\alpha + \alpha\beta C_0 e^{-\alpha t} + \gamma} + \frac{C_1}{e^{-\beta C_0 e^{-\alpha t} + \gamma t}}$$

Equation shows the number of infected persons in epidemic with carriers. Number of susceptible can be find out by using equation (17).

VI. SIR MODEL WITHVACCINATION

While the time the person become susceptible and he got proper vaccination at proper time, then he will be recovered directly without getting infected. Vaccination is assumed to be of fixed number of shots per time period ' ω '.The governing equations are

$$\frac{dS}{dt} = -\beta SI - \omega$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I + \omega$$

VII. FUZZY APPROACH

An attempt has been made to use fuzzy logic in this model. In this attempt I have prefer only SIR model i.e. Susceptible Infected Removed and the conditions are given for some restricted number of persons. This attempt of fuzzy logic is not universal; it can be treated as an example.

```
r = newfis('epidemic');

r = addvar(r,'input',
'susceptible',[0,100]);
r = addvar(r,'input', 'infected'
,[0,50]);
r = addvar(r,'output',
'removed',[0,75]);

r =
addmf(r,'input',1,'less','trimf',[-
2 2 5]);
r =
addmf(r,'input',1,'moderate','trimf
',[4 10 30]);
r =
addmf(r,'input',1,'more','trimf',[2
5 60 100]);

r =
addmf(r,'input',2,'<2','trimf',[-1
2 5]);
r = addmf(r,'input',2,'2-
7','trimf',[2 6 8]);
r =
addmf(r,'input',2,'>7','trimf',[7
12 15]);

r =
addmf(r,'output',1,'very_less','tri
mf',[-1 0.5 1]);
r =
addmf(r,'output',1,'less','trimf',[
1 2 3]);
r =
addmf(r,'output',1,'moderate','trim
f',[3 4 5]);
r =
addmf(r,'output',1,'more','trimf',[
4 6 8]);
r =
addmf(r,'output',1,'excess','trimf'
,[7 8 12]);

ruleList = [1 1 1 1 1;1 2 2 1 1; 1
3 1 1 1; 2 1 2 1 1;2 2 3 1 1; 2 3 2
1 1;3 1 5 1 1;3 2 4 1 1;3 3 3 1 1];
r = addrule(r,ruleList);
gensurf(r,[1 2]);
surfview(r)
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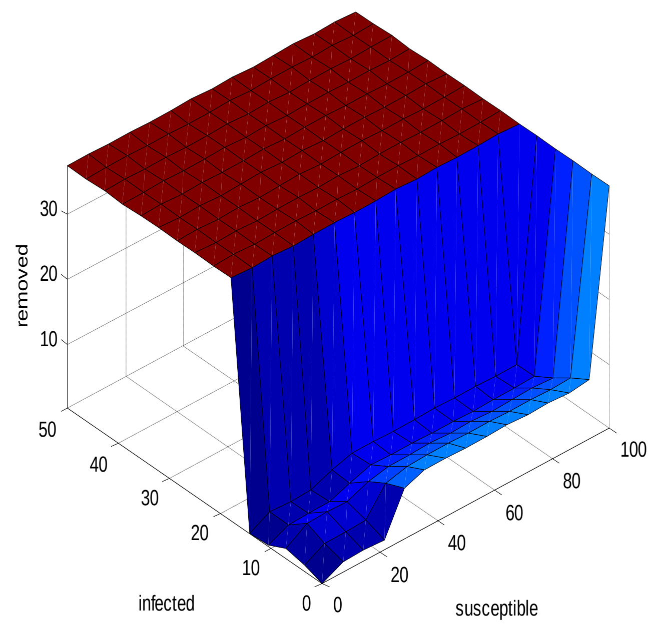

Figure (1)

VIII. CONCLUSION

Here, we have reviewed, analyzed and discuss about different stages of SIR epidemic model.Various parameters like birth, death, virus, vaccination, immunity were discussed. This model is fully depend on factors, such as infection rate 'β', rate of recovery'γ', constant numbers of carriers 'C',constant number of vaccination 'ω'. The model relies on these vital parameters because they all play a part in determining epidemic status in population.It can be known that whether there will be kind of epidemic spread out in the society or not and proper medication will be taken. The SIR model has proven to be a reliable mathematical tool for examining epidemiology in a population. A fuzzy model of epidemic is made by passing different rules, which can be viewed by figure (1) containing susceptible, infected and removed.

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