

## SLIGHTLY $v$ -OPEN MAPPINGS

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### ABSTRACT:

The aim of this paper is to introduce and study the concept of slightly  $v$ -open mappings and the interrelationship between other open maps.

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### 1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced  $\alpha$ -open and  $\alpha$ -closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semi pre-open and semi pre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semi pre-closed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced  $\beta$ -open mappings in the year 1983, S. Balasubramanian and P.A.S.Vyjayanthi [2011, 2012] introduced  $v$ -open and regular pre-open mappings. They further defined almost  $v$ -open mappings and also introduced  $v$ -closed and Almost  $v$ -closed mappings. C.W. Baker [2011] studied slightly-open and slightly-closed mappings. Inspired with these concepts and its interesting properties in this paper we tried to study a new variety of closed maps called slightly  $v$ -closed and almost slightly  $v$ -closed maps. Throughout the paper  $X$ ,  $Y$  means topological spaces  $(X, \tau)$  and  $(Y, \sigma)$  on which no separation axioms are assured.

### 2. PRELIMINARIES:

**Definition 2.1:**  $A \subseteq X$  is said to be

- a) regular open[pre-open; semi-open;  $\alpha$ -open;  $\beta$ -open] if  $A = \text{int}(\text{cl}(A))$  [ $A \subseteq \text{int}(\text{cl}(A))$ ;  $A \subseteq \text{cl}(\text{int}(A))$ ;  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ ;  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ ] and regular closed[pre-closed; semi-closed;  $\alpha$ -closed;  $\beta$ -closed] if  $A = \text{cl}(\text{int}(A))$  [ $\text{cl}(\text{int}(A)) \subseteq A$ ;  $\text{int}(\text{cl}(A)) \subseteq A$ ;  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ ;  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ ]
- b)  $v$ -open if there exists a r-open set  $U$  such that  $U \subseteq A \subseteq \text{cl}(U)$ .
- c) g-closed[rg-closed] if  $\text{cl}(A) \subset U$  [ $\text{rcl}(A) \subset U$ ] whenever  $A \subset U$  and  $U$  is open[r-open] in  $X$ .
- d) g-open[rg-open] if its complement  $X - A$  is g-closed[rg-closed].

**Remark 1:** We have the following implication diagrams for open sets.

$$\begin{array}{ccccccc}
 r\alpha\text{-open} & \rightarrow & \rightarrow & \rightarrow & v\text{-open} \\
 \uparrow & & & & \downarrow \\
 r\text{-open} & \rightarrow & \text{open} & \rightarrow & \alpha\text{-open} & \rightarrow & \text{semi-open} & \rightarrow & \beta\text{-open}
 \end{array}$$

**Definition 2.2:** A function  $f:X \rightarrow Y$  is said to be

- a) continuous [resp: semi-continuous, r-continuous,  $v$ -continuous] if the inverse image of every open set is open [resp: semi open, regular open,  $v$ -open].
- b) irresolute [resp: r irresolute,  $v$  irresolute] if the inverse image of every semi open [resp: regular open,  $v$ -open] set is semi open [resp: regular open,  $v$ -open].
- c) open [resp: semi-open, r-open] if the image of every open set is open [resp: semi-open, regular-open].
- d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed [resp: rg-closed].

**Definition 2.3:**  $X$  is said to be  $v$ -regular space (or  $v$ -T<sub>3</sub> space) if for a open set  $F$  and a point  $x \notin F$ , there exists disjoint  $v$ -open sets  $G$  and  $H$  such that  $F \subseteq G$  and  $x \in H$ .

**Definition 2.4:**  $X$  is said to be  $T_{1/2}$ [r-T<sub>1/2</sub>] if every (regular) generalized closed set is (regular) closed.

### 3. SLIGHTLY $v$ -OPEN MAPPINGS:

**Definition 3.1:** A function  $f: X \rightarrow Y$  is said to be

- (i) slightly  $v$ -open if the image of every clopen set in  $X$  is  $v$ -open in  $Y$ .
- (ii) almost slightly  $v$ -open if the image of every r-clopen set in  $X$  is  $v$ -open in  $Y$ .

**Theorem 3.1:** Every slightly  $v$ -open map is almost slightly  $v$ -open but not conversely.

**Proof:** Let  $A \subseteq X$  be r-clopen, then  $A \subseteq X$  is clopen  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f$  is  $v$ -open  $\Rightarrow f(A)$  is  $v$ -open in  $Y$ . Hence  $f$  is almost slightly  $v$ -open.

**Note 1:** We have the following implication diagrams for slightly open mappings.

$$\begin{array}{ccccccc}
 i) sl.r\alpha.\text{open.map} & \rightarrow & \rightarrow & \rightarrow & sl.v.\text{open.map} \\
 \uparrow & & & & \downarrow \\
 sl.r.\text{open.map} & \rightarrow & sl.\alpha.\text{open.map} & \rightarrow & sl.s.\text{open.map} & \rightarrow & sl.\beta.\text{open.map}
 \end{array}$$

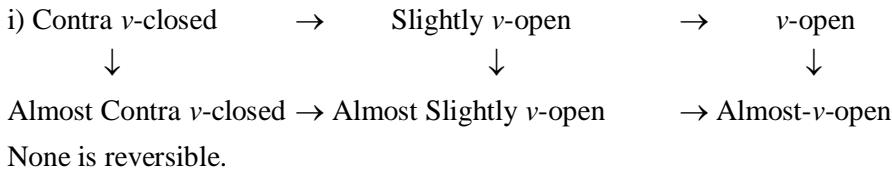
$$\begin{array}{ccccccc}
 ii) al.sl.r\alpha.\text{open.map} & \rightarrow & \rightarrow & \rightarrow & al.sl.v.\text{open.map} \\
 \uparrow & & & & \downarrow \\
 Al.sl.r.\text{open.map} & \rightarrow & al.sl.\text{open.map} & \rightarrow & al.sl.\alpha.\text{open.map} & \rightarrow & al.sl.s.\text{open.map} \rightarrow al.sl.\beta.\text{open.map}
 \end{array}$$

None is reversible.

**Example 1:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = c, f(b) = b$  and  $f(c) = a$ . Then  $f$  is slightly  $v$ -open and almost slightly  $v$ -open.

**Example 2:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ ;  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b, f(b) = c$  and  $f(c) = a$ . Then  $f$  is not slightly  $\nu$ -open and not almost slightly  $\nu$ -open.

**Note 2:** We have the following implication diagram.



**Example 3:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = c, f(b) = b$  and  $f(c) = a$ . Then  $f$  is slightly  $\nu$ -open, almost slightly  $\nu$ -open but not  $\nu$ -closed,  $\nu$ -open, contra- $\nu$ -open and contra- $\nu$ -closed.

**Example 4:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b, f(b) = c$  and  $f(c) = a$ . Then  $f$  is slightly  $\nu$ -open, almost slightly  $\nu$ -open but not almost  $\nu$ -closed, almost  $\nu$ -open, almost contra- $\nu$ -open and almost contra- $\nu$ -closed.

### Theorem 3.2:

- (i) If  $R\alpha O(Y) = \nu O(Y)$  then  $f$  is [almost]-slightly  $r\alpha$ -open iff  $f$  is [almost]-slightly  $\nu$ -open.
- (ii) If  $\nu O(Y) = RO(Y)$  then  $f$  is [almost]-slightly  $r$ -open iff  $f$  is [almost]-slightly  $\nu$ -open.
- (iii) If  $\nu O(Y) = \alpha O(Y)$  then  $f$  is [almost]-slightly  $\alpha$ -open iff  $f$  is [almost]-slightly  $\nu$ -open.

**Example 5:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = c, f(b) = b$  and  $f(c) = a$ . Then  $f$  is slightly pre-open, slightly  $\nu$ -open, slightly  $\beta$ -open, slightly open, slightly  $\alpha$ -open, slightly  $r\alpha$ -open, slightly semi-open and slightly  $r$ -open.

**Example 6:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = a, f(b) = c$  and  $f(c) = b$ . Then  $f$  is slightly open, slightly pre-open, slightly  $\beta$ -open, slightly  $\alpha$ -open, slightly  $r\alpha$ -open, slightly semi-open, slightly  $\nu$ -open and slightly  $r$ -open.

**Example 7:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$ . Assume  $f: X \rightarrow Y$  be the identity map. Then  $f$  is slightly open, slightly pre-open, slightly  $\beta$ -open, slightly  $\alpha$ -open, slightly semi-open, slightly  $r\alpha$ -open, slightly  $\nu$ -open and slightly  $r$ -open.

**Theorem 3.3:** If  $f$  is [almost]-slightly open and  $g$  is  $\nu$ -open then  $gof$  is [almost]-slightly  $\nu$ -open.

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen  $\Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $\nu$ -open in  $Z \Rightarrow g \circ f(A)$  is  $\nu$ -open in  $Z$ . Hence  $gof$  is slightly  $\nu$ -open.

**Theorem 3.4:** If  $f$  is [almost-]slightly open and  $g$  is  $r$ -open then  $gof$  is [almost-]slightly  $v$ -open.

**Proof:** Follows from Remark 1 and Theorem 3.3.

**Theorem 3.5:** If  $f$  and  $g$  are [almost-]slightly  $r$ -open then  $gof$  is [almost-]slightly  $v$ -open.

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen  $\Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $gof$  is [almost-]slightly  $v$ -open.

**Theorem 3.6:** If  $f$  is [almost-]slightly  $r$ -open and  $g$  is  $v$ -open then  $gof$  is [almost-]slightly  $v$ -open.

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen  $\Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $gof$  is [almost-]slightly  $v$ -open.

**Theorem 3.7:** If  $f$  is [almost-]slightly open and  $g$  is  $r\alpha$ -open then  $gof$  is [almost-]slightly  $v$ -open.

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen  $\Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $gof$  is [almost-]slightly  $v$ -open.

### Corollary 3.1:

- a) If  $f$  is [almost-]slightly open[[almost-]slightly  $r$ -open] and  $g$  is [almost-] $v$ -open then  $gof$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.
- b) If  $f$  is [almost-]slightly  $r$ -open and  $g$  is [almost-] $r\alpha$ -open then  $gof$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.

**Theorem 3.8:** If  $f: X \rightarrow Y$  is [almost-]slightly  $v$ -open, then  $f(A^\circ) \subset v(f(A))^\circ$

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen and  $f: X \rightarrow Y$  is slightly  $v$ -open gives  $f(A^\circ)$  is  $v$ -open in  $Y$  and  $f(A^\circ) \subset f(A)$  which in turn gives  $v(f(A^\circ))^\circ \subset v(f(A))^\circ$  - - - (1)

Since  $f(A^\circ)$  is  $v$ -open in  $Y$ ,  $v(f(A^\circ))^\circ = f(A^\circ)$  - - - - - - - - - (2)

combining (1) and (2) we have  $f(A^\circ) \subset v(f(A))^\circ$  for every subset  $A$  of  $X$ .

**Remark 2:** Converse is not true in general.

**Corollary 3.2:** If  $f: X \rightarrow Y$  is [almost-]slightly  $r$ -open, then  $f(A^\circ) \subset v(f(A))^\circ$

**Theorem 3.9:** If  $f: X \rightarrow Y$  is [almost-]slightly  $v$ -open and  $A \subseteq X$  is  $r$ -clopen,  $f(A)$  is  $\tau_v$ -open in  $Y$ .

**Proof:** Let  $A \subseteq X$  be  $r$ -clopen and  $f: X \rightarrow Y$  is slightly  $v$ -open  $\Rightarrow f(A^\circ) \subset v(f(A))^\circ \Rightarrow f(A) \subset v(f(A))^\circ$ , since  $f(A) = f(A^\circ)$ . But  $v(f(A))^\circ \subset f(A)$ . Combining we get  $f(A) = v(f(A))^\circ$ . Therefore  $f(A)$  is  $\tau_v$ -open in  $Y$ .

**Corollary 3.3:** If  $f: X \rightarrow Y$  is [almost-]slightly  $r$ -open, then  $f(A)$  is  $\tau_v$  open in  $Y$  if  $A$  is  $r$ -clopen set in  $X$ .

**Proof:** Follows from Remark 1, Theorem 3.8 and Theorem 3.9.

**Theorem 3.10:** If  $\nu(A)^\circ = r(A)^\circ$  for every  $A \subset Y$ , then the following are equivalent:

- a)  $f: X \rightarrow Y$  is [almost-]slightly  $\nu$ -open map
- b)  $f(A^\circ) \subset \nu(f(A))^\circ$

**Proof:** (a)  $\Rightarrow$ (b) follows from theorem 3.8.

(b)  $\Rightarrow$  (a) Let  $A$  be any  $r$ -clopen set in  $X$ , then  $f(A) = f(A^\circ) \subset \nu(f(A))^\circ$  by hypothesis. We have  $f(A) \subset \nu(f(A))^\circ$ . Combining we get  $f(A) = \nu(f(A))^\circ = r(f(A))^\circ$  [ by given condition] which implies  $f(A)$  is  $r$ -open and hence  $\nu$ -open. Thus  $f$  is  $\nu$ -open. Therefore  $f$  is [almost-]slightly  $\nu$ -open.

**Theorem 3.11:**  $f: X \rightarrow Y$  is [almost-]slightly  $\nu$ -open iff for each subset  $S$  of  $Y$  and each  $r$ -clopen set  $U$  containing  $f^{-1}(S)$ , there is an  $\nu$ -open set  $V$  of  $Y$  such that  $S \subseteq V$  and  $f^{-1}(V) \subseteq U$ .

**Proof:** Assume  $f: X \rightarrow Y$  is [almost-]slightly  $\nu$ -open. Let  $S \subseteq Y$  and  $U$  be a  $r$ -clopen set of  $X$  containing  $f^{-1}(S)$ . Then  $X - U$  is open in  $X$  and  $f(X - U)$  is  $\nu$ -open in  $Y$  as  $f$  is [almost-]slightly  $\nu$ -open and  $V = Y - f(X - U)$  is  $\nu$ -open in  $Y$ .  $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$  and  $f^{-1}(V) = f^{-1}(Y - f(X - U)) = f^{-1}(Y) - f^{-1}(f(X - U)) = f^{-1}(Y) - (X - U) = X - (X - U) = U$

Conversely Let  $F$  be  $r$ -clopen in  $X \Rightarrow F^c$  is clopen. Then  $f^{-1}(f(F^c)) \subseteq F^c$ . By hypothesis there exists an  $\nu$ -open set  $V$  of  $Y$ , such that  $f(F^c) \subseteq V$  and  $f^{-1}(V) \supseteq F^c$  and so  $F \subseteq [f^{-1}(V)]^c$ . Hence  $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$ . Thus  $f(F)$  is  $\nu$ -open in  $Y$ . Therefore  $f$  is [almost-]slightly  $\nu$ -open.

**Remark 3:** Composition of two [almost-]slightly  $\nu$ -open maps is not [almost-]slightly  $\nu$ -open in general.

**Theorem 3.12:** Let  $X, Y, Z$  be topological spaces and every  $\nu$ -open set is open [ $r$ -open] in  $Y$ . Then the composition of two [almost-]slightly  $\nu$ -open[[almost-]slightly  $r$ -open] maps is [almost-]slightly  $\nu$ -open.

**Proof:** (a) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be slightly  $\nu$ -open maps. Let  $A$  be any clopen set in  $X \Rightarrow f(A)$  is open in  $Y$  (by assumption)  $\Rightarrow g(f(A))$  is  $\nu$ -open in  $Z \Rightarrow g \circ f(A)$  is  $\nu$ -open in  $Z$ . Therefore  $g \circ f$  is slightly  $\nu$ -open.

**Example 8:** Let  $X = Y = Z = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a, c\}, Y\}$  and  $\eta = \{\emptyset, \{a\}, \{b, c\}, Z\}$ .  $f: X \rightarrow Y$  be defined  $f(a) = c, f(b) = b$  and  $f(c) = a$  and  $g: Y \rightarrow Z$  be defined  $g(a) = b, g(b) = a$  and  $g(c) = c$ , then  $g, f$  and  $g \circ f$  are slightly  $\nu$ -open.

**Theorem 3.13:** If  $f: X \rightarrow Y$  is [almost-]slightly  $g$ -open,  $g: Y \rightarrow Z$  is  $\nu$ -open [ $r$ -open] and  $Y$  is  $T_{1/2}$  [ $r-T_{1/2}$ ] then  $g \circ f$  is [almost-]slightly  $\nu$ -open.

**Proof:** (a) Let  $A$  be a clopen set in  $X$ . Then  $f(A)$  is  $g$ -open set in  $Y \Rightarrow f(A)$  is open in  $Y$  as  $Y$  is  $T_{1/2} \Rightarrow g(f(A))$  is  $\nu$ -open in  $Z$  since  $g$  is  $\nu$ -open  $\Rightarrow g \circ f(A)$  is  $\nu$ -open in  $Z$ . Hence  $g \circ f$  is slightly  $\nu$ -open.

**Corollary 3.4:** If  $f: X \rightarrow Y$  is [almost-]slightly  $g$ -open,  $g: Y \rightarrow Z$  is  $\nu$ -open [ $r$ -open] and  $Y$  is  $T_{1/2}$  [ $r-T_{1/2}$ ] then  $g \circ f$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.

**Theorem 3.14:** If  $f: X \rightarrow Y$  is [almost-]slightly  $rg$ -open,  $g: Y \rightarrow Z$  is  $\nu$ -open [ $r$ -open] and  $Y$  is  $r-T_{1/2}$ , then  $g \circ f$  is [almost-]slightly  $\nu$ -open.

**Corollary 3.5:** If  $f:X \rightarrow Y$  is [almost]-slightly rg-open,  $g:Y \rightarrow Z$  is [almost]- $v$ -open [[almost]- $r$ -open] and  $Y$  is  $r$ - $T_{1/2}$ , then  $g \circ f$  is [almost]-slightly semi-open and hence [almost]-slightly  $\beta$ -open.

**Theorem 3.15:** If  $f:X \rightarrow Y$ ,  $g:Y \rightarrow Z$  be two mappings such that  $g \circ f$  is [almost]-slightly  $v$ -open [[almost]-slightly  $r$ -open] then the following statements are true.

- a) If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is [almost]-slightly  $v$ -open.
- b) If  $f$  is  $g$ -continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is [almost]-slightly  $v$ -open.
- c) If  $f$  is rg-continuous, surjective and  $X$  is  $r$ - $T_{1/2}$  then  $g$  is [almost]-slightly  $v$ -open.

**Proof:** (a) Let  $A$  be a clopen set in  $Y \Rightarrow f^{-1}(A)$  is open in  $X \Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -open in  $Z \Rightarrow g(A)$  is  $v$ -open in  $Z$ . Hence  $g$  is slightly  $v$ -open.

Similarly one can prove the remaining parts and hence omitted.

**Corollary 3.6:** If  $f:X \rightarrow Y$ ,  $g:Y \rightarrow Z$  be two mappings such that  $g \circ f$  is  $v$ -open [ $r$ -open] then the following statements are true.

- a) If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is [almost]-slightly semi-open and hence [almost]-slightly  $\beta$ -open.
- b) If  $f$  is  $g$  continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is [almost]-slightly semi-open and hence [almost]-slightly  $\beta$ -open.
- c) If  $f$  is rg-continuous, surjective and  $X$  is  $r$ - $T_{1/2}$  then  $g$  is [almost]-slightly semi-open and hence [almost]-slightly  $\beta$ -open.

To prove the following theorem we use the following Definition.

**Theorem 3.16:** If  $X$  is  $v$ -regular,  $f:X \rightarrow Y$  is  $r$ -open,  $r$ -continuous, [almost]-slightly  $v$ -open surjective and  $A^o = A$  for every  $v$ -open set in  $Y$  then  $Y$  is  $v$ -regular.

**Proof:** Let  $p \in U \in vO(Y)$ ,  $\exists$  a point  $x \in X \ni f(x) = p$  by surjection. Since  $X$  is  $v$ -regular and  $f$  is nearly-continuous,  $\exists V \in RC(X) \ni x \in V^o \subset V \subset f^{-1}(U)$  which implies  $p \in f(V^o) \subset f(V) \subset U$  (1)  
for  $f$  is  $v$ -open,  $f(V^o) \subset U$  is  $v$ -open. By hypothesis  $f(V^o)^o = f(V^o)$  and  $f(V^o)^o = \{f(V)\}^o$  (2)  
combaining (1) and (2)  $p \in f(V)^o \subset f(V) \subset U$  and  $f(V)$  is  $r$ -closed. Hence  $Y$  is  $v$ -regular.

**Corollary 3.7:** If  $X$  is  $v$ -regular,  $f:X \rightarrow Y$  is  $r$ -open,  $r$ -continuous, [almost]-slightly  $v$ -open, surjective and  $A^o = A$  for every  $r$ -open set in  $Y$  then  $Y$  is  $v$ -regular.

**Theorem 3.17:** If  $f:X \rightarrow Y$  is [almost]-slightly  $v$ -open [[almost]-slightly  $r$ -open] and  $A$  is a open set of  $X$  then  $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$  is [almost]-slightly  $v$ -open.

**Proof:** Let  $F$  be a clopen set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(A)$  is  $v$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is slightly  $v$ -open.

**Corollary 3.8:** If  $f:X \rightarrow Y$  is [almost-]slightly  $\nu$ -open [[almost-]slightly  $r$ -open] and  $A$  is a open set of  $X$  then  $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.

**Theorem 3.18:** If  $f:X \rightarrow Y$  is [almost-]slightly  $\nu$ -open [[almost-]slightly  $r$ -open],  $X$  is  $T_{1/2}$  and  $A$  is g-open set of  $X$  then  $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$  is [almost-]slightly  $\nu$ -open.

**Proof:** Let  $F$  be a clopen set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(F)$  is  $\nu$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is slightly  $\nu$ -open.

**Corollary 3.9:** If  $f:X \rightarrow Y$  is [almost-]slightly  $\nu$ -open [[almost-]slightly  $r$ -open],  $X$  is  $T_{1/2}$ ,  $A$  is g-open set of  $X$  then  $f_A:(X,\tau(A)) \rightarrow (Y,\sigma)$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.

**Theorem 3.19:** If  $f_i:X_i \rightarrow Y_i$  be [almost-]slightly  $\nu$ -open [[almost-]slightly  $r$ -open] for  $i=1, 2$ . Let  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ . Then  $f:X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is [almost-]slightly  $\nu$ -open.

**Proof:** Let  $U_1 \times U_2 \subseteq X_1 \times X_2$  where  $U_i$  is clopen in  $X_i$  for  $i = 1, 2$ . Then  $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$  is  $\nu$ -open set in  $Y_1 \times Y_2$ . Hence  $f$  is slightly  $\nu$ -open.

**Corollary 3.10:** If  $f_i:X_i \rightarrow Y_i$  be [almost-]slightly  $\nu$ -open [[almost-]slightly  $r$ -open] for  $i=1, 2$ . Let  $f:X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ , then  $f:X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open.

**Theorem 3.20:** Let  $h:X \rightarrow X_1 \times X_2$  be [almost-]slightly  $\nu$ -open. Let  $f_i:X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i:X \rightarrow X_i$  is [almost-]slightly  $\nu$ -open for  $i=1, 2$ .

**Proof:** Let  $U_1$  be r-clopen in  $X_1$ , then  $U_1 \times X_2$  is r-clopen in  $X_1 \times X_2$ , and  $h(U_1 \times X_2)$  is  $\nu$ -open in  $X$ . But  $f_1(U_1) = h(U_1 \times X_2)$ , therefore  $f_1$  is slightly  $\nu$ -open. Similarly we can show that  $f_2$  is also slightly  $\nu$ -open and thus  $f_i:X \rightarrow X_i$  is slightly  $\nu$ -open for  $i = 1, 2$ .

**Corollary 3.11:** Let  $h:X \rightarrow X_1 \times X_2$  be [almost-]slightly  $\nu$ -open. Let  $f_i:X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i:X \rightarrow X_i$  is [almost-]slightly semi-open and hence [almost-]slightly  $\beta$ -open for  $i=1, 2$ .

## CONCLUSION:

In this paper we introduced the concept of slightly  $\nu$ -open and almost slightly  $\nu$ -open mappings, studied their basic properties and the interrelationship between other slightly open maps.

## REFERENCES :

- [1.] D. Andrijevic, Semi pre-open sets, Mat. Vesnik 38 (1986) 24-32.
- [2.] Asit Kumar Sen and Bhattacharya.P., On preclosed mappings, Bull.Cal.Math.Soc.,85, 409-412(1993).
- [3.] Baker.C.W., On weak forms of contra-open and contra-closed mappings, Internationsl Journal of Pure and Applied Mathematics, Vol.73,No.3(2011)281-287.

- [4.] Balasubramanian.S., and Vyjayanthi.P.A.S.,  $\nu$ -open Mappings – Scientia Magna Vol 6. No. 4(2010) 118 – 124.
- [5.] Balasubramanian.S., Sandhya.C., and Vyjayanthi.P.A.S., Almost  $\nu$ -open Mappings – Inter. J. Math. Archive, Vol 2, No.10 (2011) 1943 – 1948.
- [6.] Balasubramanian S., Aruna S. Vyjayanthi P. and Sandhya C., “Regular Pre-open Mappings Aryabhatta Journal of Maths. & Info. Vol. 5 [1] 2013 pp 31-38.
- [7.] Balasubramanian.S., Sandhya.C., and Aruna Swathi Vyjayanthi.P., Contra  $\nu$ -open map – International Journal of Engineering Sciences and Research Technology, Vol.2,Issue 3,(2013)561 – 566
- [8.] Di Maio. G., and Noiri.T, I. J. P. A. M., 18(3) (1987) 226-233.
- [9.] S. Balasubramanian “Somewhat M $\nu$ g open Functions” Aryabhatta J. of Maths & Informatics vol. 4 (2) 2012 pp 315-320.
- [10.] Dunham.W.,  $T_{1/2}$  Spaces, Kyungpook Math. J.17(1977), 161-169.
- [11.] Long.P.E., and Herington.L.L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.
- [12.] Mashhour.A.S., Hasanein.I.A., and El.Deep.S.N.,  $\alpha$ -continuous and  $\alpha$ -open mappings, Acta Math. Hungar.,41(1983), 213-218.
- [13.] Norman Levine, Semi open sets and semi continuity in topological spaces, Amer. math. monthly 70 (1963). 36-41.
- [14.] Norman Levine, Generalized closed sets in topological spaces, Rend. Civc. Mat. Palermo 19 (1970). 89-96.
- [15.] Palaniappan.N., Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).
- [16.] M.E. Abd. El. Monsef. S.N. El. Deep and R.A. Mohmoud,  $\beta$ -open sets and  $\beta$ -continuous mappings, Bull. Fac. Sci. Assiut Univ. A12 (1983) No.1, 77-90.
- [17.] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A. Math 12 (1991); 5-13.
- [18.] A.S. Mashhour M.E. Abd. El- Monsef and S.N. El. Deep, On pre continuous and Weak pre continuous mappings, Proc. Math. Phy. Soc. Egypt 53 (1982) 47-53.
- [19.] O. Njastad, On some classes of nearly open sets, Pacific J. math. 109(1984) (2) 118-126.