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ALMOST gpr-CLOSED AND ALMOST gpr-OPEN MAPPINGS

S. Balasubramania*and M. Lakshmi Sarada**
*Department of Mathematics, Govt. Arts College(A), Karur-639 005 (Tamilnadu)

**Department of Mathematics, A.M.G. Degree College, Chilakaluripet (A.P.)

ABSTRACT :
In this paper we discuss new type of closed and open mappings called almost gpr-closed and almost gpr-open mappings; its properties and interrelation with other continuous functions are studied.
Keywords: ***closed mapping; semi-closed mapping; pre-closed mapping; β-closed mapping; γ-closed mapping, ν-closed mapping, open mapping; semi-open mapping; pre-open mapping; β-open mapping; γ-open mapping and ν-open mapping***
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1. INTRODUCTION:

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Levine (1970) introduced the notion of generalized closed sets. After him different mathematicians worked and studied on different versions of generalized closed sets and related topological properties. Recently Balasubramanian, etal. (2012) studied, somewhat gp-continuous, somewhat gp-open function and almost *gpr* continuous mapping and their basic properties. In this paper we are going to further study weak form of closed and open mappings namely almost *gpr*-closed and almost *gpr*-open mappings using *gpr*-closed and *gpr*-open sets. Basic properties are verified by relevant theorems. Throughout the paper X, Y means a topological spaces (X, τ) and (Y, σ) unless otherwise mentioned without any separation axioms.

2. PRELIMINARIES

Definition 2.1: $A\subset X$ is called

(i) closed[semi-closed] if its complement is open[semi-open].
(ii) Regular closed if A = cl(int(A))
(iii) g-closed[rg-closed] if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(iv) pg-closed[gp-closed; gpr-closed] if pcl(A) $\subseteq U$ whenever $A\subseteq U$ and U is pre-open[open; regular-open] in X.
(v) αg-closed if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.2: A function f**:**$X\rightarrow Y$ is said to be

(i) continuous[resp**:** nearly-continuous; pre-continuous; g-continuous; rg-continuous] if inverse image of each open set is open[resp**:** regular-open; preopen; g-open; rg-open].
(ii) nearly-irresolute; [resp**:** g-rresolute; rg-irresolute] if inverse image of each regular-open set[resp**:** g-open; rg-open] is regular-open; [resp**:** g-open; rg-open].
(iii) closed[resp**:** nearly-closed; g-closed; rg-closed] if inverse image of each closed set is closed[resp**:** regular-closed; g-closed; rg-closed].

Definition 2.03: X is said to be $T_{1/2}$[r-$T_{1/2}$] if every [regular-]generalized closed set is [regular-]closed.

Note 1: From Definition 2.1 we have the following interrelations among the closed and open sets.

Closed → → g-closed → → αg-closed → → gp-closed → → gpr-closed
↑ ↑
r-closed → → → → → → → → → → → → → rg-closed

Open → → g-open → → αg-open → → gp-open → → gpr-open
↑ ↑
r-open → → → → → → → → → → → → → rg-open

3. ALMOST GPR-CLOSED MAPPINGS:

Definition 3.01:

A function *f***:** X→ Y is said to be almost *gpr*-closed if image of every regular closed set in X is *gpr*-closed in Y

Note 2: By note 1 we have the following implication diagram.

Closed map → g-closed map → αg-closed map → gp-closed map → *gpr*-closed map
↑ ↑
r-irresolute map → → → → → → → → → → → → rg-closed map

However, we have the following converse part**:**

Note 3: If GPRC(Y) = RC(Y) we have the following implication diagram.

Almost closed map ↔ Almost g-closed map ↔ Almost αg-closed map ↔ Almost gp-closed map ↔ Almost *gpr* closed map
↑↓ ↑↓
r-irresolute map ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ Almost rg-closed map

Example 1: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X}. and σ = {ϕ, {a}, Y} and let *f***:** X→ Y be defined as *f*(a) = b; *f*(b) = c; *f*(c) = a, then *f* is al.***gpr*-closed**; al. rg-closed but not al.g-closed; al.gp-closed; al.r-closed; al.closed; al.αg-closed.

Theorem 3.01: If (Y, σ) is a discrete space, then *f* is almost closed of all types**:**

Example 2: Let X = Y = {a, b, c} and τ = {ϕ, {b}, {a, b}, {b, c}, X}; σ = ℘(Y) and let *f***:**X→ Y be defined as *f*(a) = b; *f*(b) = a; *f*(c) = c, then *f* is al.***gpr*-closed**; al.rg-closed; al.g-closed; al.gp-closed; al.αg-closed; al.r-closed; al.closed.

Example 3: Let X = Y = {a, b, c} and τ = ℘(X); σ = {ϕ, {b}, {a, b}, {b, c}, Y} and let *f***:**X→ Y be defined as *f*(a) = c; *f*(b) = a; *f*(c) = b, then *f* is al.***gpr*-closed**; al.rg-closed; but not al.g-closed; al.r-closed; al.closed.

Theorem 3.02: (i) If *f* is almost closed and *g* is almost *gpr*-closed[almost rg-closed] then *g*•*f* is almost *gpr*-closed

(ii) If f and g are r-irresolute then $g \bullet f$ is almost *gpr*-closed

(iii) If f is r-irresolute and g is almost *gpr*-closed then $g \bullet f$ is almost *gpr*-closed

Theorem 3.03: If f**:** X→ Y is almost *gpr*-closed, then $gpr(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

Proof: Let A⊂ X and f:X→ Y is *gpr*-closed gives $f(\text{cl}\{A\})$ is *gpr*-closed in Y and $f(A) \subset f(\text{cl}\{A\})$ which in turn gives $gpr(\text{cl}\{f(A)\}) \subset gpr\text{cl}(f(\text{cl}\{A\}))$ (1)

Since $f(\text{cl}\{A\})$ is *gpr*-closed in Y, $gpr\text{cl}(f(\text{cl}\{A\})) = f(\text{cl}\{A\})$ (2)

combining (1) and (2) we have $gpr(\text{cl}\{f(A)\}) \subset (f(\text{cl}\{A\}))$ for every subset A of X.

Remark 1: converse is not true in general.

Theorem 3.04: If f**:** X→ Y is almost *gpr*-closed[almost rg-closed] and A⊂ X is regular-closed, then $f(A)$ is τ_{gpr}-r-closed in Y.

Proof: Let A⊂ X and f:X→ Y is almost *gpr*-closed ⇒ $gpr(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$ which in turn implies $gpr(\text{cl}\{f(A)\}) \subset f(A)$, since $f(A) = f(\text{cl}\{A\})$. But $f(A) \subset gpr(\text{cl}\{f(A)\})$. Combining we get $f(A) = gpr(\text{cl}\{f(A)\})$. Hence $f(A)$ is τ_{gpr}-closed in Y.

Corollary 3.01: (i) If f**:** X→ Y is almost rg-closed, then $gpr(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

(ii) If f**:** X→ Y is r-closed, then $gpr(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

(iii) If f**:** X→ Y is r-closed, then $f(A)$ is τ_{gpr}-closed in Y if A is closed[r-closed] set in X.

Theorem 3.05: If $gpr(\text{cl}\{A\}) = \text{rg}(\text{cl}\{A\})$ for every A⊂ Y, then the following are equivalent**:**

(i) f:X→ Y is *gpr*-closed map

(ii) $gpr(\text{cl}\{f(A)\}) \subset f(\text{cl}\{A\})$

Proof: (i) ⇒ (ii) follows from Theorem 3.2

(ii) ⇒ (i) Let A be any closed set in X, then $f(A) = f(\text{cl}\{A\}) \supset gpr(\text{cl}\{f(A)\})$ by hypothesis. We have $f(A) \subset gpr(\text{cl}\{f(A)\})$. Combining we get $f(A) = gpr(\text{cl}\{f(A)\}) = \text{rg}(\text{cl}\{f(A)\})$[by given condition] which ⇒ $f(A)$ is rg-closed and hence *gpr*-closed. Thus f is *gpr*-closed.

Theorem 3.06: f**:** X→ Y is almost *gpr*-closed iff for each subset S of Y and each regular open set U containing $f^{-1}(S)$, there is a *gpr*-open set V of Y such that S⊂ V and $f^{-1}(V) \subset U$.

Proof: Assume f is almost *gpr*-closed, S⊂Y and U∈RO(X) containing $f^{-1}(S)$, then $f(X - U)$ is *gpr*-closed in Y and $V = Y - f(X - U)$ is *gpr*-open in Y. $f^{-1}(S) \subset U \Rightarrow S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$.

Conversely let F∈RO(X), then $f^{-1}(f(F^c)) \subset F^c$. By hypothesis, there exists V∈GPRO(Y) such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f[(f^{-1}(V))^c] \subset V^c \Rightarrow f(F) \subset V^c$, which ⇒ $f(F) = V^c$. Thus $f(F)$ is *gpr*-closed in Y and therefore f is almost *gpr*-closed.

Remark 2: composition of two almost *gpr*-closed maps is not almost *gpr*-closed.

Theorem 3.07: Let X, Y, Z be topological spaces and every *gpr*-closed set is r-closed in Y, then the composition of two almost *gpr*-closed maps is almost *gpr*-closed.

Proof: Let A be regular closed in X $\Rightarrow$ *f*(A) is *gpr*-closed in Y $\Rightarrow$ *f*(A) is regular-closed in Y[by assumption] $\Rightarrow$ *g*(*f*(A)) is *gpr*-closed in Z $\Rightarrow$ *g•f*(A) is *gpr*-closed in Z $\Rightarrow$ *g•f* is almost *gpr*-closed.

Theorem 3.08: If *f* is almost rg-closed; *g* is almost *gpr*-closed[almost rg-closed] and Y is r-$T_{1/2}$, then *g•f* is almost *gpr*-closed.

Proof: (i) Let A be r-closed in X $\Rightarrow$ *f*(A) is rg-closed in Y $\Rightarrow$ *f*(A) is r-closed in Y[since Y is r-$T_{1/2}$] $\Rightarrow$ *g*(*f*(A)) is *gpr*-closed in Z $\Rightarrow$ *g•f*(A) is *gpr*-closed in Z $\Rightarrow$ *g•f* is almost *gpr*-closed.

(ii) Since every g-closed set is rg-closed, this part follows from the above case.

Corollary 3.02: If *f*:X$\rightarrow$ Y is almost rg-closed; *g*:Y$\rightarrow$ Z is r-open and Y is r-$T_{1/2}$, then *g•f* is almost *gpr*- closed.

Theorem 3.09: If *f* and *g* be two mappings such that *g•f* is almost *gpr*-closed[almost rg-closed]. The following are true

(i) If *f* is continuous[r-irresolute] and surjective, then *g* is almost *gpr*-closed

(ii) If *f* is rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is almost *gpr*-closed

Corollary 3.03: If *f* and *g* be two mappings such that *g•f* is r-irresolute. Then the following are true

(i) If *f* is continuous[r-irresoloute] and surjective, then *g* is almost *gpr*-closed

(ii) If *f* is rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is almost *gpr*-closed

Theorem 3.10: If X is *gpr*-regular, *f*:X$\rightarrow$ Y is r-open, almost rg-continuous, almost *gpr*-closed surjection and cl{A} = A for every *gpr*-closed set in Y, then Y is *gpr*-regular.

Proof: Let p$\in$ U$\in$*GPRO*(Y), $\exists$ x$\in$X such that *f*(x) = p by surjection. Since X is *gpr*-regular and *f* is rg-continuous $\exists$ V$\in$ RGO(X) such that x$\in$ V$\subset$ cl(V)$\subset$ f^{-1}(U) which implies p$\in$*f*(V)$\subset$*f*(cl(V))$\subset$ U (1)

Since *f* is *gpr*-closed, *f*(cl(V))$\subset$ U is *gpr*-closed and cl{*f*(cl(V))} = *f*(cl(V)) and cl{*f*(cl(V))} = cl{*f*(V)} (2)

combining (1) and (2) p$\in$ *f*(V) $\subset$ cl{*f*(V)} $\subset$ U and *f*(V) is rg-open. Hence Y is *gpr*-regular.

Corollary 3.04: If X is *gpr*-regular, *f*:X$\rightarrow$ Y is r-open, almost rg-continuous, almost *gpr*-closed surjection and cl{A} = A for every rg-closed set in Y, then Y is *gpr*-regular.

Theorem 3.11: (i) If *f* is almost *gpr*-closed[almost rg-closed] and A$\in$*RC*(X), then f_A is almost *gpr*-closed.

(ii) If *f* is almost *gpr*-closed[almost rg-closed], X is r$T_{1/2}$ and A$\in$*RGC*(X), then f_A is almost *gpr*-closed.

Corollary 3.05: (i) If *f* is r-closed and A$\in$*RC*(X), then f_A is almost *gpr*-closed.

(ii) If *f* is r-closed, X is r$T_{1/2}$ and A$\in$*RGC*(X), then f_A is almost *gpr*-closed.

Theorem 3.12: If f_i: $X_i \rightarrow Y_i$ be almost *gpr*-closed [almost rg-closed] for i = 1, 2. Let *f*: $X_1\times X_2 \rightarrow Y_1\times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f*: $X_1\times X_2 \rightarrow Y_1\times Y_2$ is almost *gpr*-closed.

Proof: Let $U_1\times U_2 \subset X_1\times X_2$ where U_i is r-closed in X_i for i = 1, 2. Then $f(U_1\times U_2) = f_1(U_1) \times f_2(U_2)$ a *gpr*-closed set in $Y_1 \times Y_2$. Thus $f(U_1 \times U_2)$ is *gpr*-closed and hence f is almost *gpr*-closed.

Theorem 3.13: Let h**:** $X \to X_1 \times X_2$ be almost *gpr*-closed[almost rg-closed]. Let f_i**:** $X \to X_i$ be defined as**:** $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i**:** $X \to X_i$ is almost *gpr*-closed for i = 1, 2.

Proof: Let $U_1 \times X_2$ is r-closed in $X_1 \times X_2$, and $h(U_1 \times X_2)$ is *gpr*-closed in X. But $f_1(U_1) = h(U_1 \times X_2)$, therefore f_1 is *gpr*-closed. Similarly we can show that f_2 is also *gpr*-closed and thus f_i**:** $X \to X_i$ is almost *gpr*-closed for i = 1, 2.

Corollary 3.06: (i) If f_i**:** $X_i \to Y_i$ be r-closed for i = 1, 2. Let f**:** $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as follows**:** $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f**:** $X_1 \times X_2 \to Y_1 \times Y_2$ is almost *gpr*-closed.

(ii) Let h**:** $X \to X_1 \times X_2$ be r-closed. Let f_i**:** $X \to X_i$ be defined as**:** $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i**:** $X \to X_i$ is almost *gpr*-closed for i = 1, 2.

3. ALMOST GPR-OPEN MAPPINGS:

Definition 4.01: A function f**:** X→ Y is said to be almost *gpr*-open if image of every r-open set in X is *gpr*-open in Y

Note 4: By note 1 we have the following implication diagram.

Open map → g-open map → αg-open map → gp-open map → *gpr*-open map
↑ ↑
r-open map → → → → → → → → → rg-open map

However, we have the following converse part**:**

Note 5: If GPRO(Y) = RO(Y) we have the following diagram

Almost open map ↔ Almost g-open map ↔ Almost αg- open map ↔ Almost gp-open map ↔ Almost *gpr*-open map
↑↓ ↑↓
r-irresolute map ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ ↔ Almost rg-open map

Example 4: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X}. and σ = {ϕ, {a}, Y} and let f**:** X→ Y be defined as f(a) = b; f(b) = c; f(c) = a, then f is al.***gpr*-open**; al. rg-open but not al.g-open; al.gp-open; al.r-open; al.open; al.αg-open.

Theorem 3.01: If (Y, σ) is a discrete space, then f is almost open of all types**:**

Example 5: Let X = Y = {a, b, c} and τ = {ϕ, {b}, {a, b}, {b, c}, X}; σ = ℘(Y) and let f**:**X→ Y be defined as f(a) = b; f(b) = a; f(c) = c, then f is al.***gpr*-open**; al.rg-open; al.g-open; al.gp-open; al.αg-open; al.r-open; al.open.

Example 6: Let X = Y = {a, b, c} and τ = ℘(X); σ = {ϕ, {b}, {a, b}, {b, c}, Y} and let f**:**X→ Y be defined as f(a)

= c; f(b) = a; f(c) = b, then f is al.***gpr*-open**; al.rg-open; but not al.g-open; al.r-open; al.open.

Theorem 4.02: (i) If f is almost open and g is almost *gpr*-open[almost rg-open] then $g{\bullet}f$ is almost *gpr*-open

(ii) If f and g are r-irresolute then $g{\bullet}f$ is almost *gpr*-open

(iii) If f is r-irresolute and g is almost *gpr*-open then $g{\bullet}f$ is almost *gpr*-open

Theorem 4.03: If f**:** $X\rightarrow Y$ is almost *gpr*-open, then $f(A^o) \subset gpr(\{f(A)\}^o)$

Proof: Let $A\subset X$ and f**:**$X\rightarrow Y$ is almost *gpr*-open gives $f(A^o)$ is *gpr*-open in Y and $f(A^o) \subset f(A)$ which in turn gives$gpr(f(\{A\}^o))^o \subset gpr(\{f(A)\}^o)$ - - - - - - - - - - - - - - - - - -- (1)

Since $f(\{A\}^o)$ is *gpr*-open in Y, $gpr(f(\{A\}^o))^o = f(\{A\}^o)$- (2)

combaining (1) and (2) we have $f(A^o) \subset gpr(\{f(A)\}^o)$ for every subset A of X.

Remark 3: converse is not true in general.

Theorem 4.04: If f**:** $X\rightarrow Y$ is almost *gpr*-open[almost rg-open] and $A\subset X$ is r-open, then $f(A)$ is τ_{gpr}-open in Y.

Proof: Let $A\subset X$ and f**:**$X\rightarrow Y$ is almost *gpr*-open $\Rightarrow gpr(\{f(A)\}^o) \subset f(\{A\}^o)$ which in turn implies $gpr(\{f(A)\}^o) \subset f(A)$, since $f(A) = f(\{A\}^o)$. But $f(A) \subset gpr(\{f(A)\}^o)$. Combining we get $f(A) = gpr(\{f(A)\}^o)$. Hence $f(A)$ is τ_{gpr}-open in Y.

Corollary 4.01: (i) If f**:** $X\rightarrow Y$ is almost rg-open, then $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$

(ii) If f**:** $X\rightarrow Y$ is r-open, then $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$

(iii) If f**:** $X\rightarrow Y$ is r-open, then $f(A)$ is τ_{gpr}-open in Y if A is r-open set in X.

Theorem 4.05: If $gpr(\{A\}^o) = rg(\{A\}^o)$ for every $A\subset Y$, then the following are equivalent**:**

(i) f**:**$X\rightarrow Y$ is almost *gpr*-open map

(ii) $f(\{A\}^o) \subset gpr(\{f(A)\}^o)$

Theorem 4.06: f**:** $X\rightarrow Y$ is *gpr*-open iff for each subset S of Y and each regular open set U containing $f^{-1}(S)$, there is a *gpr*-open set V of Y such that $S\subset V$ and $f^{-1}(V) \subset U$.

Proof: Assume f is almost *gpr*-open, $S\subset Y$ and U a regular open set of X containing $f^{-1}(S)$, then $f(X - U)$ is *gpr*-open in Y and $V = Y - f(X - U)$ is *gpr*-open in Y. $f^{-1}(S) \subset U \Rightarrow S\subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$.

Conversely let F be regular open in X, then $f^{-1}(f(F^c))\subset F^c$. By hypothesis, there exists $V\in GPRO(Y)$ such that $f(F^c)\subset V$ and $f^{-1}(V) \subset F^c$ and so $F\subset (f^{-1}(V))^c$. Hence $V^c\subset f(F) \subset f[(f^{-1}(V))^c]\subset V^c \Rightarrow f(F) \subset V^c$, which $\Rightarrow f(F) = V^c$. Thus $f(F)$ is *gpr*-open in Y and therefore f is *gpr*-open.

Remark 4: composition of two *gpr*-open maps is not *gpr*-open.

Theorem 4.07: Let X, Y, Z be topological spaces and every *gpr*-open set is r-open in Y, then the composition of two almost *gpr*-open maps is almost *gpr*-open.

Proof: Let A be r-open in X ⇒ *f*(A) is *gpr*-open in Y ⇒ *f*(A) is r-open in Y[by assumption] ⇒ *g*(*f*(A)) is *gpr*-open in Z ⇒ *g*•*f*(A) is *gpr*-open in Z ⇒ *g*•*f* is almost *gpr*-open.

Theorem 4.08: If *f* is almost rg-open; *g* is almost *gpr*-open[almost rg-open] and Y is r-$T_{1/2}$, then *g*•*f* is almost *gpr*-open.

Proof:(i) Let A be r-open in X ⇒ *f*(A) is rg-open in Y ⇒ *f*(A) is r-open in Y[since Y is r-$T_{1/2}$] ⇒ *g*(*f*(A)) is *gpr*-open in Z ⇒ *g*•*f*(A) is *gpr*-open in Z ⇒ *g*•*f* is almost *gpr*-open.

(ii) Since every g-open set is rg-open, this part follows from the above case.

Corollary 4.02: If *f* is almost g-open[almost rg-open]; *g* is r-open and Y is $T_{1/2}${r-$T_{1/2}$}, then *g*•*f* is almost *gpr*-open.

Theorem 4.09: If *f*:X→ Y; *g*:Y→ Z be two mappings such that *g*•*f* is almost *gpr*-open[almost rg-open]. Then the following are true

(i) If *f* is r-continuous[r-irresolute] and surjective, then *g* is almost *gpr*-open

(ii) If *f* is almost rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is almost *gpr*-open

Corollary 4.03: If *f* and *g* be two mappings such that *g*•*f* is r-open. Then the following are true

(i) If *f* is r-continuous and surjective, then *g* is almost *gpr*-open

(ii) If *f* is almost rg-continuous, surjective and X is r-$T_{1/2}$, then *g* is almost *gpr*-open

Theorem 4.10: If X is *gpr*-regular, *f*:X→ Y is r-open, almost rg-continuous, almost *gpr*-open surjection and $A^o = A$ for every *gpr*-open set in Y, then Y is *gpr*-regular.

Proof: Let p∈ U∈*GPRO*(Y), ∃ x∈X such that *f*(x) = p by surjection. Since X is *gpr*-regular and *f* is rg-continuous ∃ V∈ RGO(X) such that x∈ V^o⊂ V⊂ f^{-1}(U) which implies p∈*f*(V^o)⊂*f*(V)⊂ U (1)

Since *f* is *gpr*-open, *f*($(V)^o$)⊂ U is *gpr*-open and $\{f(V^o)\}^o = f(V^o)$ and $\{f(V^o)\}^o = \{f(V)\}^o$ (2)

combining (1) and (2) p∈$\{f(V)\}^o$ ⊂ *f*(V) ⊂ U and *f*(V) is rg-open. Hence Y is *gpr*-regular.

Corollary 4.04: If X is *gpr*-regular, *f*:X→ Y is r-open, almost rg-continuous, almost *gpr*-open surjection and A^o = A for every rg-open set in Y, then Y is *gpr*-regular.

Theorem 4.11: (i) If *f* is almost *gpr*-open[almost rg-open] and A∈*RO*(X), then f_A is almost *gpr*-open.

(ii) If *f* is almost *gpr*-open[almost rg-open], X is r$T_{1/2}$ and A∈*RGO*(X), then f_A is almost *gpr*-open.

Corollary 4.05: (i) If *f* is r-open and A∈*RO*(X), then f_A is almost *gpr*-open.

(ii) If *f*: X→ Y is r-open, X is r$T_{1/2}$ and A∈*RGO*(X), then f_A is almost *gpr*-open.

Theorem 4.12: If f_i: $X_i \to Y_i$ be almost *gpr*-open [almost rg-open] for i = 1, 2. Let *f*: $X_1\times X_2 \to Y_1\times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f*: $X_1\times X_2 \to Y_1\times Y_2$ is almost *gpr*-open.

Proof: Let $U_1\times U_2 \subset X_1\times X_2$ where U_i is r-open in X_i for i = 1, 2. Then $f(U_1\times U_2) = f_1(U_1) \times f_2(U_2)$ a *gpr*-open set in $Y_1\times Y_2$. Thus $f(U_1\times U_2)$ is *gpr*-open and hence *f* is almost *gpr*-open.

Theorem 4.13: Let h**:** $X \to X_1 \times X_2$ be almost *gpr*-open[almost rg-open]. Let f_i**:** $X \to X_i$ be defined as**:** $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i**:** $X \to X_i$ is almost *gpr*-open for i = 1, 2.

Proof: Let $U_1 \times X_2$ is r-open in $X_1 \times X_2$, and $h(U_1 \times X_2)$ is *gpr*-open in X. But $f_1(U_1) = h(U_1 \times X_2)$, therefore f_1 is almost *gpr*-open. Similarly we can show that f_2 is also *gpr*-open and thus f_i**:** $X \to X_i$ is almost *gpr*-open for i = 1, 2.

Corollary 4.06: (i) If f_i**:** $X_i \to Y_i$ be r-open for i = 1, 2. Let f**:** $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as follows**:** $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f**:** $X_1 \times X_2 \to Y_1 \times Y_2$ is almost *gpr*-open.

(ii) Let h**:** $X \to X_1 \times X_2$ be r-open. Let f_i**:** $X \to X_i$ be defined as**:** $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i**:** $X \to X_i$ is almost *gpr*-open for i = 1, 2.

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