Aryabhatta Journal of Mathematics & Informatics Vol. 6, No. 1, Jan-July, 2014 ISSN : 0975-7139
Journal Impact Factor (2013) : 0.489

AN ECONOMIC ORDER QUANTITY MODEL OF ITEMS WITH TWO WARE-HOUSES AND STOCK DEPENDENT CONSUMPTION RATE

Naresh Kumar Aggarwal*, Chander Bhan Gupta, Anil Kumar Sharma*** and Sanjay Kumar******

*Department of Mathematics, G S S S School, Bikaner (Rewari)

**Department of Mathematics, Birla Institute of Tech. & Science, Pilani (Rajasthan)

***Department of Mathematics, R R Government College, Alwar (Rajasthan)

****Department of Mathematics, K.L.P.College, Rewari (Haryana)

ABSTRACT:

In this paper, we have developed, an EOQ model for two ware houses taking different form of consumption rate, which depends on stock and shortages are not allowed, considering infinite rate of replenishment and transportation cost, dependent on the quantity to be transferred from RW to OW . Some additional replenishment cost incurred when L_2 system is used. This cost has been taken into account for the development of the model. Choosing appropriate value of the parameter, the developed model reduces to well known results. A numerical example is also provided at the end .

Key Words: ***Stock dependent consumption rate, EOQ, L_2 system.***

INTRODUCTION

A number of inventory models have been developed by many researchers as it usually assumed that the warehouse owned by the management has sufficient place to stock the goods purchased or produced. But this assumption is not true for items where the demand is influenced by stock maintained. However in some cases management is more interesting to purchase more goods that can not be accommodated in own warehouse (OW) excess goods are stocked in another warehouse known as rented warehouse (RW). There are many reasons for this, such as to get an attractive price discount and bulk purchase or production or the acquisition costs are higher than the cost in RW . Usually the holding cost is higher than the holding OW and customer are served from OW only. So it is economical to empty RW first.

Easy availability of items motivates the customers to purchase more and the behavior of the customers is termed as stock dependent. Many authors have considered L_2 – system{Two ware house} under different assumptions. Hartly (5) first considered this type of model without considering transportation cost. Sarma (11) extended Hartly (5) model by considering transpiration cost of a unit from RW to OW and called optimum released rule. Sarma (12) again considered the model developed by Sarma (11) and provide its improved version. Murdeshwar and Sathe (7) has Presented some aspects of lot size model with two level of storage by considering finite production rate. Gupta and Vrat (4) considered inventory for stock dependant consumption rate and presented inventory model for instantaneous and finite rate of replenishment . taking different from of consumption rate. Dave (2) discussed model developed by Sarma (13) , Murdeshwar and Sathe (7) and point out some questions rectifying at the suitable place an provide improved solution Sarma (13) reconsidered the models developed by Sarma (11, 12) & Murdeshwar and Sathe (7) and deva (2) and gave an improved algorithm for the optimal solution. Goswami and Chaudhuri (3) developed models with and without shortages for linear trend in demand.

Bhunia and Mati (1) has suggested a two level storage model for linear trend in demand. Anil and Manoj (15) developed an Inventory Model when the demand rate is a function of selling price and is non linear form and the holding cost is time dependent. Mandal and Phaujdar (6) has considered an inventory model for deteriorating items and stock dependent consumption rate Jacob and Antony (8) considered the transportation cost of K units as constant from RW to OW and fixed set up cost. Actually transportation cost depends on the quantity to be transported and constant set up cost may not be true for two warehouse. Recently Hari Kishan etal. (16) discussed an inventory model with variable demand rate shortage and constant deterioration.

In this model it was assumed that there are ‘n’ shipment from RW to OW each of K-units. We have developed an economic order quantity model for items with two warehouses and stock dependent demand rate by considering the transporting cost as a function of K. It is also assumed that an additional set up/ordering cost must be incurred for two warehouses and K units are transferred in each of (n-1) shipment from RW to OW and in last shipment remaining $\{S-(n-1)K\}$ units are transferred

i.e. $n = \begin{cases} \frac{S}{K} & \text{If } \frac{S}{K} \text{ is an integer} \\ \left[\frac{S}{K}\right]+1 & \text{Otherwise} \end{cases}$

[S/K] denotes the integer part of S/K.

ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used for the development of the model.

1. The demand rate is assumed as D, where

$$D = \alpha Q^{\beta}$$
$$= \alpha e^{\beta Q}$$
$$= \alpha + \beta a^{Q}$$
$$= \alpha + \frac{\beta}{Q}$$
$$= \alpha + \beta Q^{\lambda}$$

Where α, β, λ are constants.

2. Replenishment rate is infinite.
3. Lead time is zero.
4. Shortages are not allowed.
5. Consumption takes place only in OW.
6. The items of RW are transferred to OW in ‘n’ shipment of which $K \le W$ (Capacity of OW) units are transported in each shipment.
7. L_1 identifies an inventory system with a single storage facility.
8. L_2 identifies an inventory system with two storage facilities.
9. ‘A’ is replenishment cost (order cost) per replenishment with different values for L_1 and L_2 system;
   Where
   A = A_1 for L_1- system.
   = A_1 + A_2 for L_2- system.
   A_2 being the additional replenishment cost incurred when L_2- system is used such as loading, unloading costs etc. for excess items in the RW.
10. The unit holding cost is H for items in OW and F for items in RW and F>H.

11. The capacity of OW is limited to W units and RW has unlimited capacity.
12. The transportation cost of K units from RW to OW is x+y(K-P) where P(<K) is the maximum number of units which can be transported under a fixed charge 'x' and for charge 'y' is to be paid.
13. T_K is the time taken for the consumption of 'K' units.

MODEL DEVELOPMENT AND ANALYSIS

In the development of the model, it is assumed that a company purchases Q (Q>W) units out of which W units are kept in OW and S=Q-W units are kept in RW. If Q>W and S=0 if Q<W. Figure 1 represents the pictorial representation of the system under study. Initially the demand is satisfied using the stocks of OW until the stock level drops to (W-K) units. At this stage, $K(K \leq W)$ units are transported from RW to OW to meet further demand. As a result the stock level of OW again becomes W. This process is continued until the stocks of OW is fully exhausted.

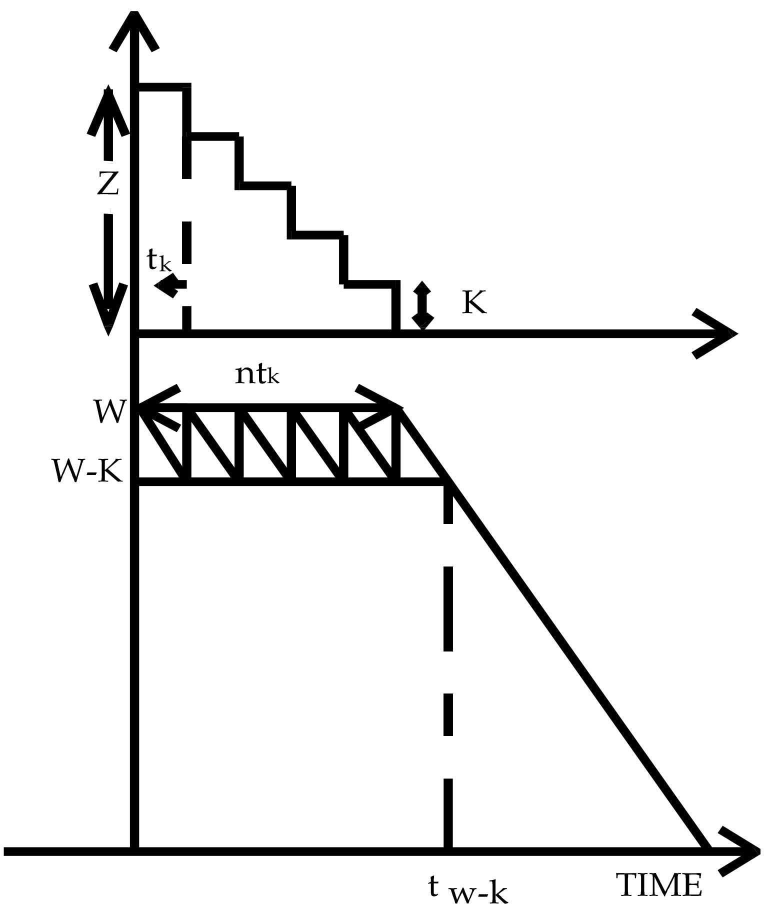

Fig (ii)

Case (A) (Before Consumption)

Initially, we have the total average cost as

$$C(Q)=\frac{AD}{Q}+\frac{HW}{2}+\frac{F(Q-W)}{2} \qquad (1)$$

Sub case (1) when $D=\alpha Q^{\beta}$

$$C(Q)=\frac{A\alpha\, Q^{\beta}}{Q}+\frac{HW}{2}+\frac{F(Q-W)}{2}$$

$$=A\alpha\ Q^{\beta-1}+\frac{HW}{2}+\frac{F(Q-W)}{2}$$

For optimum value Q^* of Q, which minimize total average cost is obtained by solving $d\,C(Q)\,/\,dQ=O$

$$A\alpha(\beta-1)Q^{\beta-2}+\frac{F}{2}=0$$

which gives $Q^*=\sqrt{\frac{2A\alpha}{F}}$ (If $\beta=0$)

which is same as Classical EOQ formula.

Also $\frac{d^2C(Q)}{dQ^2}=A\alpha(\beta-1)(\beta-2)Q^{\beta-3}>0$ if $\beta>2$

Sub Case (ii) When $D=\alpha\, e^{\beta Q}$

We have $C(Q)=\frac{A\alpha\ e^{\beta Q}}{Q}+\frac{HW}{2}+\frac{F(Q-W)}{2}$ .... (3)

The optimal value of Q , which minimize (3) is obtained by solving $\frac{dC(Q)}{dQ}=0$

$$A\alpha\left(-\frac{e^{\beta Q}}{Q^2}+\frac{\beta\, e^{\beta Q}}{Q}\right)+\frac{F}{2}=0$$

If $\beta=0$ Then $Q^*=\sqrt{\frac{2A\alpha}{F}}$

Which is same as classical EOQ formula

Also $\frac{d^2C(Q)}{dQ^2}=\frac{A\alpha\, e^{\beta Q}}{Q^3}\left[2-2Q\beta+Q^2\beta^2\right]>0$ $[\text{for } (Q\neq 0)]$

Sub Case (iii) When $D=\alpha+\beta\, a^{Q}$

We have $C(Q)=\frac{A\alpha}{Q}+\frac{A\beta\, a^{Q}}{Q}+\frac{HW}{2}+\frac{F(Q-W)}{2}$ .... (4)

For optimality, we have

$$\frac{dC(Q)}{dQ} = 0 \Rightarrow -\frac{A\alpha}{Q^2} + \frac{A\beta\, a^Q}{Q^2}(Q\log a - 1) + \frac{F}{2} = 0$$

$$\Rightarrow \quad Q^* = \sqrt{\frac{2A\alpha}{F}} \qquad \text{(if } \beta = 0)$$

Also $$\frac{d^2C(Q)}{dQ^2} = \frac{2A\alpha}{Q^3} + \frac{2A\beta\, a^Q}{Q^3} + \frac{A\beta\, a^Q \log a}{Q^2}[Q\log a - 2] > 0 \qquad \text{.... (5)}$$

Sub Case (iv) $D = \alpha + \dfrac{\beta}{Q}$

We have $$C(Q) = \frac{A\alpha}{Q} + \frac{A\beta}{Q^2} + \frac{HW}{2} + \frac{F(Q-W)}{2} \qquad \text{.... (6)}$$

For optimal value of Q , which minimize (6) Solve $\dfrac{dC(Q)}{dQ} = 0$

$$\frac{dC(Q)}{dQ} = -\frac{A\alpha}{Q^2} - \frac{2A\beta}{Q^3} + \frac{F}{2} = 0$$

If $\beta = 0$ Then $Q^* = \sqrt{\dfrac{2A\alpha}{F}}$

Which same as classical EOQ formula

$$\frac{d^2C(Q)}{dQ^2} = \frac{2A\alpha}{Q^3} + \frac{6A\beta}{Q^4} > 0 \qquad \text{.... (7)}$$

Sub Case (v) When $D = \alpha + \beta Q^{\lambda}$

We have $$C(Q) = \frac{A\alpha}{Q} + \frac{A\beta Q^{\lambda-1}}{1} + \frac{HW}{2} + \frac{F(Q-W)}{2}$$

For optimal value of Q, put $\dfrac{dC(Q)}{dQ} = 0$

$$\frac{dC(Q)}{dQ} = -\frac{A\alpha}{Q^2} - \frac{(1-\lambda)A\beta}{Q^{2-\lambda}} + \frac{F}{2}$$

If $\beta = 0$ Then $Q^* = \sqrt{\dfrac{2A\alpha}{F}}$

Also $\dfrac{d^2C(Q)}{dQ^2} > 0$

Case (B) (During consumption)

From figure (1), the inventory unit is RW is given by

$$S_1 = T_k[S + (S-K) + (S-2K) + \ldots\ldots\ldots\ldots + S - \overline{n-1}\,k] = T_k\frac{S(n+1)}{2}$$

Where $T_k = \dfrac{K}{D}$, also $S = Q - W$, holding cost in RW is F, There for we have

$$\text{F.S}\, S_1 = \frac{F.K(n+1)(Q-W)}{2} \quad \text{.... (8)}$$

The transportation cost in shipment is given by

$$C_t = \begin{cases} n[x + y(K-p)] & \text{When K > p} \\ nx & \text{When K} \le \text{p} \end{cases}$$

Where n = S/k

When K units are drawn from *RW* in each shipment more are carried in *OW* for a period of T_k and hence holding cost of these items is given by $KHT_k/2$.

Since there are '*n'* such shipment and taking the initial K units into account, we have the total holding cost of these items

$= \{ (n+1) HKT_k \}/2 = \{(n+1) HK^2 \}/2D$

A quantity of $(W - K)$ units is kept unused in *OW* for a period of $T_{w-k} = (n+1)\, T_k$ and during usage the average inventory in *OW* is given by $(W - K)/2$ units and for a period $(T\text{-}T_{w-k})$. Hence inventory holding cost in *OW* of these items is given by

$H \{ K (W\text{-}K) (n+1)/D + (W\text{-}K)^2 / 2D \}$

Taking ordering cost A into consideration . Hence the total cost of inventory for the system using (2) to (7) is given by

$$C = A + (n+1) \left[\frac{FK(Q-W)}{2D} + \frac{K^2 H}{2D} + \frac{HK(W-K)}{2D} + C_t + \frac{(W-K)^2 H}{2D} \right]$$

Average inventory cost is given by

$C(Q, K) = C/T$

Also we have $T = Q/D$, $S = (Q - W)$, $n = S/K$ and $\phi = F - H$,

Therefore, average cost becomes

$$C(Q,K) = \frac{AD}{Q} + \frac{FQ}{2} + \frac{W^2\phi}{2Q} - \frac{KW\phi}{2Q} + \frac{K\phi}{2} - W\phi + \frac{D}{K}(1 - \frac{W}{Q})\{x + y(K-p)\}$$

Sub Case (i) When $D = \alpha\, Q^{\beta}$

Substituting this value of D in Equation (8) we get

$$C(Q,K) = \frac{A\alpha\, Q^{\beta}}{Q} + \frac{FQ}{2} + \frac{W^2\phi}{2Q} - \frac{KW\phi}{2Q} + \frac{K\phi}{2} - W\phi + \frac{\alpha\, Q^{\beta}}{K}(1 - \frac{W}{Q})\{x + y(K-p)\} \quad \text{.... (9)}$$

For optimum value of Q and K, substituting $C(Q,K)/\ Q = 0$ and $C(Q,K)/\ K = 0$

$C(Q,K)/\ Q = 0$ implies

$$2A\alpha(\beta-1)Q^{\beta} + FQ^2 - W\phi(W-K) + 2\alpha\beta Q^{\beta+1}\{(x-yp)/K + y\} - 2\alpha W(\beta-1)Q^{\beta}\{(x-yp)/K + y\} = 0$$

which can not be solved for Q analytically and can be solved with the help of Newton Raphson method.

Also if $\beta = 0$

$$Q^{*}=\left[\frac{2A\alpha}{F}+\frac{W\phi(W-K)}{F}-\frac{2W\alpha}{F}\{(x-yp)/K+y\}\right]^{1/2} \quad .... \ (10)$$

Also *C (Q,K) / Q = 0* implies

$$\left(1-\frac{W}{Q}\right)\frac{\phi}{2}=\alpha\left(1-\frac{W}{Q}\right)(x-yp)/K^{2}$$

$$\Rightarrow K^{*}=\left[\frac{2\alpha(x-yp)}{\phi}\right]^{1/2} \quad .... \ (11)$$

Also $\frac{\partial^{2}C(Q,K)}{\partial Q^{2}}>0$ $\qquad$ $\frac{\partial^{2}C(Q,K)}{\partial K^{2}}>0$

Sub Case 2

When $D=\alpha e^{\beta Q}$

Substituting this value of 'D' in equation (9), we get

$$C(Q,K)=\frac{A\alpha\, e^{\beta Q}}{Q}+\frac{FQ}{2}+\frac{W^{2}\phi}{2Q}-\frac{KW\phi}{2Q}+\frac{K\phi}{2}-W\phi+\frac{\alpha\, e^{\beta Q}}{K}(1-\frac{W}{Q})\{x+y(K-p)\} \quad .... \ (12)$$

For optimality of Q and K, Solve *C (Q,K) / Q = 0* and *C (Q,K) / K=0*

C (Q,K) / Q = 0 implies

$$\frac{A\alpha\, e^{\beta Q}}{Q^{2}}(\beta Q-1)+\frac{F}{2}-\frac{W\phi(W-K)}{2Q^{2}}+\alpha\, e^{\beta Q}\{(x-yp)/K+y\}\left[\frac{W}{Q^{2}}+\left(1-\frac{W}{Q}\right)\beta\right]=0 \quad .... \ (13)$$

If $\beta=0$, We get $Q^{*}=\left[\frac{2A\alpha}{F}+\frac{W\phi(W-K)}{F}-\frac{2W\alpha}{F}\{(x-yp)/K+y\}\right]^{1/2}$ .... (14)

which is the same equation of Deva (2).

C (Q,K) / K=0

$$\Rightarrow K^{*}=\left[\frac{2\alpha(x-yp)}{\phi}\right]^{1/2} \quad .... \ (15)$$

Sub Case 3

When $D=\alpha+\beta a^{Q}$

Using this value of 'D' in equation (9), we get

$$C(Q,K)=\frac{A\alpha}{Q}+\frac{A\beta a^{Q}}{Q}+\frac{FQ}{2}+\frac{W^{2}\phi}{2Q}(W-K)+\phi\left(\frac{K}{2}-W\right)+(\alpha+\beta a^{Q})(1-\frac{W}{Q})\{(x-yp)/K+y\} \quad .... \ (16)$$

For optimality of Q, we have $\frac{\partial C(Q,K)}{\partial Q}=0$

$$\Rightarrow -\frac{A\alpha}{Q^2}+\frac{A\beta\, a^Q}{Q^2}(Q\log a-1)+\frac{F}{2}-\frac{W\phi(W-K)}{2Q^2}+\{(x-yp)/K+y\}\left\{\begin{matrix}(\alpha+\beta\, a^Q)\dfrac{W}{Q^2}\\ +\left(1-\dfrac{W}{Q}\right)(\beta\, a^Q\log a)\end{matrix}\right\}=0 \quad \text{.... (17)}$$

If $\beta=0$, We get

$$Q^*=\left[\frac{2A\alpha}{F}+\frac{W\phi(W-K)}{F}-\frac{2W\alpha}{F}\{(x-yp)/K+y\}\right]^{1/2} \quad \text{.... (18)}$$

$\dfrac{\partial C(Q,K)}{\partial K}=0$, this implies

$$-\frac{W\phi}{2Q}+\frac{\phi}{2}-(\alpha+\beta\, a^Q)\left(1-\frac{W}{Q}\right)(x-yp))\frac{1}{K^2}=0$$

If $\beta=0$, then

$$\Rightarrow K^*=\left[\frac{2\alpha(x-yp)}{\phi}\right]^{1/2} \quad \text{.... (19)}$$

<u>Sub Case 4</u>

When $D=\alpha+\beta/Q$

Substituting this value of 'D' , in equation (8), we get

$$C(Q,K)=\frac{A\alpha}{Q}+\frac{A\beta}{Q^2}+\frac{FQ}{2}+\frac{W^2\phi}{2Q}(W-K)+\phi\left(\frac{K}{2}-W\right)+(\alpha+\beta/Q)(1-\frac{W}{Q})\{(x-yp)/K+y\} \quad \text{.... (20)}$$

For which the optimal condition is

$$\frac{\partial C(Q,K)}{\partial Q}=0 \quad \text{and} \quad \frac{\partial C(Q,K)}{\partial K}=0$$

Now $$\frac{\partial C(Q,K)}{\partial Q}=-\frac{A\alpha}{Q^2}-\frac{2A\beta}{Q^3}+\frac{F}{2}-\frac{W^2\phi(W-K)}{2Q^2}+\left\{\frac{(x-yp)/K+y}{Q^2}\right\}\left\{W\left(\alpha+\frac{\beta}{Q}\right)-\left(1-\frac{W}{Q}\right)\beta\right\}=0 \quad \text{.... (21)}$$

Which can be solved by N.R. Method & if $\beta=0$, this gives the following equation.

$$Q^*=\left[\frac{2A\alpha}{F}+\frac{W\phi(W-K)}{F}-\frac{2W\alpha}{F}\{(x-yp)/K+y\}\right]^{1/2} \quad \text{.... (22)}$$

Also $\dfrac{\partial C(Q,K)}{\partial K}=0$ gives

$$\Rightarrow K^*=\left[\frac{2\alpha(x-yp)}{\phi}\right]^{1/2}$$

Also $\dfrac{\partial^2 C(Q,K)}{\partial Q^2}>0$ and $\dfrac{\partial^2 C(Q,K)}{\partial K^2}>0$

NUMERICAL EXAMPLE

To illustrate the developed model, a hypothetical example is considered. For a given inventory system, let

H=2, F=4, A_1=80, A_2=20, W=150, x=2, y= 0.10, $\alpha = 2000,\ \beta = 0\ ,\ p = 30$

We have $\phi = F - H = 2, \quad A = A_1 + A_2 = 100$

$$\Rightarrow K^* = \left[\frac{2\alpha(x - yp)}{\phi}\right]^{1/2}$$

$$= \left[\frac{2x2000(2 - 0.05x30)}{2}\right]^{1/2}$$

= 31.62

$$Q^* = \left[\frac{2A\alpha}{F} + \frac{W\phi(W - K)}{F} - \frac{2W\alpha}{F}\{(x - yp)/K + y\}\right]^{1/2}$$

$$= \left[\frac{2x100x2000}{4} + \frac{(150^2)x2}{4} - \frac{2x150}{4}\sqrt{2x2000(2 - 0.05x30)} - \frac{2x150x2000x0.05}{4}\right]^{1/2}$$

= 314.6

REFERENCES

[1] BHUNIA, A.K. AND MAITI. M(1994), A two warehouse inventory model for a linear trend in demand - ***Opsearch,*** 31(4), 318-329.

[2] DAVE, UPENDRA (1988), On the *EOQ* model with two levels of storage, ***Opsearch,*** 25(3), 190-196.

[3] GOSWAMI AND CHAUDHURI, K.S. (1992), An economic order quantity model for items with two levels of storage for a linear trend in demand. ***J.Opl.Res.Soc,*** 43(2), 157-167.

[4] GUPTA, R. AND P. Vrat (1986). Inventory model for stock dependent consumption rate-***Opsearch,*** 23(1), 19-24.

[5] HARTLEY, RONALD. V, (1976) Operations Research-A Managerial Emphasis, ***Good Year Publishing Company, California.***

[6] MANDAL, B.N. AND PHAUJDAR, S (1989) An inventory model for deteriorating items and stock dependent consumption rate. ***J.Opl.Res. Soc.*** 40, 483-488.

[7] MURDESHWAR, T.M. AND SATHE, Y.S. (1985) Some aspects of lot size models with two levels of storage-***Opsearch,*** 22(4), 255-262.

[8] J.MENDUS AND ANTONY, D.K.(1998). A two warehouse inventory model for stock dependent consumption rate-***Acta Ciencia Indica.*** 24(M) (1), 1-6.

[9] NADDOR, E(1966) Inventory Systems,***John Wiley, New York.***

[10] NARESH KUMAR AND SHARMA, ANIL KUMAR, On deterministic production inventory model for deteriorating items with an exponential declining demand Presented at 11th Annual Conference ***R.G.P*** held at S.G.N.Khalsa College, Sri Ganga Nagar (Dated 21, 22 Nov. 1999)

[11] SARMA, K.V.S. (1983). A deterministic inventory model with two levels of storage and an optimum release rule, ***Opsearch,*** 20(3), 175-180.

[12] SARMA, K.V.S. (1984) On K-release rule for an inventory system with two warehouses-***Opsearch***, 21 (1), 38-40.

[13] SARMA, K.V.S. (1990) A note on the EOQ model with two levels of storage-***Opsearch,*** 27(4), 269-272.

[14] SU, C.T., TONG. L.I. AND LIAO, H.C. (1996) An inventory model under inflation for stock dependent consumption rate and exponential decay-***Opsearch***, 33, 71-82.

[15] Anil Kumar Sharma & Manoj Kumar [2012] "A Non Linear Price Dependent Inventory Model for Non Instantaneous Deteriorating Items with Time Varying Holding Cost Aryabhatta J. of Maths. & Info. Vol. 4 (1) pp 133-138.

[16] Hari Kishan, Mega & Deepshikha [2013] "Inventory Model with Variable Demand, Shortages and Deterioration" Aryabhatta J. of Maths. & Info. Vol. 5 (1) pp 1-10.