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PROBABILISTIC ANALYSIS OF A SEVEN UNIT DESALINATION PLANT WITH MINOR / MAJOR FAILURES AND PRIORITY GIVEN TO REPAIR OVER MAINTENANCE

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ABSTRACT:
Desalination is a water treatment process that removes the salt from sea water or brackish water. Multi stage flash desalination process is used for sea water purification. The desalination plant operates round the clock and during the normal operation; six of the seven evaporators are in operation for water production while one evaporator is always under scheduled maintenance and used as standby. The paper presents a probabilistic analysis of the plant with two failure categories viz. minor and major and priority being given to repair over maintenance. The complete plant is shut down for about one month during winter season for annual maintenance. The water supply during shutdown period is maintained through ground water and storage system. Any major failure or annual maintenance brings the evaporator/plant to a complete halt and the plant goes under forced outage state. For the current analysis, seven years failure data have been extracted from the operations and maintenance department of the plant located in Oman. Various measures of the plant effectiveness have been obtained probabilistically. Semi-Markov processes and regenerative point techniques are used in the entire analysis.
Keywords – ***Desalination plant, failures, maintenance, shutdown, major/minor repairs, Semi – Markov, regenerative processes***

NOTATIONS

O	Operative state of evaporator
U_{ms}	Under Maintenance during summer
U_{mwb}	Under Maintenance during winter before service
U_{mwa}	Under Maintenance during winter after service
W_{ms}	Waiting for Maintenance during summer
W_{mwa}	Waiting for Maintenance during winter after service
W_{mwb}	Waiting for Maintenance during winter before service
$F_{r_{1s}}$	Failed unit is under minor repair during summer
$F_{r_{2s}}$	Failed unit is under major repair during summer
$F_{r_1 wb}$	Failed unit is under minor repair during winter before service
$F_{r_2 wb}$	Failed unit is under major repair during winter before service
$F_{r_1 wa}$	Failed unit is under minor repair during winter after service
$F_{r_2 wa}$	Failed unit is under major repair during winter after service
β_1	Rate of the unit moving from summer to winter

β_2	Rate of the unit moving from winter to summer
Λ	Rate of failure of any component of the unit
Γ	Maintenance Rate
γ_1	Rate of shutting down
γ_2	Rate of recovery after shut down during winter
α_1	Repair rate for minor repairs
α_2	Repair rate for major repairs
p_1	Probability of occurrence of minor repair
p_2	Probability of occurrence of major repair
©	Symbol for Laplace Convolution
Ⓢ	Symbol for Stieltje's convolution
*	Symbol for Laplace Transforms
**	Symbol for Laplace Stieltje's transforms
C_0	Revenue per unit uptime
C_1	Cost per unit uptime for which the repairman is busy for maintenance
C_2	Cost per unit uptime for which the repairman is busy for repair
C_3	Cost per unit uptime for which the repairman is busy during shutdown
C_4	Cost per unit repair require replacement
A_0	Steady state availability of the system
B_0^M	Expected busy period of the repairman for maintenance
B_0^R	Expected busy period of the repairman for repair
B_0^S	Expected busy period of the repairman during shutdown
R_0	Expected number of repairs require replacement
$\emptyset_i(t)$	c.d.f. of first passage time from a regenerative state i to a failed state j
$p_{ij}(t)$, $Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in (0, t]
$g_m(t)$, $G_m(t)$	p.d.f. and c.d.f. of maintenance rate
$g_{sr}(t)$, $G_{sr}(t)$	p.d.f. and c.d.f. of recovery rate
$g_1(t)$, $G_1(t)$	p.d.f. and c.d.f. of repair rate for minor repairs
$g_2(t)$, $G_2(t)$	p.d.f. and c.d.f. of repair rate for major repairs

(All Costs have been considered in Omani Riyal)

1. INTRODUCTION

Desalination is a water treatment process that removes the salt from sea water or brackish water. It is the only option in arid regions, since the rainfall is marginal. In many desalination plants, multi stage flash desalination process is normally used for water purification which is very expensive and involves sophisticated systems. Since, desalination plants are designed to fulfil the requirement of water supply for a larger sector in arid regions, they are normally kept in continuous production mode especially during summer except for emergency/forced/planned outages. It is therefore, very important that the efficiency and reliability of such a complex system is maintained in order to avoid big loses. Many researchers have spent a great deal of efforts in analysing industrial systems to achieve the reliability results that are useful in understanding the system behaviour. Munoli & Suranagi [1] predicted the reliability indices in fatal and non-fatal shock model, Singh & Satyavati [2] analysed a screening system in paper industry, Mathew et al. [3] analysed an identical two-unit parallel CC plant system operative with full installed capacity, Singh and Taneja [5] developed a reliability model for a power generative system having one gas and one steam turbine with the concept of scheduled inspection time for maintenance of three types — Minor, Path and Major Inspection, Padmavathi et al. [4] carried out an analysis for desalination plant with online repair and emergency shutdowns. Recently, some more case studies have been reported by Rizwan et al. [6] & Padma et al. [7] for desalination plants under various failure and repair situations. Thus, the methodology for system analysis under various failure and repair assumptions has been widely presented in the literature and the novelty of this work lies in its case study. The numerical results of various reliability indices are extremely helpful in understanding the significance of these failures/maintenances on plant availability and assess the impact of these failures on the overall profitability of the plant.

Thus, the paper is an attempt to present a case analysis of the desalination plant where priority is given to repair over maintenance whereas in [7], the repair of minor/major failure or maintenance is carried out on first come first served basis. Failure data for seven years have been collected from operations and maintenance department of the plant in Oman. Component failure, maintenance, and plant shutdown rates, and various maintenance costs involved are estimated from the data. The desalination plant operates round the clock and during the normal operation; six of the seven evaporators are in operation for water production while one evaporator is always under scheduled maintenance and used as standby evaporator. This ensures the continuous water production with minimum possible failures of the evaporators. The complete plant is shut down for about a month during winter season because of the low consumption of water for annual maintenance; the water supply during this period is maintained through ground water and storage system. The evaporator fails due to any one of the two types of failure viz., minor and major. Repairable and serviceable failures are categorised as minor failures, whereas the replaceable failures are categorised as major failure. Any major failure or annual maintenance brings the evaporator/plant to a complete halt and goes under forced outage state.

Using the data, following values of rates and various costs are estimated:

- Estimated rate of failure of any component of the unit (λ) = 0.00002714 per hour
- Estimated rate of the unit moving from summer to winter (β_1) =0.0002315 per hour
- Estimated rate of the unit moving from winter to summer (β_2) = 0.0002315 per hour
- Estimated rate of Maintenance (γ) = 0.0014881
- Estimated rate of shutting down (γ_1) = 0.00011416 per hour
- Estimated rate of recovery after shut down during winter (γ_2) = 0.00138889 per hour
- Estimated value of repair rate of Type I repairs (α_1) = 0.099216per hour
- Estimated value of repair rate of Type II repairs (α_2) = 0.059701per hour
- Probability of occurrence of Minor Repair (p_1) = 0.7419
- Probability of occurrence of Major Repair (p_2) = 0.2581

- The Revenue per unit uptime (C_0) = RO 596.7 per hour
- Cost per unit uptime for which the repairman is busy for maintenance(C_1) = RO0.0626 per hour
- The Cost per unit uptime for which the repairman is busy for repair(C_2) = RO0.003 per hour
- Cost per unit uptime for which the repairman is busy during shutdown(C_3) = RO 16.378 per hour
- The Cost per unit repair require replacement (C_4) = RO 13.246 per hour

The plant is analyzed probabilistically by using semi-Markov processes and regenerative point techniques. Measures of plant effectiveness such as mean times to failure of the plant, availability, busy period analysis of repairman during maintenance, expected busy period during repair, expected busy period during shut down, expected number of repairs and profitability of the system are estimated numerically.

2. MODEL DESCRIPTION AND ASSUMPTIONS

- There are seven evaporators in the desalination plant; of which 6 operate at any given time and one is always under scheduled maintenance.
- The priority is given to repair over maintenance.
- Maintenance of no evaporator is done if the repair of any other evaporator is going on.
- The plant goes into shutdown for annual maintenance during winter season for one month.
- On completion of maintenance/repair, the repairman inspects to detect the type failure i.e. minor or major before putting the repaired unit into operation.
- A unit failed in a season gets repaired in that season only.
- Not more than two units fail at a time.
- During the maintenance of one unit, not more than one out of the other units can get failed.
- All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram showing the possible states of transition of the plant is shown in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are regeneration points. The transition probabilities are given by:

$$dQ_{00} = \gamma\, e^{-(6\lambda+\beta_1+\gamma)t}\,dt;\ \ dQ_{01} = \beta_1\, e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt$$

$$dQ_{02} = p_1 6\lambda e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt;\ dQ_{04} = p_2 6\lambda e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt$$

$$dQ_{11} = e^{-(6\lambda+\gamma_1)t}\,g_m(t)dt;\ \ dQ_{13} = \gamma_1\, e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt$$

$$dQ_{15} = 6\lambda p_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt;\ \ dQ_{17} = 6\lambda p_2 e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt$$

$$dQ_{36} = \gamma_2 e^{-\gamma_2 t}\,dt$$

$$dQ_{20} = \alpha_1\, e^{-\beta_1 t}\, e^{-\alpha_1 t}\,dt;\ \ dQ_{25} = \beta_1 e^{-\beta_1 t}\, e^{-\alpha_1 t}\,dt$$

$$dQ_{40} = \alpha_2\, e^{-\beta_1 t}\, e^{-\alpha_2 t}\,dt;\ dQ_{47} = \beta_1 e^{-\beta_1 t}\, e^{-\alpha_2 t}\,dt$$

$$dQ_{51} = e^{-\gamma_1 t} g_1(t)dt;\ \ dQ_{53} = \gamma_1\, e^{-\gamma_1 t}\overline{G_1}(t)dt$$

$$dQ_{71} = e^{-\gamma_1 t} g_2(t)dt;\ dQ_{73} = \gamma_1\, e^{-\gamma_1 t}\overline{G_2}(t)dt$$

$$dQ_{60} = \beta_2\, e^{-(6\lambda+\beta_2)t}\,\overline{G_m}(t)dt;\ dQ_{66} = e^{-(6\lambda+\beta_2)t}\,g_m(t)$$

$$dQ_{68} = 6\lambda p_1\, e^{-(6\lambda+\beta_2)t}\,\overline{G_m}(t)dt;\ dQ_{69} = 6\lambda\, p_2 e^{-(6\lambda+\beta_2)t}\,\overline{G_m}(t)dt$$

$$dQ_{82} = \beta_2 e^{-\beta_2 t} e^{-\alpha_1 t} dt; \; dQ_{86} = \alpha_1 e^{-\alpha_1 t} e^{-\beta_2 t} dt$$
$$dQ_{94} = \beta_2 e^{-\beta_2 t} e^{-\alpha_2 t} dt; \; dQ_{96} = \alpha_2 e^{-\alpha_2 t} e^{-\beta_2 t} dt \qquad (1\text{-}25)$$

The transition probabilities p_{ij} are given below:

$$p_{00} + p_{01} + p_{02} + p_{04} = 1; \; p_{11} + p_{13} + p_{15} + p_{17} = 1; \; p_{20} + p_{25} = 1, \; p_{36} = 1; \; p_{40} + p_{47} = 1$$
$$p_{51} + p_{53} = 1; \; p_{60} + p_{66} + p_{68} + p_{69} = 1; \; p_{71} + p_{73} = 1, \; p_{82} + p_{86} = 1; \; p_{94} + p_{96} = 1 \qquad (26\text{-}35)$$

The mean sojourn time (μ_i) in the regenerative state '*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state '*i*', then:

$$\mu_i = E(T) = P(T > t)$$

$$\mu_0 = \int_0^\infty e^{-(6\lambda+\beta_1+\gamma)t} dt = \frac{1}{(6\lambda+\beta_1+\gamma)}; \; \mu_1 = \int_0^\infty e^{-(6\lambda+\gamma_1+\gamma)t} dt = \frac{1}{(6\lambda+\gamma_1+\gamma)}$$
$$\mu_2 = \int_0^\infty e^{-(\beta_1+\alpha_1)t} dt = \frac{1}{(\beta_1+\alpha_1)}; \qquad \mu_3 = \int_0^\infty e^{-\gamma_2 t} dt = \frac{1}{\gamma_2}$$
$$\mu_4 = \int_0^\infty e^{-(\beta_1+\alpha_2)t} dt = \frac{1}{(\beta_1+\alpha_2)}; \qquad \mu_5 = \int_0^\infty e^{-(\gamma_1+\alpha_1)t} dt = \frac{1}{(\gamma_1+\alpha_1)}$$

$$\mu_6 = \int_0^\infty e^{-(6\lambda+\beta_2+\gamma)t} dt = \frac{1}{(6\lambda+\beta_2+\gamma)}; \; \mu_7 = \int_0^\infty e^{-(\gamma_1+\alpha_2)t} dt = \frac{1}{(\gamma_1+\alpha_2)}$$
$$\mu_8 = \int_0^\infty e^{-(\alpha_1+\beta_2)t} dt = \frac{1}{(\alpha_1+\beta_2)}; \qquad \mu_9 = \int_0^\infty e^{-(\beta_2+\alpha_2)t} dt = \frac{1}{(\beta_2+\alpha_2)} \qquad (36\text{—}45)$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entry into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0), \; \sum_j m_{ij} = \mu_i$$

$$m_{00} + m_{01} + m_{02} + m_{04} = \mu_0; \; m_{11} + m_{13} + m_{15} + m_{17} = \mu_1; \; m_{20} + m_{25} = \mu_2 \; ; m_{36} = \mu_3,$$
$$m_{40} + m_{47} = \mu_{4;} \; m_{51} + m_{53} = \mu_{5;} \; m_{60} + m_{66} + m_{68} + m_{69} = \mu_6$$
$$m_{71} + m_{73} = \mu_7 \; ; m_{82} + m_{86} = \mu_8; \; m_{94} + m_{96} = \mu_9 \qquad (46\text{—}55)$$

4. THE MATHEMATICAL ANALYSIS

4.1 Mean time to System Failure

To determine the Mean time to system failure, the failed states are considered as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ is obtained:

$$ø_0(t) = Q_{00}(t) \, Ⓢ \, ø_0(t) + Q_{01}(t) \, Ⓢ \, ø_1(t) + Q_{02}(t) + Q_{04}(t)$$
$$ø_1(t) = Q_{11}(t) \, Ⓢ \, ø_1(t) + Q_{13}(t) + Q_{15}(t) + Q_{17}(t)$$
$$ø_3(t) = Q_{36}(t) \, Ⓢ \, ø_6(t)$$
$$ø_6(t) = Q_{60}(t) \, Ⓢ \, ø_0(t) + Q_{66}(t) \, Ⓢ \, ø_6(t) + Q_{68}(t) + Q_{69}(t) \qquad (56\text{—}59)$$

Solving the above equation for $\phi_o^{**}(s)$, the mean time to system failure when the unit started at the beginning of state 0 is,

$$MTSF = \lim_{s\to 0} \frac{1-\emptyset_0^{**}(s)}{s} = \frac{N}{D}$$

4.2 Availability Analysis of the Plant

Using the probabilistic arguments and defining the steady state availability $A_i(t)$ as the probability of unit entering into upstate at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $A_i(t)$:

$A_0(t) = M_0(t) + q_{00}(t) \copyright A_0(t) + q_{01}(t) \copyright A_1(t) + q_{02}(t) \copyright A_2(t) + q_{04}(t) \copyright A_4(t)$
$A_1(t) = M_1(t) + q_{11}(t) \copyright A_1(t) + q_{13}(t) \copyright A_3(t) + q_{15}(t) \copyright A_5(t) + q_{17}(t) \copyright A_7(t)$
$A_2(t) = M_2(t) + q_{20}(t) \copyright A_0(t) + q_{25}(t) \copyright A_5(t)$
$A_3(t) = q_{36}(t) \copyright A_6(t)$
$A_4(t) = M_4(t) + q_{40}(t) \copyright A_0(t) + q_{47}(t) \copyright A_7(t)$
$A_5(t) = M_5(t) + q_{51}(t) \copyright A_1(t) + q_{53}(t) \copyright A_3(t)$
$A_6(t) = M_6(t) + q_{60}(t) \copyright A_0(t) + q_{66}(t) \copyright A_6(t) + q_{68}(t) \copyright A_8(t) + q_{69}(t) \copyright A_9(t)$
$A_7(t) = M_7(t) + q_{71}(t) \copyright A_1(t) + q_{73}(t) \copyright A_3(t)$

$A_8(t) = M_8(t) + q_{82}(t) \copyright A_2(t) + q_{86}(t) \copyright A_6(t)$
$A_9(t) = M_9(t) + q_{94}(t) \copyright A_4(t) + q_{96}(t) \copyright A_6(t)$

Where $M_0(t) = e^{-(6\lambda+\beta_1+\gamma)t}$; $M_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$; $M_2(t) = e^{-(\beta_1+\alpha_1)t}$
$M_4(t) = e^{-(\gamma_1+\lambda_1)t}$; $M_5(t) = e^{-(6\lambda+\alpha_1)t}$; $M_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$
$M_7(t) = e^{-(6\lambda+\alpha_2)t}$; $M_8(t) = e^{-(6\lambda+\alpha_1+\gamma_1)t}$; $M_9(t) = e^{-(6\lambda+\alpha_2+\gamma_1)t}$

On taking Laplace Transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability is given by,

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1(0)}{D_1'(0)}$$

4.3 Busy period analysis for Maintenance

Using the probabilistic arguments and defining $B_i^M(t)$ as the probability of unit is busy for maintenance at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^M(t)$:

$B_0^M(t) = W_0(t) + q_{00}(t) \copyright B_0^M(t) + q_{01}(t) \copyright B_1^M(t) + q_{02}(t) \copyright B_2^M(t) + q_{04}(t) \copyright B_4^M(t)$,
$B_1^M(t) = W_1(t) + q_{11}(t) \copyright B_1^M(t) + q_{13}(t) \copyright B_3^M(t) + q_{15}(t) \copyright B_5^M(t) + q_{17}(t) \copyright B_7^M(t)$,
$B_2^M(t) = q_{20}(t) \copyright B_0^M(t) + q_{25}(t) \copyright B_5^M(t)$,
$B_3^M(t) = q_{36}(t) \copyright B_6^M(t)$

$B_4^M(t) = q_{40}(t)$ ©$B_0^M(t) + q_{47}(t)$ ©$B_7^M(t)$
$B_5^M(t) = q_{51}(t)$ © $B_1^M(t) + q_{53}(t)$ ©$B_3^M(t)$,
$B_6^M(t) = W_6(t) + q_{60}(t)$ © $B_0^M(t) + q_{66}(t)$ © $B_6^M(t) + q_{68}(t)$ © $B_8^M(t) + q_{69}(t)$ © $B_9^M(t)$,
$B_7^M(t) = q_{71}(t)$ © $B_1^M(t) + q_{73}(t)$ © $B_3^M(t)$,
$B_8^M(t) = q_{82}(t)$ © $B_2^M(t) + q_{86}(t)$ © $B_6^M(t)$,
$B_9^M(t) = q_{94}(t)$ © $B_4^M(t) + q_{96}(t)$ © $B_6^M(t)$,

Where $W_0(t) = e^{-(6\lambda+\beta_1+\gamma)t}$; $W_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$; $W_2(t) = e^{-(\beta_1+\alpha_1)t}$
$W_4(t) = e^{-(\gamma_1+\lambda_1)t}$; $W_5(t) = e^{-(6\lambda+\alpha_1)t}$; $W_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$
$W_7(t) = e^{-(6\lambda+\alpha_2)t}$; $W_8(t) = e^{-(6\lambda+\alpha_1+\gamma_1)t}$; $W_9(t) = e^{-(6\lambda+\alpha_2+\gamma_1)t}$

Taking Laplace Transforms of the above equations and solving them for $B_0^{M*}(s)$, using the determinants method the following is obtained:

$$B_0^M = \lim_{s\to 0} sB_0^{M*}(s) = \frac{N_2(0)}{D_1'(0)}$$

4.4 Busy period analysis for Repair

Using the probabilistic arguments and defining $B_i^R(t)$ as the probability of unit is busy for repair at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^R(t)$:

$B_0^R(t) = q_{00}(t)$ ©$B_0^R(t) + q_{01}(t)$ ©$B_1^R(t) + q_{02}(t)$ ©$B_2^R(t) + q_{04}(t)$ ©$B_4^R(t)$,
$B_1^R(t) = q_{11}(t)$ ©$B_1^R(t) + q_{13}(t)$ ©$B_3^R(t) + q_{15}(t)$ ©$B_5^R(t) + q_{17}(t)$ ©$B_7^R(t)$,
$B_2^R(t) = W_2(t) + q_{20}(t)$ © $B_0^R(t) + q_{25}(t)$ © $B_5^R(t)$,
$B_3^R(t) = q_{36}(t)$ ©$B_6^R(t)$,
$B_4^R(t) = W_4(t) + q_{40}(t)$ ©$B_0^R(t) + q_{47}(t)$ ©$B_7^R(t)$,
$B_5^R(t) = W_5(t) + q_{51}(t)$ © $B_1^R(t) + q_{53}(t)$ ©$B_3^R(t)$,
$B_6^R(t) = q_{60}(t)$ © $B_0^R(t) + q_{66}(t)$ © $B_6^R(t) + q_{68}(t)$ © $B_8^R(t) + q_{69}(t)$ © $B_9^R(t)$,
$B_7^R(t) = W_7(t) + q_{71}(t)$ © $B_1^R(t) + q_{73}(t)$ © $B_3^R(t)$,
$B_8^R(t) = W_8(t) + q_{82}(t)$ © $B_2^R(t) + q_{86}(t)$ © $B_6^R(t)$,
$B_9^R(t) = W_9(t) + q_{94}(t)$ © $B_4^R(t) + q_{96}(t)$ © $B_6^R(t)$,

Where, $W_2(t) = e^{-(\beta_1+\alpha_1)t}$, $W_4(t) = e^{-(\gamma_1+\lambda_1)t}$, $W_5(t) = e^{-(6\lambda+\alpha_1)t}$,
$W_7(t) = e^{-(6\lambda+\alpha_2)t}$, $W_8(t) = e^{-(6\lambda+\alpha_1+\gamma_1)t}$, $W_9(t) = e^{-(6\lambda+\alpha_2+\gamma_1)t}$

Taking Laplace Transforms of the above equations and solving them for $B_0^{R*}(s)$, the following is obtained:

$$B_0^R = \lim_{s\to 0} sB_0^{R*}(s) = \frac{N_3(0)}{D_1'(0)}$$

4.5 Expected Busy Period during Shut Down

Using the probabilistic arguments and defining $B_i^S(t)$ as the probability of unit is busy for maintenance at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained for $B_i^S(t)$:

$B_0^S(t) = q_{00}(t) © B_0^S(t) + q_{01}(t) © B_1^S(t) + q_{02}(t) © B_2^S(t) + q_{04}(t) © B_4^S(t)$,
$B_1^S(t) = q_{11}(t) © B_1^S(t) + q_{13}(t) © B_3^S(t) + q_{15}(t) © B_5^S(t) + q_{17}(t) © B_7^S(t)$,
$B_2^S(t) = q_{20}(t) © B_0^S(t) + q_{25}(t) © B_5^S(t)$,
$B_3^S(t) = W_3(t) + q_{36}(t) © B_6^S(t)$,
$B_4^S(t) = q_{40}(t) © B_0^S(t) + q_{47}(t) © B_7^S(t)$,
$B_5^S(t) = q_{51}(t) © B_1^S(t) + q_{53}(t) © B_3^S(t)$,
$B_6^S(t) = q_{60}(t) © B_0^S(t) + q_{66}(t) © B_6^S(t) + q_{68}(t) © B_8^S(t) + q_{69}(t) © B_9^S(t)$,
$B_7^S(t) = q_{71}(t) © B_1^S(t) + q_{73}(t) © B_3^S(t)$,
$B_8^S(t) = q_{82}(t) © B_2^S(t) + q_{86}(t) © B_6^S(t)$,
$B_9^S(t) = q_{94}(t) © B_4^S(t) + q_{96}(t) © B_6^S(t)$,

Where, $W_3(t) = e^{-\gamma t}$

Taking Laplace Transforms of the above equations and solving them for $B_0^{S*}(s)$, the following is obtained:

$$B_0^S = \lim_{s\to 0} s\ B_0^{S*}(s) = \frac{N_4(0)}{D_1'(0)}$$

4.6 Expected number of repairs

$R_0(t) = Q_{00}(t) Ⓢ R_0(t) + Q_{01}(t) Ⓢ R_1(t) + Q_{02}(t) Ⓢ R_2(t) + Q_{04}(t) Ⓢ R_4(t)$,
$R_1(t) = Q_{11}(t) Ⓢ R_1(t) + Q_{13}(t) Ⓢ R_3(t) + Q_{15}(t) Ⓢ R_5(t) + Q_{17}(t) Ⓢ R_7(t)$,
$R_2(t) = Q_{20}(t) Ⓢ [1 + R_0(t)] + Q_{25}(t) Ⓢ R_5(t)$,
$R_3(t) = Q_{36}(t) Ⓢ R_6(t)$,
$R_4(t) = Q_{40}(t) Ⓢ [1 + R_0(t)] + Q_{47}(t) Ⓢ R_7(t)$,
$R_5(t) = Q_{51}(t) Ⓢ [1 + R_1(t)] + Q_{53}(t) Ⓢ R_3(t)$,
$R_6(t) = Q_{60}(t) Ⓢ R_0(t) + Q_{66}(t) Ⓢ R_6(t) + Q_{68}(t) Ⓢ R_8(t) + Q_{69}(t) Ⓢ R_9(t)$,
$R_7(t) = Q_{71}(t) Ⓢ [1 + R_1(t)] + Q_{73}(t) Ⓢ R_3(t)$,
$R_8(t) = Q_{82}(t) Ⓢ R_2(t) + Q_{86}(t) Ⓢ [1 + R_6(t)]$,
$R_9(t) = Q_{94}(t) Ⓢ R_4(t) + Q_{96}(t) Ⓢ [1 + R_6(t)]$,

Taking Laplace Transforms of the above equations and solving them for $R_0^*(s)$, the following is obtained:

$$R_0 = \lim_{s\to 0} s\ R_0^*(s) = \frac{N_5(0)}{D_1'(0)}$$

5. PROFIT ANALYSIS

One of the main objectives of the reliability analysis is to have cost-effective and profitable maintenance strategies. In order to reflect this, the overall profit of the system could be defined; by incorporating the steady-state solutions and various costs:

$$P = C_0 A_0 - C_1 B_0^M - C_2 B_0^R - C_3 B_0^S - C_4 R_0$$

6. PARTICULAR CASE

For the particular case, it is assumed that the failure rates are exponentially distributed whereas other rates are general. Using the values estimated from the data as summarized in section1, the following are obtained:

$$p_{00} = g_m^*(6\lambda + \beta_1) = \frac{\gamma}{(6\lambda + \beta_1 + \gamma)}$$

$$p_{01} = \frac{\beta_1}{(6\lambda + \beta_1)}\left(1 - g_m^*(6\lambda + \beta_1)\right) = \frac{\beta_1}{(6\lambda + \beta_1 + \gamma)}$$

$$p_{02} = \frac{6\lambda p_1}{(6\lambda + \beta_1)}\left(1 - g_m^*(6\lambda + \beta_1)\right) = \frac{6\lambda p_1}{(6\lambda + \beta_1 + \gamma)}$$

$$p_{04} = \frac{6\lambda p_2}{(6\lambda + \beta_1)}\left(1 - g_m^*(6\lambda + \beta_1)\right) = \frac{6\lambda p_2}{(6\lambda + \beta_1 + \gamma)}$$

$$p_{11} = g_m^*(6\lambda + \gamma_1) = \frac{\gamma}{(6\lambda + \gamma_1 + \gamma)}$$

$$p_{13} = \frac{\gamma_1}{(6\lambda + \gamma_1)}\left(1 - g_m^*(6\lambda + \gamma_1)\right) = \frac{\gamma_1}{(6\lambda + \gamma_1 + \gamma)}$$

$$p_{15} = \frac{6\lambda p_1}{(6\lambda + \gamma_1)}\left(1 - g_m^*(6\lambda + \gamma_1)\right) = \frac{6\lambda p_1}{(6\lambda + \gamma_1 + \gamma)}$$

$$p_{17} = \frac{6\lambda p_2}{(6\lambda + \gamma_1)}\left(1 - g_m^*(6\lambda + \gamma_1)\right) = \frac{6\lambda p_2}{(6\lambda + \gamma_1 + \gamma)}$$

$$p_{25} = \left(1 - g_1^*(\beta_1)\right) = \frac{\beta_1}{(\alpha_1 + \beta_1)};\ p_{20} = g_1^*(\beta_1) = \frac{\alpha_1}{(\alpha_1 + \beta_1)};\ p_{36} = g_{sr}^*(\gamma_2) = 1,$$

$$p_{40} = g_2^*(\beta_1) = \frac{\alpha_2}{(\alpha_2 + \beta_1)};\ p_{47} = \left(1 - g_2^*(\beta_1)\right) = \frac{\beta_1}{(\alpha_2 + \beta_1)}$$

$$p_{51} = g_1^*(\gamma_1) = \frac{\alpha_1}{(\gamma_1 + \alpha_1)};\ p_{53} = 1 - g_1^*(\gamma_1) = \frac{\gamma_1}{(\gamma_1 + \alpha_1)}$$

$$p_{60} = \frac{\beta_2}{(6\lambda + \beta_2)}\left(1 - g_m^*(6\lambda + \beta_2)\right) = \frac{\beta_2}{(6\lambda + \beta_2 + \gamma)}$$

$$p_{66} = g_m^*(6\lambda + \beta_2) = \frac{\gamma}{(6\lambda + \beta_2 + \gamma)}$$

$$p_{68} = \frac{6\lambda p_1}{(6\lambda + \beta_2)}\left(1 - g_m^*(6\lambda + \beta_2)\right) = \frac{6\lambda p_1}{(6\lambda + \beta_2 + \gamma)}$$

$$p_{69} = \frac{6\lambda p_2}{(6\lambda + \beta_2)}\left(1 - g_m^*(6\lambda + \beta_2)\right) = \frac{6\lambda p_2}{(6\lambda + \beta_2 + \gamma)}$$

$$p_{71} = g_2^*(\gamma_1) = \frac{\alpha_2}{(\gamma_1 + \alpha_2)};\ p_{73} = 1 - g_2^*(\gamma_1) = \frac{\gamma_1}{(\gamma_1 + \alpha_2)},$$

$$p_{82} = 1 - g_1^*(\beta_2) = \frac{\beta_2}{(\alpha_1 + \beta_2)};\ p_{86} = g_1^*(\beta_2) = \frac{\alpha_1}{(\alpha_1 + \beta_2)}$$

$$p_{96} = g_2^*(\beta_2) = \frac{\alpha_2}{(\alpha_2 + \beta_2)};\ p_{94} = \left(1 - g_2^*(\beta_2)\right) = \frac{\beta_2}{(\alpha_2 + \beta_2)}$$

$$\mu_0 = \int_0^\infty e^{-(6\lambda + \beta_1 + \gamma)t}\, dt = \frac{1}{(6\lambda + \beta_1 + \gamma)};\ \mu_1 = \int_0^\infty e^{-(6\lambda + \gamma_1 + \gamma)t}\, dt = \frac{1}{(6\lambda + \gamma_1 + \gamma)}$$

$$\mu_2 = \int_0^\infty e^{-(\beta_1 + \alpha_1)t}\, dt = \frac{1}{(\beta_1 + \alpha_1)};\ \mu_3 = \int_0^\infty e^{-\gamma_2 t}\, dt = \frac{1}{\gamma_2}$$

$$\mu_4 = \int_0^\infty e^{-(\beta_1 + \alpha_2)t}\, dt = \frac{1}{(\beta_1 + \alpha_2)};\ \mu_5 = \int_0^\infty e^{-(\gamma_1 + \alpha_1)t}\, dt = \frac{1}{(\gamma_1 + \alpha_1)}$$

$$\mu_6 = \int_0^\infty e^{-(6\lambda + \beta_2 + \gamma)t}\, dt = \frac{1}{(6\lambda + \beta_2 + \gamma)};\ \mu_7 = \int_0^\infty e^{-(\gamma_1 + \alpha_2)t}\, dt = \frac{1}{(\gamma_1 + \alpha_2)}$$

$$\mu_8 = \int_0^\infty e^{-(\alpha_1 + \beta_2)t}\, dt = \frac{1}{(\alpha_1 + \beta_2)};\ \mu_9 = \int_0^\infty e^{-(\beta_2 + \alpha_2)t}\, dt = \frac{1}{(\beta_2 + \alpha_2)}$$

Using the data as summarized in section 1, various expressions for reliability indicators obtained as in section 4, the following measures of plant effectiveness are obtained:

Mean Time to shut down = 194 days
Availability (A_0) = 0.9603
Expected Busy period for Maintenance (B_0^M) = 0.9584
Expected Busy period for repair(B_0^R) = 0.0018
Expected Busy period during shutdown (B_0^S) = 0.0397
Expected number of repairs (R_0) = 0.0002
Profit (P) = RO 572.277 per unit uptime

7. GRAPHICAL INTERPRETATION

The above particular case has been considered for the graphical interpretation.

- Figure 2 represents the behavior of MTSF with respect to the failure rate (λ). MTSF decreases with respect to an increase in the failure rate (λ).
- Figure 3 represents the behavior of the evaporator availability (A_0) with respect to the failure rate λ. An increasing trend for Availability with the decrease in the failure rate has been observed.
- Figure 4 depicts the behavior of profit (P) with respect to revenue per unit uptime (C_0). The profit is positive or zero or negative according as the revenue per unit uptime C_0 is $> or = or <$ 0.750.
- Figure 5 illustrates the behavior of the profit (P) with respect to revenue per unit uptime (C_0) for different values of the cost of manpower during maintenance(C_1):

  - For C_1 = 0.0626, the profit is positive or zero or negative according as C_0 is $> or = or <$ 0.750.

- For C_3 =50, the profit is positive or zero or negative according as C_0 is $> or = or < 50.475$.
- For C_3 =100, the profit is positive or zero or negative according as C_0 is $> or = or < 100.475$

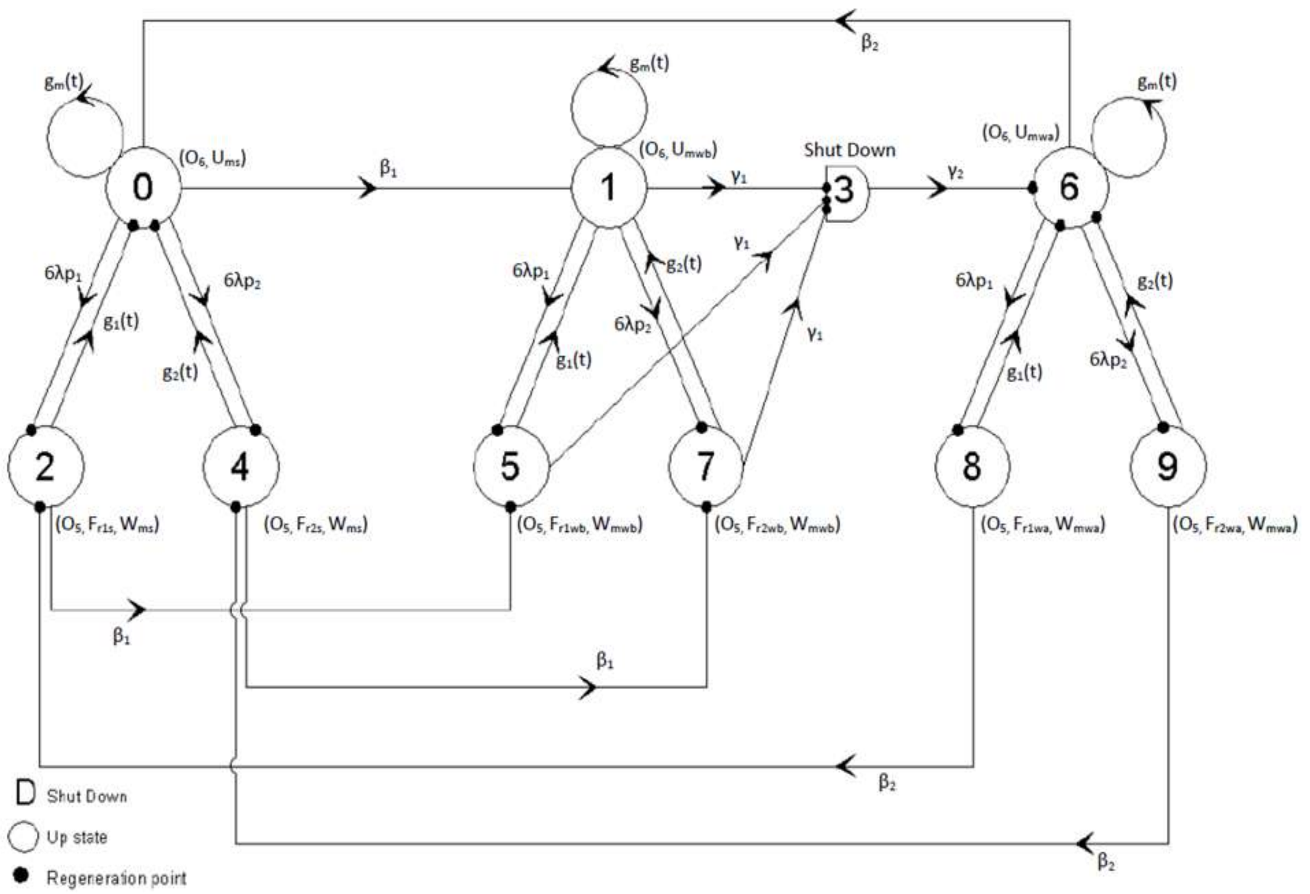

Figure 1

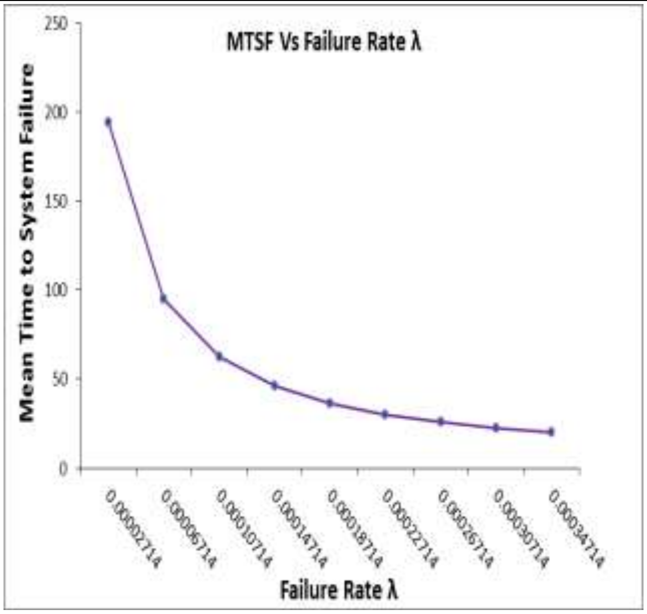

Figure 2

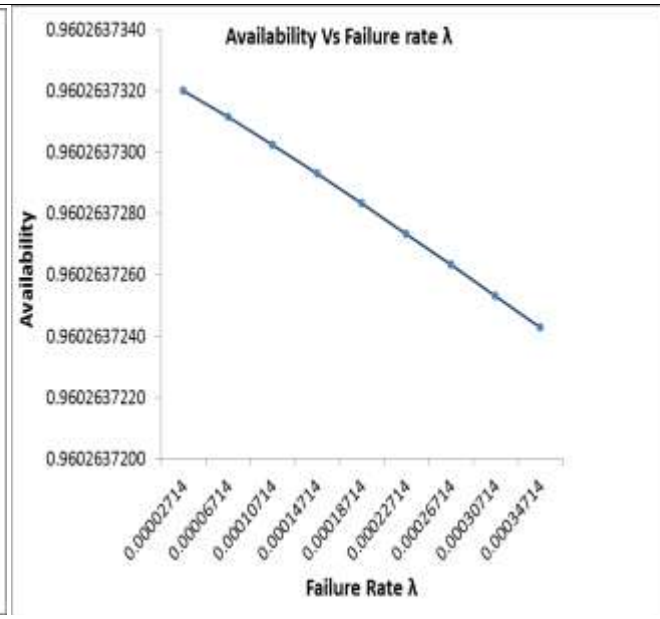

Figure 3

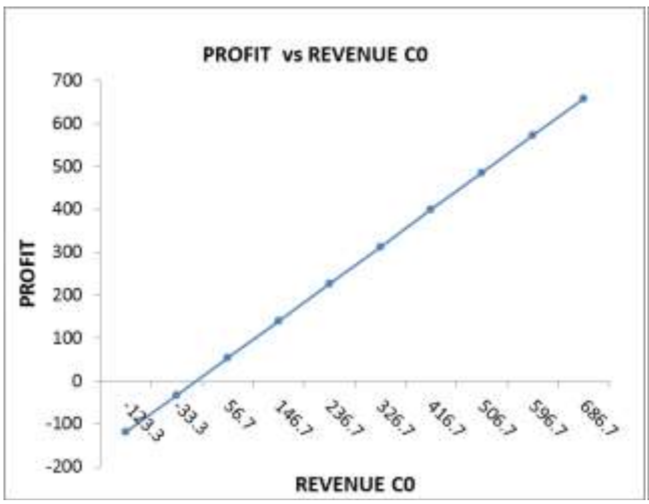

Figure 4

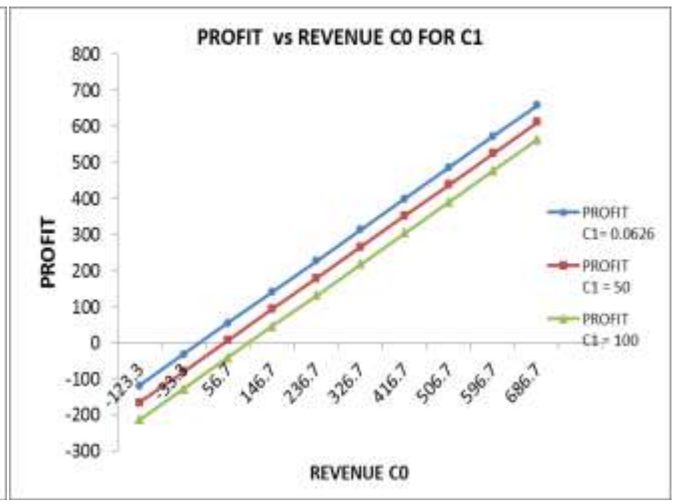

Figure 5

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