MATHEMATICAL MODELS IN TERMS OF FUZZY DIRECTED GRAPHS WITH FUZZY IF-THEN CHAIN SET LOGIC

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ABSTRACT:

In this paper we generalize the concept of the graph to fuzzy graph and define the degree of a vertex, regular fuzzy graph and fuzzy directed graph. Here we show that the mathematical modelling through fuzzy directed graphs by using fuzzy If-Then chain set logic and related problems are solved.

Keywords:*Fuzzy Directed Graph, Fuzzy If-Then Chain set logic, Mathematical Model, Regular Fuzzy Graph.*

INTRODUCTION:

A mathematical model is an abstract model that uses mathematical language to describe the behavior of a system. Mathematical models are used particularly in the natural sciences and engineering disciplines (such as physics,biology and electrical engineering),as well as in the social sciences (such as economics,sociology and political science). Physicists, engineers, computer scientists and economists use mathematical models most extensively.

Eykhoft-(1974) defined a mathematical model as 'a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form'. Mathematical models can take many forms, including but not limited to dynamical sytems, statistical models, differential equations or game theoratic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. There are six basic groups of variables: discussion variables, input variables, state variables, exogenous variables, random variables and output variables. Since there can be many variables of each type, the variables are generally represented by vectors.

In 1736 Euler first introduced the concept of grph theory. In the history of mathematics, the solution given by Euler of the well-known Konigsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation research, optimizationand computer science.

Fuzzy graph theory was introduced by AzrielRosenfield in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. The fuzzy relations between fuzzy sets were also considered by Rosenfield and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts.

2. BASIC DEFINITIONS:

Definition 2.1:

- Models are abstractions of reality!
- Models are a representation of a particular thing, idea or condition.
- Mathematical models are simplified representations of some real world entity.
    - Can be in equations or computer code.
    - Are intended to mimic essential features while leaving out inessentials.
- Mathematical models are characterized by assumptions about:
    - Variables (the things which change)
    - Parameters (the things which do not change)
    - Functional forms (the relationship between the two).

Example 2.1:

Problem:

If there are 11 coins, nickles and dimes, valuing 70 cents, how much of each are there?

Discussion:

First try to identify the variables needed, keeping in mind what the problem is asking you to find-that might be helpful when you decide on the relevant variables. This problem is asking for the number of nickles and the number of dimes.

Thus, let x=number of nickles and y=number of dimes. The natural next step is to create a system of two equations. With two unknowns: the first one describing the fact that there are 11 coins all together:$x+y=11$ and the second one describing the fact that the value of x many coins worth 5 cents each and the value of y many coins worth 10 cents each should add to the total value of 70 cents. Thus, the second equation is $5x+10y=70$.

Solution:

Using the usual methods (elimination of variables or solving a system of linear equations).

$x+y=11 \longrightarrow 1$

$5x+10y=70 \longrightarrow 2$

Solve the above equations, we get x=8 and y=3.

(i.e.) Number of nickles x=8 and

Number of dimes y=3.

We apply this values in equations (1) & (2) $\Rightarrow$ 8+3=11 & 40+30=70.

Definition 2.2: The degree of any vertex σ(vi) of a fuzzy graph is sum of degree of membership of all those edges which are incident on a vertex σ(vi) . And is denoted by d[σ(vi)]. The minimum degree of G is δ(G)=^{d(v)/vϵσ*} and the maximum degree of G is Δ(G)=˅{d(v)/vϵσ*}.

Example 2.2:

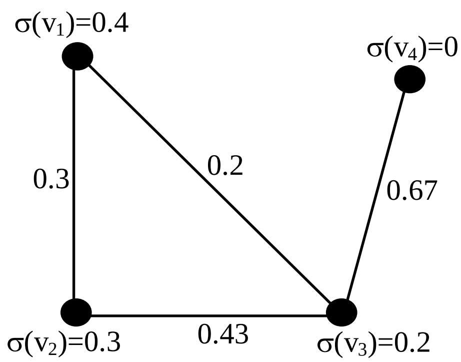

Fig1: G

Degree of vertex σ(v_3)=degree of membership of all those edges which are incident on a vertex σ(v_3)

= μ(v_1,v_3)+μ(v_2,v_3) + μ(v_3,v_4)

= 0.2+0.43 + 0.67 =1.3

(i.e.)d[σ(v_3)] = 1.3

Definition 2.3: Let G: (σ,μ) be a fuzzy graph of a graph on G^*:(V,E). If $d_G(v)$=K for all v∈V. (i.e) if each vertex has same degree K, then G is called a regular fuzzy graph of degree K or a K-Regular fuzzy graph.

Example 2.3:

d[$\sigma(v_1)$]= 0.24+0.24=0.48

d[$\sigma(v_2)$]= 0.24+0.24=0.48

d[$\sigma(v_3)$]= 0.24+0.24=0.48

d[$\sigma(v_4)$]= 0.24+0.24=0.48

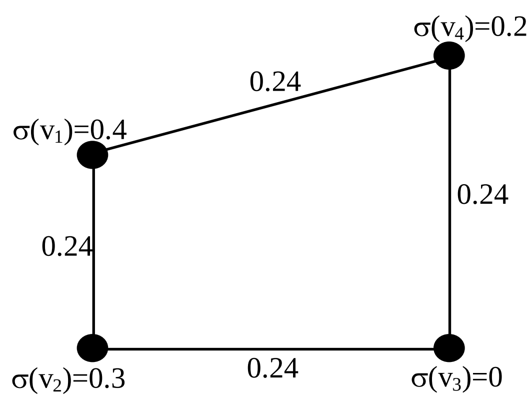

Fig2:Regular Fuzzy Graph

i.e. d[$\sigma(v_1)$]= d[$\sigma(v_2)$]= d[$\sigma(v_3)$]= d[$\sigma(v_4)$]=0.48

Definition 2.4:A fuzzy graph is called a fuzzy directed graph or a fuzzy digraph if every edge is directed with an arrow. An every edge joining onto some ordered pair of vertices $(\sigma(v_i), \sigma(v_j))$.If an edge is left undirected in a fuzzy digraph, it will be assume to be directed both ways.

Example 2.4:

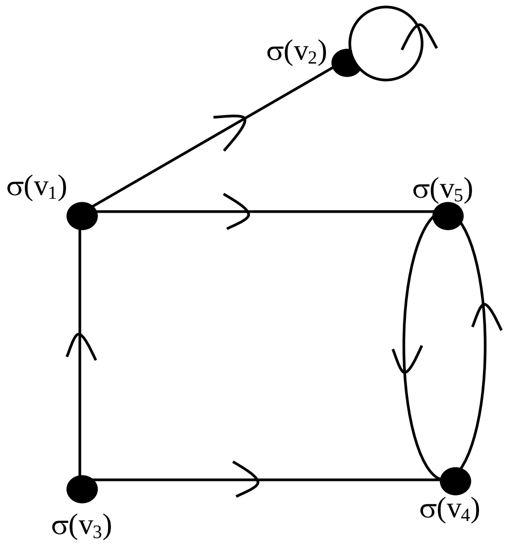

Fig4:Fuzzy Directed Graph

Definition 2.5:

A fuzzy rule is defined as an IF-THEN chain set logic in the form:

p= x is….. q = x is …..

Info-kb = (p → q)

= (IF P THEN q)

Where p and $q$be the sentences and info-kb stands for knowledge base information.

Example 2.5:

Let p and q be the sentences

P= x is aninstance of a whale,q =x is an instance of a mammal

Info-kb= (p→ q)

= (IF x is an instance of a whale THEN x is an instance of a mammal).

3.MATHEMATICAL MODELS IN TERMS OF FUZZY DIRECTED GRAPHS:

Theorem 3.1:

IfG:(σ,μ) is a fuzzy graph with G:(V,E). Then $\sum_{i=1}^{n} d[\sigma(vi)] = \sum_{i=1}^{n} 2\mu(ui, vi+1)$=Twice the sum of degree of membership of $\sum_{i=1}^{n} 2\mu(\text{ui}, \text{vi}+1)$, where $\sigma(v_i)$ is degree of all vertices v_i and $\mu(u_i,v_{i+1})$ is degree of membership $(u_i,v_{i+1}) \in E$.

Problem 3.2:

Let G:(σ,μ) be a fuzzy graph all of whose points have degree k or k+1. If G has t>0 points of degree k, show that $t=\sigma(v_i)(k+1)-2\sum_{i=1}^{n} \mu(\text{ui}, \text{vi}+1)$.

Solution:

Since G has t points of degree k, the remaining $\sigma(v_i)$-t points have degree k+1.

Hence $\sum_{v\in V} d[\sigma(\text{vi})] = tk+(\sigma(v_i)-t)(k+1)$

$2\sum\mu(u_i,v_{i+1}) = tk+\sigma(v_i)k+\sigma(v_i)-tk-t$
$=\sigma(v_i)(k+1)-t$

∴ $t =\sigma(v_i)(k+1)- 2\sum_{i=1}^{n} \mu(\text{ui}, \text{vi}+1)$.

Problem 3.3:

Representing result of tournaments in terms of fuzzy directed graph. Prove that

$\delta(G) \leq \frac{2\sum_{i=1}^{n} \mu(\text{ui},\text{vi}+1)}{\sigma(\text{vi})} \leq \Delta(G)$.

Solution:

We can prove this problem mathematical models in terms offuzzydirected graph with fuzzy IF- THEN chain set logic.

(i) Team $\sigma(v_1)$ has defeated team $\sigma(v_2)$

(ii) Team $\sigma(v_2)$has defeated teams $\sigma(v_3)$& $\sigma(v_4)$

(iii) Team $\sigma(v_3)$ has defeated team $\sigma(v_4)$

(iv)Team $\sigma(v_4)$ has defeated team $\sigma(v_5)$

(v)Team $\sigma(v_5)$ has defeated teams $\sigma(v_1)$ & $\sigma(v_2)$

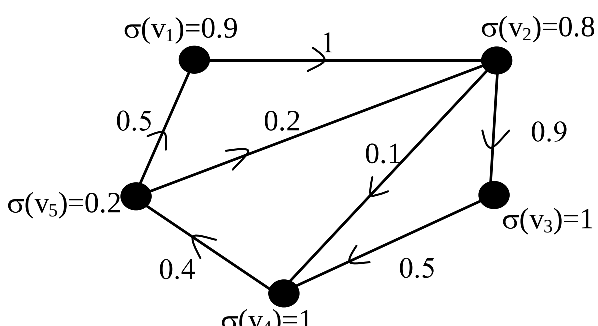

Fig5:G

(vi)Matches between $\sigma(v_1)$and $\sigma(v_3)$, $\sigma(v_1)$ and $\sigma(v_4)$,$\sigma(v_3)$ and $\sigma(v_5)$have yet to be played.

Let V(G) = $\{\sigma(v_1),\sigma(v_2),\ldots,\sigma(v_n)\}$

We have $\delta(G) \leq \deg \sigma(v_i) \leq \Delta(G)$ for all i

Hence $\sigma(v_i)\delta(G) \leq \sum_{i=1}^{n} \deg \sigma(vi) \leq \sigma(v_i)\Delta(G)$

$\therefore \sigma(v_i)\delta(G) \leq 2\sum_{i=1}^{n} \mu(ui, vi+1) \leq \sigma(v_i)\delta(G)$ (using theorem 3.1)

$\therefore \delta(G) \leq \frac{2\sum_{i=1}^{n} \mu(ui,vi+1)}{\sigma(vi)} \leq \Delta(G)$

Rule 1:

P=x is $\delta(G)$= 1, q=x is δ (G) $\leq \frac{2\sum_{i=1}^{n} \mu(ui,vi+1)}{\sigma(vi)}$which is equal to 1.8.

Info-kb = (p$\rightarrow q$)

= (IF x is $\delta(G)$ = 1 THEN x isδ (G) $\leq \frac{2\sum_{i=1}^{n} \mu(ui,vi+1)}{\sigma(vi)}$which is equal to 1.8)

Example:

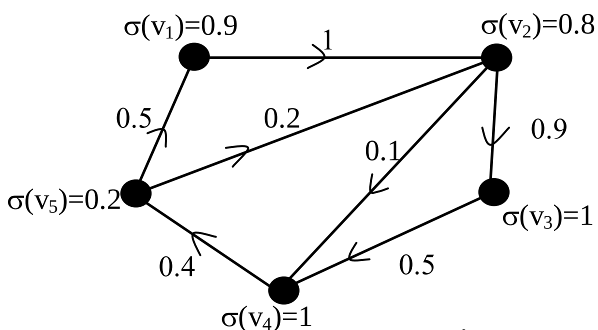

Fig5:G

$d[\sigma(v_1)]$= 1+0.5=1.5

$d[\sigma(v_2)]$= 1+0.9+0.2+0.1=2.2

$d[\sigma(v_3)]$=0.9+0.5=1.4

$d[\sigma(v_4)]$= 0.5+0.4+0.1=1

$d[\sigma(v_5)]$=0.4+0.5+0.2=1.1

$\sigma(v_i)$ =0.9+0.8+1+1+0.2=3.9

$\sum_{i=1}^{n} \mu(ui, vi+1)$=1+0.9+0.5+0.4+0.5+0.2+0.1=3.6

$2\sum_{i=1}^{n} \mu(ui, vi+1)$=2(3.6)=7.2

$\frac{2\sum_{i=1}^{n} \mu(ui,vi+1)}{\sigma(vi)} = \frac{7.2}{3.9} = 1.8$

Here$\delta(G)$ is minimum degree, (i.e.) $\delta(G)$= 1

$\therefore \delta$ (G) $\leq \frac{2\sum_{i=1}^{n} \mu(ui,vi+1)}{\sigma(vi)}$

(i.e.) $1 \leq 1.8$

Rule 2:

P =x is $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)}$ = 1.8, q=x is $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)} \leq$ Δ(G) which is equal to 2.2

Info-kb =(p→ q)

=(IF x is $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)}$ = 1.8 THEN x is $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)} \leq$ Δ(G) which is equal to 2.2)

Example:

d[$\sigma(v_1)$]= 1+0.5=1.5

d[$\sigma(v_2)$]= 1+0.9+0.2+0.1=2.2

d[$\sigma(v_3)$]=0.9+0.5=1.4

d[$\sigma(v_4)$]= 0.5+0.4+0.1=1

d[$\sigma(v_5)$]=0.4+0.5+0.2=1.

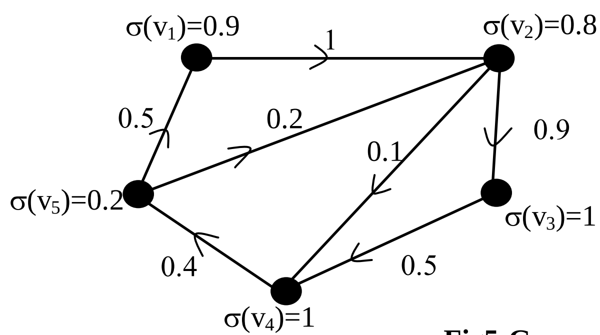

Fig5:G

HereΔ(G) is maximum degree, (i.e.)Δ(G) = 2.2

∴ $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)} \leq$ Δ(G)

(i.e.) 1.8 ≤ 2.2

Finally,

δ(G) ≤ $\frac{2\sum_{i=1}^{n}\mu(ui,vi+1)}{\sigma(vi)} \leq \Delta(G)$

(i.e.) 1 ≤ 1.8 ≤ 2.2

Hence proved.

CONCLUSION:

Finally, we have discussed regular fuzzy graph and fuzzy directed graph. The notion of fuzzy If-Then chain set logic was also discussed. Further we introduced mathematical models in terms of fuzzy directed graphs by using fuzzy If-Then chain set logic.

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