Supplier Selection using combined MCDM Approach -A Case Study for Mobile selection

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Abstract

Purchasing management is most essential in today's running world, because the profit potential of effective management of the purchasing and supply activities is enormous compared with other practical management alternatives. One of the most critical way is the selection of the right supplier. A Multi Criteria Decision Making (MCDM) techniques namely Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Fuzzy TOPSIS, Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) and Linear Programming (LP) are effectively applied in the supplier selection process.

Entropy is the measure of the disorder degree of the system, and it can also measure the effective information provided by the data. Therefore, the entropy can be used to determine the weights. ENTROPY method is used to calculate the weight and give suppliers a ranking; LP effectively allocates order quantity to each vendor. The alternatives are ranked and compared in order to arrive at an efficient result. This approach is demonstrated with a real world case study involving four main evaluation criteria and the firm has to determine the most appropriate and beneficial suppliers, which results in the great savings in both costs and man hours.

Keywords: Fuzzy TOPSIS, LP, MCDM, TOPSIS, Supplier's Selection, TORA Software, VIKOR.

1. INTRODUCTION

Fuzzy set theory provides a major paradigm in modeling and reasoning with uncertainty. In constructing a model, we always attempt to maximize its usefulness. This aim is closely constructed with relationship among these key characteristics of every model complexity credibility and uncertainty. Here uncertainty plays an important role which tends to reduce the complexity and increase credibility of the resulting model. In 1965 the fuzzy set theory was first subjected to technical scrutiny by Lotfi. A. Zadeh, in his seminar work “Fuzzy sets”. It is an extension of crisp sets, by enlarging the triple value set of ‘Grade membership’ from the two value set {0,1} to the unit interval [0,1] of real numbers.

Fuzzy sets are characterized by mapping called ‘Membership functions’ in to [0, 1] which are extension of characteristic functions of crisp sets. The capacity of fuzzy sets to express gradual transition from membership to non-membership and vice versa and has a broad utility. It not only provides representation of measurement uncertainties but also with a poignant representation of vague concepts expressed in natural language. Fuzzy set theory is a tool that gives reasonable analysis of complex, systems without making the process of analysis too complex. Also there might be situations in which a decision maker needs to consider multiple criteria in arriving at the overall best decision. Hence Fuzzy set theory solves all the inventory models in an uncertain environment.

1.2. A Survey of Preliminary Fuzzy Set Theory

The following definitions and preliminaries are required in the sequel of this thesis and hence presented in brief.

1.2.1. Fuzzy Set

If X is a collection of objects denoted generically by x, then a **fuzzy set** A in X is defined as a set of ordered pairs $A = \{(x,\mu_A(x)) \mid x \in X\}$, where, $\mu_A(x)$ is called the **membership function** (or MF for short) for the fuzzy set A. The MF maps each element of X to a membership grade (or membership value) between 0 and 1 (included) Let X = {0, 1, 2, 3, 4, 5, 6} be the set of numbers of children a family may choose to have. Then the fuzzy set B = "desirable number of children in a family" may be described as follows: B = {(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.7), (5, 0.3), (6, 0.1)}. Here we have a discrete ordered universe X. Again, the membership grades of this fuzzy set are obviously subjective.

1.2.2. α-cut

The **α -cut** or **α -level set** of a fuzzy set A is a crisp set defined by

$A_\alpha = \{x \mid \mu_A(x) \geq \alpha$

Strong α -cut or **strong α -level set** is defined by

$A_\alpha = \{x \mid \mu_A(x) > \alpha\}$.

1.3. Fuzzy Numbers

The notion of fuzzy numbers was introduced by Dubois .D and Prade. H [20]. A fuzzy subset of the real line R with membership function $\tilde{A} : R\ [0,1]$ is called a fuzzy number if

i. $\tilde{A}$ is normal,(i.e) there exists an element X_0 such that $(x_0) = 1$
ii. $\tilde{A}$ is fuzzy convex, (i.e)

$$\mu_{\tilde{A}}\ [\lambda x_1+(1-)\lambda x_2] \geq \mu\tilde{A}\,(x_1) \wedge \mu\tilde{A}(x_2),\ x_1, x_2\ R, \forall\ \lambda \in [0, 1]$$

iii. $\mu_{\tilde{A}}$, is upper continuous , and
iv. supp $\tilde{A}$ is bounded , where supp $\tilde{A} = \{x \in R: \mu_{\tilde{A}}(x) > 0\}$

1.3.1. Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions is a generalized fuzzy number $\tilde{A}$.

(i) $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_{\tilde{A}} = 0, -\infty < x \le a_1$,

(iii) $\mu_{\tilde{A}} = L(x)$ is strictly increasing on $[a_1,a_2]$

(iv) $\mu_{\tilde{A}} = 1, a_2 \le x \le a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

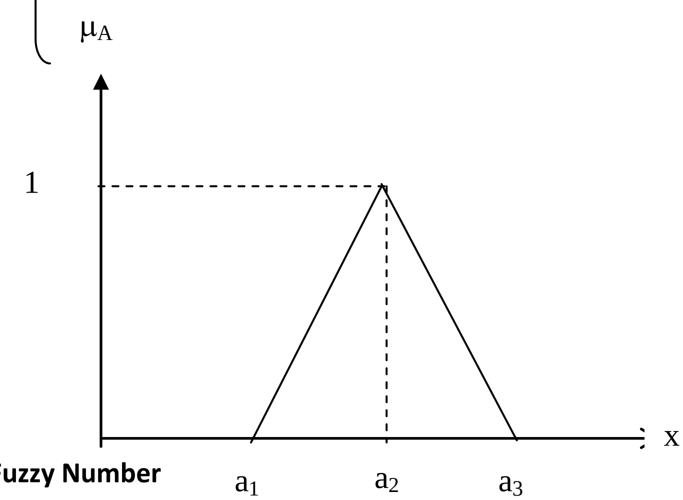
(vi) $\mu_A = 0, a_4 \le x < \infty$

where a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_{4;}\ w_A)_{LR}$; When w_A=1, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

1.3.2. Triangular Fuzzy Number

The fuzzy set $\breve{A} = (a_1, a_2, a_3)$ where $a_1 \le a_2 \le a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

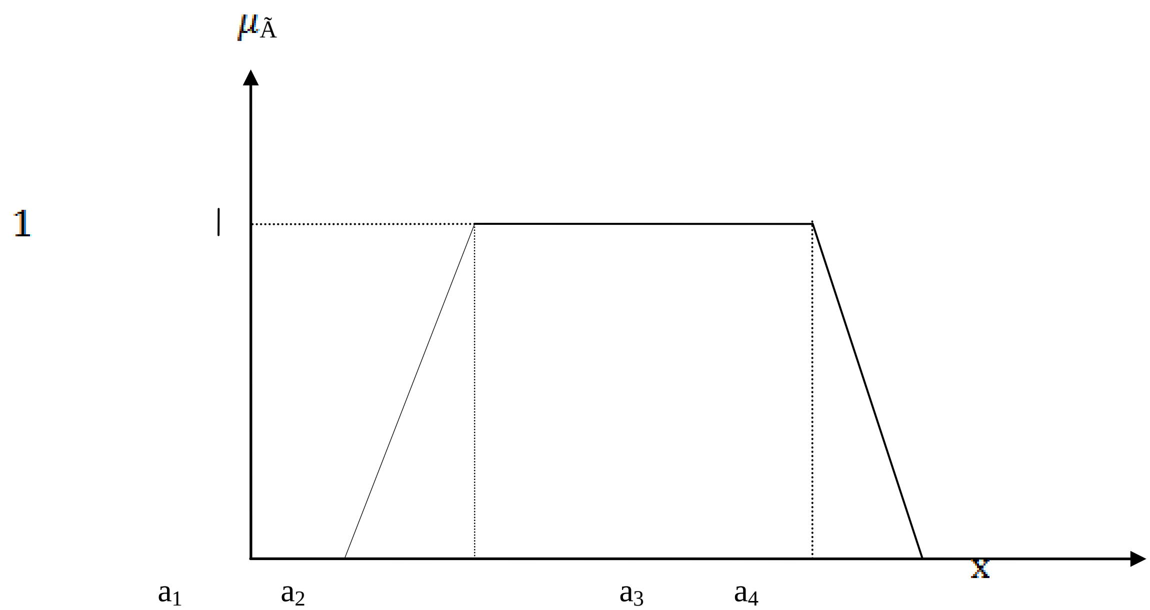
$$\mu_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \quad \text{other wise} \end{cases}$$



1.3.3. Trapezoidal Fuzzy Number

The fuzzy set $\tilde{A} = (a_1,a_2,a_3,a_4)$ where $a_1 \le a_2 \le a_3 \le a_4$ and defined on R, is called the trapezoidal number if membership function of is given by

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & \text{Otherwise} \end{cases}$$



1.4. Operations on Fuzzy Numbers

Though different methods are available for the operation of fuzzy numbers , the function principle is used for the operation of fuzzy numbers in the present thesis.

1.4.1. The Function Principle

The function principle was introduced by Chen [14] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers. Suppose $\tilde{A}$ = (a_1, a_2, a_3,) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} + \tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3) where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B}$ = (c_1, c_2, c_3)
where T = { a_1b_1, a_1b_3, a_3b_1, a_3b_3}
c_1 = min T, $c_2 = a_2b_2$, c_3 = Max T

If a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then

$\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} - \tilde{B} = (a_1-b_3, a_1-b_2, a_3-b_1)$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers

(iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 are all non zero positive real number, then the division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$

(v) For any real number K, $K\tilde{A} = (Ka_1, Ka_2, Ka_3)$ if $K > 0$

$K\tilde{A} = (Ka_3, Ka_2, Ka_1)$ if $K < 0$

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are trapezoidal fuzzy numbers. Then

(i) Addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$ where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4, are real numbers.

(ii) The product of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$ if a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers.

(iii) The subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A} - \tilde{B} = (a_1 – b_4, a_2 – b_3, a_3 – b_2, a_4 – b_1)$ where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

(iv) The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$ if a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers.

(v) For any real number K, $K\tilde{A} = (Ka_1, Ka_2, Ka_3, Ka_4)$ if $K > 0$

$K\tilde{A} = (Ka_4, Ka_3, Ka_2, Ka_1)$ if $K < 0$

1.4.2. DECISION MAKING:-

Decision-making process involves a series of identifying the problems, constructing the preferences, evaluating the alternatives, and determining the best alternative [1-3].Decision making is extremely intuitive while considering the single criterion problems, since we only need to choose the alternative with the highest preference rating. However, when decision makers evaluate the alternatives with the multiple criteria, many problems, such as weights of criteria, preference dependence, and conflicts among criteria, seem to complicate the decision problems and should be overcome by more sophisticate methods.

1.4.3. MCDM

Multi-criteria decision making (MCDM) is one of the well-known topics of decision making. Fuzzy logic provides a useful way to approach a MCDM problem. Very often in MCDM problems, data are imprecise and fuzzy. In a real-world decision situation, the application of the classic MCDM method may face serious practical constraints, because of the criteria containing imprecision or vagueness inherent in the information.

1.4.4. TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)
The principle behind TOPSIS is simple: The chosen alternative should be as close to the ideal solution as possible and as far from the negative-ideal solution as possible. The ideal solution is formed as a composite of the best performance values exhibited (in the decision matrix) by any alternative for each attribute.

1.4.5. FUZZY TOPSIS

The technique called fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Situation) can be used to evaluate multiple alternatives against the selected criteria.

1.4.6.VIKOR method

VIKOR is one of the multiple criteria decision making (MCDM) models to determine the preference ranking from a set of alternatives in the presence of conflicting criteria. The justification of VIKOR is to use the concept of the compromise programming to determine the preference ranking by the results of the individual and group regrets.

1.5. Defuzzification

Defuzzification is a process of transforming fuzzy values to crisp values. Defuzzification Methods have been widely studied for some years and were applied to fuzzy systems. The major idea behind these methods was to obtain a typical value from a given set according to some specified characters. Defuzzification method provides a correspondence from the set of all fuzzy sets into the set of all real numbers.

MCDM TECHNIQUES

A. TOPSIS

The **Technique for Order of Preference by Similarity to Ideal Solution** (**TOPSIS**) is a multi-criteria decision analysis method, which was originally developed by Hwang and Yoon in 1981 ,with further developments by Yoon in 1987, and Hwang, Lai and Liu in 1993.TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion, normalising scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. Normalisation is usually required as the parameters or criteria are often of incongruous dimensions in multi-criteria problems.

The procedure of TOPSIS can be described as follows.
Given a set of alternatives, A = {Ai | i = 1, . . . , n} and a set of criteria C = {cj | j = 1, . . . , m} where X = {xij | i = 1, . . . , n, j = 1, . . . , m} denotes the set of ratings and W = {Wj | j = 1, . . . , m} is the set of weights. Then the information able I = (A, C, X, W) can be represented as:

I	C_1	C_2	....	C_m
A_1	x_{11}	x_{12}	....	x_{1m}
A_2	x_{21}	x_{22}	....	x_{2m}
.	.	.		.
.	.	.		.
.	.	.		.
A_n	x_{n1}	x_{n2}	....	x_{nm}
W	w_1	w_2	....	w_m

Step 1: Calculate normalized ratings by

$$r_{ij}(x) \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^{2}}}; \quad \begin{array}{l} \mathrm{i}=1,....\mathrm{n} \\ j=1,....,m \end{array}$$

Step 2: Calculate weighted normalized ratings by

$$V_{ij}(x) = w_j r_{ij}(x) \quad \mathrm{i}=1,....,\mathrm{n}; \quad \mathrm{j}=1,....,\mathrm{m};$$

Step 3: Calculate PIS (positive ideal solution) and negative ideal solution (NIS) by

$$\mathrm{PIS} = \mathrm{A}^{+} = \left\{ \max_i V_{ij}(x) \mid j \in J_1, (\min_i V_{ij} \mid j \in J_2 \right\}$$
$$= \left\{ v_1^{+}, v_2^{+}, ...v_n^{+} \right\}$$

$$\mathrm{NIS} = \mathrm{A}^{-} = \left\{ \min_i V_{ij}(x) \mid j \in J_1, (\max_i V_{ij} \mid j \in J_2 \right\}$$
$$= \left\{ v_1^{-}, v_2^{-}, ...v_n^{-} \right\}$$

Step 4: Calculate separation from PIS and NIS between the alternatives. The separation values can be measured using the Euclidean distance which is given by

$$S_i^+ = \sqrt{\sum_{j=1}^{m} \left[V_{ij}(x) - V_j^+(x)\right]^2} \quad i = 1,\ldots\ldots, n$$

$$S_i^- = \sqrt{\sum_{j=1}^{m} \left[V_{ij}(x) - V_j^-(x)\right]^2} \quad i = 1,\ldots\ldots, n$$

Step 5: Similarities to the PIS can be derived as

$$c_i^* = \frac{S_i^-}{S_i^+ + S_i^-} \quad i = 1,\ldots\ldots, n \quad c_i^* \in [0,1] \forall \quad i = 1,\ldots\ldots, n$$

Finally, the preferred order can be obtained according to the similarities to the PIS (C_i^*) in descending order to choose the best alternatives.

NUMERICAL EXAMPLE

Consider a problem of selecting suppliers for a best mobile for a particular specifications of the mobile phones under 4 different criteria namely RAM, Inbuilt memory, Camera Pixels, Memory card,(supported up to) . The problem is to find out the best mobile to use:

HERE LET US KEEP THE COST OF EACH PRODUCT TO BE BETWEEN Rs.13500 – Rs.14000

1. Topsis Method-TABLE 1. INFORMATION TABLE

Alternatives	RAM	Camera Pixels	Memory card	Inbuilt memory
Sony Xperia C	0.05	0.610	0.5	0.25
Panasonic	1	0.99	0.25	1
Samsung galaxy grand	0.05	0.610	1	0.5
Alcatel One Touch	1	1	0	1

TABLE 2. NORMALIZED RATINGS

Alternatives	***r1***	***r2***	***r3***	***r4***
S1	0.0353	0.5	0.4082	0.1644
S2	0.7062	0.5	0.4082	0.6576
S3	0.0353	0.5	0.8165	0.3288
S4	0.7062	0.5	0	0.6576
WEIGHT	0.3088	0.0266	0	0.6576

TABLE 3. WEIGHTED NORMALIZED RATINGS

	V1	V2	V3	V4
S1	0.0109	0.0133	0.1298	0.0567
S2	0.2180	0.0133	0.1298	0.2270
S3	0.0109	0.0133	0.33329	0.1135
S4	0.2180	0.0133	0	0.004

MAX VJ^{+}	0.2180	0.0133	0.3332	0.2270
MIN VJ^{-}	0.0109	0.0133	0	0.1135

TABLE 4. THE PIS AND THE NIS

Alternatives	S^+	S^-	*C**	***Rank***
S1	0.3365	0.14168	0.2961	4
S2	0.2034	0.2694	0.5697	2
S3	0.2361	0.3332	0.5852	1
S4	0.3332	0.2361	0.4147	3

The preferred order of alternatives are S3>S2>S4>S1.On the basis of preferred order, Alternative – III (ie) Supplier – III should be the best choice.

2. VIKOR

The VIKOR method is a multicriteria decision making (MCDM) or Multi-criteria decision analysis method. It was originally developed by Serafim Opricovic to solve decision problems with conflicting and non commensurable (different units) criteria, assuming that compromise is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria. VIKOR ranks alternatives and determines the solution named compromise that is the closest to the ideal. The VIKOR method of compromise ranking determines a compromise solution, providing a maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent". The TOPSIS method determines a solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances. A comparative analysis of these two methods is illustrated with a numerical example, showing their similarity and some differences.

VIKOR algorithm was posed by Opricovic (1998) which is a multi-attribute decision making method for complex system based on ideal point method. The basic view of VIKOR is determining positive – ideal solution and negative – ideal solution. The positive ideal solution is the best value of alternatives under assessment criteria and the negative ideal solution is the worst value of alternatives under assessment criteria. The procedure for evaluating the best solution to an MCDM problem include computation the utilities of alternatives and ranking these alternatives. The alternative solution with the highest utility is considered to be the optical solution.

Step 1: Representation of normalized decision matrix.

The normalized decision matrix can be expressed as follows.

$$F = [f_{ij}]_{mxn} \quad where \;\; f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}; \quad \mathrm{i = 1,2,...., m}$$

And x_{ij} -performance of alternative A_i with respect to the j^{th} criterion.

Step 2 : Determination of positive ideal and negative ideal solution.

The positive ideal solution A^+ and negative ideal solution A^- are determined as

$$\mathrm{A}^{+} = \{(\max f_{ij})\, j \in J \quad \text{or} \quad (\min f_{ij} \mid j \in J) \;\; i = 1,...., m\}$$

$$= \{f_1^{+}, f_2^{+}, ...\, f_n^{+}\}$$

$$\mathrm{A}^{-} = \{(\min f_{ij})\, j \in J \quad \text{or} \quad (\max f_{ij} \mid j \in J) \;\; i = 1,...., m\}$$

$$= \{f_1^{-}, f_2^{-}, ...\, f_n^{-}\}$$

Where J is the attributes

Step 3": Calculation of utility measure and regret measure by

$$S_i = \sum_{j=1}^{n} W_j \frac{(f_j^{+} - f_{ij})}{(f_j^{+} - f_j^{-})} \qquad\qquad R_i = \max_j \left[W_j \frac{(f_j^{+} - f_{ij})}{(f_j^{+} - f_j^{-})} \right]$$

S_i - Utility measure $\qquad$ R_i - Regret measure

Step 4: Computation of VIKOR index

The VIKOR index can be expressed as

$$Q_i = V\left[\frac{(S_i - S^{+})}{S^{-} - S^{+})}\right] + (1-V)\left[\frac{(R_j - R^{+})}{R^{-} - R^{+})}\right] -$$

Where $S^{+} = \min_{i}(S_i)$; $S^{-} = \max_{i}(S_i)$; $R^{+} = \min_{i}(R_i)$; $R^{-} = \max_{i}(R_i)$

V- Weight of maximum group utility (usually it is to be set fo 0.5) **The alternative having smallest VIKOR value is determined to be the best solution.**

Entropy method is used to determine the weight of each indicator.

Step 1 : Calculate $P_{ij} = \frac{r_{ij}}{\sum_{j=1}^{m} r_{ij}}$; r_{ij} - i^{th} scheme, j^{th} indicator value

Step 2: Calculate the , j^{th} indicator entropy value e_j

$$e_j = -K\sum_{i=1}^{m} P_{ij} In P_{ij} ; K = \frac{1}{\ln m}$$ *m* is the number of assessments.

Step 3 : Calculate w_j

$$w_j = \frac{(1\text{-}e_j)}{\sum_{j=1}^{m}(1\text{-}e_j)}$$ n is the no. of indicators and $0 \le w_j \le 1$;

$$\sum_{i=1}^{m} W_j = 1$$

$$V_{ij}(x) = w_j r_{ij}(x) \quad \mathrm{i} = 1,...., \mathrm{n}; \quad \mathrm{j} = 1,....,$$

NUMERICAL EXAMPLE- TABLE 5. INFORMATION TABLE

Alternatives	RAM	Camera Pixels	Memory card	Inbuilt memory
S1	0.05	0.610	0.5	0.25
S2	1	0.99	0.25	1

S3	0.05	0.610	1	0.5
S4	1	1	0	1
WEIGHT	0.48	0.48	0.03	0.01

TABLE 6. NORMALIZED RATINGS

Alternatives	***r1***	***r2***	***r3***	***r4***
S1	0.0353	0.5	0.4082	0.1644
S2	0.7062	0.5	0.4082	0.6576
S3	0.0353	0.5	0.8165	0.3288
S4	0.7062	0.5	0	0.6576

TABLE 7. UTILITY MEASURES (Si)

Alternatives	***Sj***	***Rj***
S1	0.8130 = S+	0.3452= R+
S2	0.1590 = S-	0.1590=R-
S3	0.5389	0.3088
S4	0.3181	0.2597

	VIKOR Index	*Ranking*
S1	$Q_1 = 0$	1
S2	$Q_2 = 1$	4
S3	$Q_3 = 0.3073$	2
S4	$Q_4 = 0.6081$	3

AS, VIKOR INDEX of SUPPLIER I is the least,

SUPPLIER I is to be selected FIRST followed by S3, S4 and S2

3.Fuzzy TOPSIS

Since the preferred ratings usually refer to the subjective uncertainty, it is natural to extend TOPSIS to consider the situation of fuzzy numbers. Fuzzy TOPSIS can be intuitively extended by using the fuzzy arithmetic operations as follows:

Given a set of alternatives, $\mathrm{A} = \{\mathrm{A}_i \mid \mathrm{i} = 1,\ldots., \mathrm{n}\}$ and a set of criteria $\mathrm{C} = \{\mathrm{C}_j \mid \mathrm{j} = 1,\ldots., \mathrm{m}\}$

Where $\tilde{X} = \{\tilde{x}_{ij} \mid i = 1,\ldots., n;\ j = 1,\ldots, m\}$ denotes the set of fuzzy ratings and $\tilde{W} = \{\tilde{w}_{ij} \mid j = 1,\ldots, m\}$ is a set of fuzzy weights.

The first step of TOPSIS is to calculate normalized ratings by

$$\tilde{r}_{ij}(x) = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{x}_{jn}^{2}}}, \quad i = 1,\ldots., n; \quad j = 1,\ldots., m$$

And then to calculate the weighted normalized ratings by

$$\tilde{v}_{ij}(x) = \tilde{w}_j \tilde{r}_{ij}(x);\ i = 1,\ldots., n; \quad j = 1,\ldots., m$$

Next the positive ideal point (PIS) and the negative ideal point (NIS) are derived as

$$PIS = A^{+} = \{\tilde{v}_1^{+}(x), \tilde{v}_2^{+}(x),\ldots., \tilde{v}_j^{+}(x),\ldots., \tilde{v}_m^{+}(x),$$

$$= \{\max \tilde{v}_{ij}(x) \mid j \in J_1), \min \tilde{v}_{ij}(x) \mid j \in J_2) \mid i = 1,...., n\}$$

$$PIS = A^- = \{\tilde{v}_1^-(x), \tilde{v}_2^-(x),...., \tilde{v}_j^-(x),...., \tilde{v}_m^-(x),$$

$$= \{\min \tilde{v}_{ij}(x) \mid j \in J_1), \max \tilde{v}_{ij}(x) \mid j \in J_2) \mid i = 1,...., n\}$$

Where J_1 and J_2 are the benefit and the cost attributes respectively.

Similar to the crisp solution, the following step is to calculate the separation from the PIS and the NIS between the alternatives. The separation values cal also be measured using the Euclidean distance given as :

$$\tilde{S}_i^+ = \sqrt{\sum_{j=1}^{m} [\tilde{v}_{ij}(x) - \tilde{v}_j^+(x)]^2}, \quad 1,...., n$$

And

$$\tilde{S}_o^- = \sqrt{\sum_{j=1}^{m} [\tilde{v}_{ij}(x) - \tilde{v}_j^-(x)]^2}, \quad 1,...., n$$

Where

$$\max\{\tilde{v}_{ij}(x) - \tilde{v}_j^+(x) = \min\{\tilde{v}_{ij}(x)\} - \tilde{v}_j^-(x) = 0$$

Next, the similarities to the PIS is given as

$$C_i^* = \frac{D(S_i^-)}{[D(S_i^+) + D(S_i^-)} \quad i = 1,...., n$$

Where $\mathrm{C}_i^* \in [0,1] \forall \quad i = 1,....., n$

Finally, the preferred orders are ranked according to C_i^* in descending order to choose the best alternatives.

NUMERICAL EXAMPLE

TABLE 8. INFORMATION TABLE

Alternatives	RAM			Camera Pixels			Memory card			Inbuilt memory		
	L	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*
S1	0.04	0.05	0.06	0.99	1	1	0.4	0.5	0.6	0.24	0.25	0.26
S2	0.99	1	1	0.99	1	1	0.4	0.5	0.6	0.99	1	1
S3	0.04	0.05	0.07	0.99	1	1	0.9	1	1	0.049	0.5	0.51
S4	0.99	1	1	0.99	1	1	0	0	0.1	0.99	1	1

TABLE 9. NORMALIZED MATRIX

Alternatives	RAM			Camera Pixels			Memory card			Inbuilt memory		
	L	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*
S1	0.0231	0.0353	0.043	0.431	0.5	0.6	0.5	0.408	0.52	0.02	0.164	0.2
S2	0.621	0.7062	0.862	0.41	0.5	0.62	0.3	0.408	0.54	0.523	0.65	0.7
S3	0.022	0.035	0.042	0.41	0.5	0.63	0.7	0.816	0.92	0.2	0.32	0.41
S4	0.623	0.706	0.84	0.48	0.5	0.66	0	0	0	0.5	0.65	0.7

TABLE 10.WEIGHTED NORMALIZED MATRIX

Alternatives	RAM			Camera Pixels			Memory card			Inbuilt memory		
	L	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*	*L*	*M*	*U*
S1	0.004	0.010	0.021	0.001	0.013	0.02	0.089	0.129	0.2	0.024	0.056	0.069
S2	0.629	0.706	0.8	0.001	0.013	0.02	0.08	0.129	0.2	0.199	0.227	0.3
S3	0.0024	0.035	0.076	0.001	0.013	0.024	0.21	0.332	0.4	0.049	0.113	0.231
S4	0.619	0.706	0.8	0.001	0.013	0.024	0	0.00	0	0.0001	0.004	0.01

TABLE 11:

DEFUZZIFICATION TECHNIQUE TO FIND THE MAXIMUM & MINIMUM OF FUZZY VALUES

	V1	V2	V3	V4
S1	0.010	0.013	0.1298	0.0567
S2	0.218	0.013	0.1298	0.2270
S3	0.010	0.013	0.3332	0.1135
S4	0.218	0.013	0	0.004

MAX V_J^+	0.2180	0.0133	0.3332	0.2270
MIN V_J^-	0.0109	0.0133	0	0.1135

TABLE 12. THE PIS AND THE NIS

Alternatives	S^+	S^-	***C****	***Rank***
S1	0.3365	0.14168	0.2961	4
S2	0.2034	0.2694	0.5697	2
S3	0.2361	0.3332	0.5852	1
S4	0.3332	0.2361	0.4147	3

The preferred order of alternatives are S3>S2>S4>S1.On the basis of preferred order, Alternative – III (ie) Supplier – III should be the best choice.

4. Linear Programming

After calculation of ENTROPY for four suppliers, the weights for the suppliers are 0.3088, 0.0266, 0, and 0.657 respectively.

Then, the optimal order quantity can be obtained through the LP technique. Suppose the customer needs to buy a single piece of mobile from any of the best product. We can obtain the best optimum solution by using the following information.

Max z = 0.3088X1 + 0.0266 X2 +0.657 X4

Subject to:

X1 + x2 +X3+X4=1

13900X1 + 14000X2 + 13500X3+ 14600X4 <=15000

X1+ 2X2 +X3 + 2X4 <=2

8X1+ 13X2 + 8X3 + 13 X4 < = 13

This LP optimization problem is solved by using the software TORA. The optimal order quantities for each supplier are 0, 0, 1 and 0 pieces. And the best mobile that can be selected is Samsung galaxy grand. It can be obtained either using TORA software or LINGO software.

CONCLUSION:

In this paper we have use TOPSIS, LP, MCDM, TOPSIS, Supplier's Selection, TORA software. The ENTROPY method is used to obtain dependence weights of the criteria for TOPSIS, Fuzzy TOPSIS and VIKOR. In TOPSIS and Fuzzy TOPSIS ranking is the same when entropy weights are used which results the selection of Supplier - 3, whereas Supplier - 1 is the best choice in VIKOR. We found that supplier 3 obtains the most order quantity and supplier 1, 3 and 4 obtain no orders by Linear Programming Model.

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