SCOPE AND LIMITATION OF THEORY OF PROBABILITY

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ABSTRACT

In any random experiment there is always uncertainty as to whether a particular event will or will not occur. In the objective sense the probability is that which is supported by a number of objective arguments. Probabilities are numbers between O and 1 inclusive, that reflect the chances of a physical event occurring. There are different methods by means of which we can estimate the probability of an event.

In this paper different aspects of theory of probability and its axiomatic structure have been studied in detail and attempt has been made to review the scope and limitations of conditional probability. In addition to objective interpretation of probability there is a quite different approach known as subjective or personal probability. The main advantage of subjective probability is that it is always applicable in all types of random experiments.

Keywords:- axioms of probability, conditional probability, subjective or personal probability.

1. **Introduction:-** Probability is a branch of Mathematics concerned with the estimation of uncertain events. In the objective sense the probable is that which is supported by a number of objective arguments. The classical theory of probability defines the numerical value of probability as the ratio of the number of favourable cases to the total number of equally likely cases. The main objection to this definition has been raised due to the phrase "equally likely cases". This definition would mean the reduction of all distribution to uniform distribution which is not feasible. The relative frequency approach was proposed by R. Von mises. Probability for von mises is the "limit of relative frequency in a collective'. The idea of probability is therefore applicable only to sequence of events, objections have been raised on the ground that it is inadmissible to apply mathematical concept of limit to a sequence which by definition must not be subject to any

mathematical rule due to randomness. We can not really arrive at the limiting value of relative frequency, how large the sequence might be. The true probability of 'head' in a coin tossing might be ½, however in a particular sequence of 1000 throws may yield relative frequency of "head" to be 0.4. yet it is possible that another sequence of 1000 throws might yield relative frequency of 'head' to be only 0.2. Do we need a million or a billion throws before we can be certain that we can use relative frequency to evaluate probability? It is not possible to find out the exact number of throws so to make certain about the value assigned to probability.

Both the classical and frequency approaches have serious draw backs, the first because the word ' equally likely' is vague and the second because the very large number involved is absurd. Because of these difficulties, axiomatic approach to probability is developed by A. Kolmogrov which relates probability theory to the theory of sets and measure of real variables.

2. **Axioms of Probability:-** Let S denote a sample space for an experiment. If S is discrete, all subsets correspond to events, but if S is non discrete only special subsets called measurable corresponds to events. To each event A in the class C of events we associate a real number P(A), which is called Probability of event A, if the following axioms are satisfied.

Axioms:

1. For every event A is the class C,
   $P(A) \geq O$
2. For Certain event S in the class C,
   $P(S) = 1$
3. For any number of mutually exclusive events
   A_1, A_2-------------------- in the class C,
   $P(A_1 U A_2$----------------$) = P(A_1) + P(A_2) +$-----------------

with the help of these axioms we can deduce some of the important results.

- for every event A, $O \le P(A) \le 1$

  thus probability is between 0 and 1

- The probability of impossible event is Zero

  $P(\emptyset)=0$

- The probability of Complement of A is given by

  $P(A') = 1 - P(A)$

- If $A= A_1 \cup A_2 \cup \text{------------} \cup A_n$, where $A_1, A_2, \text{---------} A_n$

are mutually exclusive events, then

$P(A) = P(A)+ P(A_2) + \text{--------------} P(A_n)$

If A= S(the whole sample space)

then $P(A_1)+ P(A_2) + \text{------------} P(A_n) =1$

- If A and B are any two events, then

  $P(A \cup B)= P(A)+ P(B) - P(A \cap B)$

  Generalization to n events can also be made.

3. **Conditional Probability:-** We wish to consider a situation where knowledge of occurrence of an event influences the occurrence of another event. If A and B are two events such that P(B)>0, the conditional probability of event A given B is denoted by P(A/B). The event B is sometimes called 'conditioning event' we define it as

   $$P(A/B)= \frac{P(A\cap B)}{P(B)} \quad ; P(B)>0$$

   or, $P(A\cap B)= P(A/B). P(B)$.

This is called 'the product rule' By induction we obtain the general formula as

$P(A_1 \; A_2 \text{---------------------} \; A_n) = P(A_1)\, P(A_2/A_1)\, P(A_3/A_1A_2)$

$\text{-----------}\; P(A_n/A_1.A_2\text{------}\; A_n)$

It P(A/B) = P(A) then we say that A and B are independent events. This is equivalent to $P(A\cap B) = P(A)\, P(B)$.

3.1 **Bayes` Theorem:** This theorem was formulated by Thomas Bayes in 1761. It deals with the conditional probability. Let A_1, A2, A3 ------An be collection of mutually exclusive events whose union is S. If B is any event then for any of the event A_i, i=1, 2, 3------------n

$$P(A_i/B) = \frac{P(B/A_i)\, P(A_i)}{\sum_{i=1}^{n} P(B/A_i)\, P(A_i)}$$

4. **Scope of the conditional Probability:-** We know that nothing is absolute in our nature. All phenomenons are basically a relative phenomenon. Our expectation for future event is based on our past experiences. The probability is also a relative phenomenon of uncertain events. Thus we consider all probability to be conditional probabilities. In this respect, the so called absolute probability is a kind of conditional probability relative to the whole sample space. Conditional probability satisfies all the axioms of probability and for fixed B, P(*/B) satisfies the following axioms.

1. $P(A/B)>0$
2. $P(B/B)=1$
3. $P(A\cap C/B) = P(A/B).\, P(C/A\cap B)$. Provided $A\cap B$ is possible.
4. $P(AUC/B) = P(A/B) + P(C/B)$

It we put B = S(the whole sample space) in axiom (3) we get the definition of conditional probability.

4.1 **Limitations:-**

(a) The conditional probability of two events in a random experiment can be obtained only with respect to a fixed given event. Thus if A, B, C, D are events of a random experiment, then conditional probability can be obtained only for some fixed event B, i.e. P(*/B) where * stands for events, A, C, D.

(b) Two conditional probabilities defined on different conditioning events can not be compared with each other.

4.2 **Case study:-**

(i) Let P(A/B) and P(A/C) are two conditional probabilities defined on the conditioning events B and C respectively then P(A/B) and P(A/C) cannot be compared. For example let us consider throw with a die let A= Four will appear, B= (1, 2, 4, 6) and C= (2, 4, 5). We find the conditional probability as follows: P(A/B)= ¼ and P(A/C)= 1/3, but we do not say that P(A/C)> P(A/B). For they are measured with respect to different conditioning events, hence they must have different probabilities irrelevant to be compared.

(ii) The set interpretation of the probability doest not define conditional probability logically. Let us consider again tosses with a die. Let A, B, C, are events such that A= (1, 2, 3, 4, 5, 6), B=(3) and C= (1, 2, 3) We obtain the following conditional probabilities. P(A/B)=1 and P(C/B)=1 But it is far from logical point of view that occurrence of B makes nearly always, the considerably larger set A. For one can never predict about the family if only one member is present.

(iii) In coin tossing experiment, we compute the conditional probability P(H, T/H)=1, where H and T stands for 'head' and 'tail' respectively. But we observe that it is not true in logical sense. For the occurrence of 'head' in no way allows to predict that ' head and tail' will occur nearly always. If the coin is biased, the result will be even worse.

Hence we find that probability should not be dealt with sets. Objections have been raised against the kolmolgrov`s axiomatic system as it leans heavily on set theory and hence shares the difficulty of the set theory.

5. **Subjective or Personal Probability:**

In addition to the interpretation of probability as objective, there is a quite different interpretation according to which the probability of a statement represent a certain numerical measure of a person`s degree of belief in the statement. A method of calculating a person's degree of belief in the occurrence of an event is obtained by knowing how much money he is willing to risk as a bet in return of Rs. 100 say in case the event occurs. If the person is certain about its occurrence he can deposit upto Rs. 100 and even in that case he is not losing. Such an event is a sure event for the person and the measure of his degree of belief is the ratio of total amount deposited to total amount offered and for sure event his degree of belief is 1. on the other hand if the person is not willing to deposit even one rupee than it shows that his total disbelief in the occurrence of event and measure of his degree of belief is 0. The only requirement for the subject is that his behavior in betting matters should be coherent. By coherence, we mean that the subject is free to risk any amount not exceeding Rs. 100 on the basis of his degree of belief. On the basis of this subjective approach, we can say that

1. The measure of degree of belief always lies between O and 1 inclusive which means that it is non-negative.
2. It is also additive. Let E be given situation and let there be two mutually exclusive alternatives E_1 and E_2 of E. So that $E= E_1+E_2$. Then naturally if the subject is willing to bet X rupees for the occurrence of E_1 in return of Rs. 100 he would be willing to bet 100-X for E_2, so that $1=$ degree of belief in $E= \frac{X}{100} + \frac{100-X}{100} = P_1+P_2$
   where $P_1=$ degree of belief in E_1 and $P_2=$ degree of belief in E_2.

This law can be easily extended to finite number of alternative events.

Thus subjective probability i.e., degree of belief of a person obeys the same law (addition law, multiplication law, Bayes law etc.) as the usual probability in the finite case.

6. **Conclusion:-** There are many interpretations of theory of probability by means of which we can estimate the probability of random events. We also find that there are some limitations of theory of probability. In a classical approach the word 'equally likely' is vague similarly in a relative frequency approach the word 'very large number' is absurd. The inadequacy of axiomatic approach, as discussed in the case study, is due to objective concept of probability which is based on the theory of sets. The paradoxes can be resolved with the help of subjective probability which is a numerical measure of person`s degree of belief in the statement . Subjective probability obeys the same laws as the usual probability in the finite case, however it does not depend upon the set theory.

References:-

1. Kolmogorov, A.N., 'Foundation of Theory of Probability'
   Mac-Millan & Co. London (1956)
2. Von-Mises, R., 'Probability, Statistics and Truth',
   Mac-Millan & Co. (1957)
3. De Finetti, B., 'Theory of Probability', John wiley,
   Newyork (1974)
4. Murray R. Spiegal, J. schiller., 'Probability and Statistics'
   Tata Mc-graw Hill, New Delhi (2005)
5. Milton, J.S. & Arnold, J.C. 'Introduction to probability and statistics
   Tata Mc-graw Hill (2007)