Cosmological Models in *f*(R,T) Theory of Gravitation

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Abstract: Tilted cosmological models in $f(R,T)$ theory of gravitation are investigated without taking any relation between density ρ and pressure p .We have solved the field equations by considering $C_{2323}=0$ & $\beta = k\alpha$, where k is constant. The tiltedness is also considered. Some geometric aspects of the model are discussed.

Key Words: Tilted models, conformally flat, $f(R,T)$ theory.

1. Introduction:

A spatially homogeneous cosmology is said to be tilted if the fluid velocity vector is not orthogonal to the group orbits, otherwise the model is said to be non-tilted. The tilted models are more complicated than those of non-tilted one. The general dynamics of tilted cosmological models are investigated by King and Ellis [1]; Ellis and King [2]; Collins and Ellis [3]. Dunn and Tupper [4] have obtained tilted Bianchi type-I cosmological model for perfect fluid. Lorentz [5] has studied tilted electromagnetic Bianchi type-I cosmological model in General Relativity. Bianchi type-I cosmological model with heat flux in General Relativity has examined by Mukherjee [6].Tilted Bianchi type V cosmological models in the scale- covariant theory developed by Beesham [7]. Hewitt et al.[8,9], Horwood et al [10], Apostolopoulos [11] presented different aspects of tilted cosmological models. Coley and Hervik [12] have constructed Bianchi cosmologies a Tale of two tilted fluids. Tilted Bianchi type-I dust fluid discussed by Bali and Sharma [13]. Bali and Meena [14] have evaluated tilted cosmological models filled with disordered radiation in General Relativity. Tilted Bianchi type I cosmological models filled with disordered radiation in general relativity examined by Pradhan and Rai [15]. Bianchi type-I models with two tilted fluids calculated by Sandin and Uggla [16]. Pawar et al. [17] have studied bulk viscous fluid with plane symmetric string dust magnetized cosmological model in general relativity. Sandin [18] has constructed tilted two fluid Bianchi type-I models. Verma [19] has obtained Qualitative analysis of two fluids FRW cosmological models. Pawar et al. [20] have investigated tilted plane symmetric cosmological models with heat conduction and disordered radiation. Tilted plane symmetric bulk viscous cosmological model with varying Λ–Term have presented by Bhaware et al. [21]. Sahu and Kumar [22] have discussed tilted Bianchi type-I cosmological model in Lyra Geometry. Tilted Bianchi type-I barotropic cosmological model and some Bianchi type-I magnetized bulk viscous fluid tilted cosmological models obtained by Bagora and purohit [23, 24]. Recently two fluids tilted cosmological model in General Relativity and tilted plane symmetric magnetized cosmological models developed by Pawar and Dagwal [25, 26].

Bali and Meena [27] have derived conformally flat tilted Bianchi type V cosmological models filled with perfect fluid and conduction. Tilted Bianchi type I cosmological model for perfect fluid distribution in the presence of magnetic field discussed by Bali and Sharma [28]. Pradhan and Rai [29] have evaluated conformally flat tilted Bianchi type V cosmological models filled with disordered radiation in the presence of a bulk viscous fluid and heat flow. Pawar and Dagwal [30-32] have investigated Conformally flat tilted cosmological models, tilted Kantowski-Sachs cosmological models with disordered radiation in scalar tensor theory of gravitation proposed by Saez and Ballester and tilted Kasner-type cosmological model in B-D theory.

Noteworthy amongst them are $f(R)$ theory of gravity and a general scheme for the modified $f(R)$ gravity reconstruction from any realistic FRW cosmology developed by Nojiri and Odintsov [33, 34]. Carroll et al. [35] have derived the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. Shamir [36] has developed a physically viable $f(R)$ gravity model, which showed the unification of early time inflation and late time acceleration. FRW models in $f(R)$ gravity discussed by Paul et al. [37].Some new exact static spherically symmetric interior solutions of metric $f(R)$ gravitational theories evaluated by Ali Shojai and Fatimah Shojai [38]. `

Recently, many researchers have formulated several aspects of $f(R,T)$ modified theory of gravity. In order to describe the early universe, the $f(R,T)$ theory of gravity is considered as fundamental theory of gravitational theories. The $f(R,T)$ theory of gravity is the generalization of $f(R)$ and $f(T)$ theories of gravity. Harko et al [39] have proposed $f(R,T)$ modified theory of gravity, where *T* denotes the trace of the energy momentum tensor and *R* is the curvature scalar. Kaluza–Klein cosmological model in $f(R,T)$ gravity with a negative constant deceleration parameter and a spatially homogeneous Bianchi type-III cosmological model in the presence of a perfect fluid source in $f(R,T)$

theory with negative constant deceleration parameter are calculated by Reddy et al. [40,41]. Adhav [42] has examined Bianchi type-I cosmological model in $f(R,T)$ gravity. Bianchi type-VIo universes and perfect fluid Einstein-Rosen in $f(R,T)$ gravity are derived by Rao and Neelima [43, 44]. Anisotropic cosmological models in $f(R,T)$ theory of gravitation studied by Shri Ram et al.[45].LRS Bianchi type-II Universe in $f(R,T)$ theory of gravity evaluated by Reddy and Kumar [46]. Bainchi type IX two fluids cosmological models in General Relativity presented by Pawar and Dagwal [47]. Naidu et al. [48] have derived FRW viscous fluid cosmological model in $f(R,T)$ gravity. A new class of Bianchi cosmological models in $f(R,T)$ gravity is obtained by Chaubey and Shukla [49]. Pawar et al. [50,51] have investigated Axially Bianchi type-I Mesonic cosmological models with two fluid sources in Lyra Geometry and LRS Bianchi type I cosmological model in the framework of the $f(R,T)$ theory of gravity in the presence of a perfect fluid.

2. Field Equation:

We consider the metric in the form

$$ds^2 = -dt^2 + e^{2\alpha}dx^2 + e^{2\beta}\left(dy^2 + dz^2\right), \tag{1}$$

where $\alpha \; and \; \beta$ are functions of *t* alone.

The Einstein's field equation in $f\left(R,T\right)$ theory of gravity for the function given by

$$f(R,T) = R + 2f(T), \tag{2}$$

as

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} + 2f'T_{ij} + \left[2p\, f'(T) + f\left(T\right)\right]g_{ij}, \tag{3}$$

The energy momentum tensor for perfect fluids given by

$$T_{ij} = \left(p+\rho\right)u_i u_j - pg_{ij} + q_i u_j + q_j u_i \;, \tag{4}$$

together with $g^{ij} u_i u_j = -1$, (5)

$$q_i q^i > 0 \quad , \quad q_i u^j = 0 \;, \tag{6}$$

where p is the pressure, ρ is the energy density, q_i is the heat conduction vector orthogonal to u_i. The fluid vector u_i has the components $\left(e^{\alpha}\sinh\lambda, 0, 0, \cosh\lambda\right)$ satisfying Equation (5) and λ is the tilt angle.

The prime denotes differentiation with respect to the argument.

We choose the function $f(T)$ as the trace of the stress energy tensor of the matter so that

$$f(T) = vT \ , \tag{7}$$

where v is an arbitrary constant.

The field equation (3) for metric (1) reduce to

$$2\beta_{44} + 3\beta_4^2 = (1+2v)\left[(\rho+p)\sinh^2\lambda - p + 2q_1\frac{\sinh\lambda}{e^{\alpha}}\right] - (3p+\rho)v \ , \tag{8}$$

$$\alpha_{44} + \beta_{44} + \alpha_4\beta_4 + \alpha_4^2 + \beta_4^2 = -(1+2v)p - (3p+\rho)v \ , \tag{9}$$

$$\beta_4^2 + 2\alpha_4\beta_4 = -(1+2v)\left[(\rho+p)\cosh^2\lambda + p + 2q_1\frac{\sinh\lambda}{e^{\alpha}}\right] - (3p+\rho)v \ , \tag{10}$$

$$(1+2v)\left[(\rho+p)e^{\alpha}\sinh\lambda\cosh\lambda + q_1\cosh\lambda + q_1\frac{\sinh^2\lambda}{\cosh\lambda}\right] = 0 \, . \tag{11}$$

Here the index 4 after a field variable denotes the differentiation with respect to time t.

We consider that the space-time is conformally flat, which gives

$$C_{2323} = \frac{e^{4\beta}}{3}\left[\alpha_{44} + \alpha_4^2 - \beta_{44} - \beta_4\alpha_4\right]. \tag{12}$$

The shear scalar is proportional to the expansion scalar which envisages a linear relationship between α and β

$$\beta = k\alpha \ , \tag{13}$$

where k is constant.

Solving equation (12) and (13) we get

$$\alpha = \log c_2\left(t-c_1\right) \; and \quad \beta = \log c_3\left(t-c_1\right)^k . \tag{14}$$

Equation (14) can be rewritten as

$$e^{\alpha} = c_2\left(t-c_1\right) \; and \quad e^{\beta} = c_3\left(t-c_1\right)^k . \tag{15}$$

Hence the line element (1) reduced to

$$ds^2 = -dt^2 + \left[c_2\left(t-c_1\right)\right]^2 dx^2 + \left[c_3\left(t-c_1\right)^k\right]^2\left(dy^2+dz^2\right) , \tag{16}$$

where c_1, c_2 & c_3 are integration constant.

Some Physical and Geometrical Property

Solving equation (8), (9) and (10) we get

$$\left[1+k-\frac{2vk}{(1+4v)}\right]\alpha_{44} + \left[1+k+k^2-\frac{2vk(2k+1)}{(1+4v)}\right]\alpha_4^2 = -(1+2v)\,p . \tag{17}$$

From equation (17) we get

$$p = -\frac{k^2}{(1+2v)(1+4v)\left(t-c_1\right)^2} . \tag{18}$$

Using equation (8), (10) and (18) we get

$$\rho = -\frac{(1+8v)k^2}{(1+4v)(1+2v)(t-c_1)^2} . \tag{19}$$

For large value of t, the pressure p and density ρ are vanishing but at $t = c_1$, the pressure p and density ρ are infinite.The density ρ is zero at $v = -\frac{1}{8}$ .When $v = -\frac{1}{2}$ *or* $v = -\frac{1}{4}$; the pressure p and density ρ are infinite. At $k = 0$, the pressure p and density ρ are zero therefore the model is empty space.

The tilt angle λ , flow vectors u_i and heat conduction vectors q_i for the model (16) are given by

$$\cosh \lambda = \sqrt{\frac{2}{(2-k)}} \quad , \tag{20}$$

$$\sinh \lambda = \sqrt{\frac{k}{(2-k)}} \quad , \tag{21}$$

$$u_1 = c_2 \left[\frac{k}{(2-k)} \right]^{1/2} (t - c_1) \ , \tag{22}$$

$$u_4 = \left[\frac{2}{(2-k)} \right]^{1/2} , \tag{23}$$

$$q_1 = \frac{4k^2 c_2}{(1+2v)\left[k(2-k)\right]^{1/2}(t-c_1)} \quad , \tag{24}$$

$$q_4 = \frac{4k^2}{(1+2v)\left[2(2-k)\right]^{1/2}(t-c_1)^2} \ . \tag{25}$$

Tiled angle λ and flow vectors u_4 are constant. When $k = 2$, the tilt angle λ, the flow vectors u_1, u_4 and heat conduction vectors q_1, q_4 are infinite. For large value of t, the flow vector u_1 is infinite and heat conduction vectors q_1, q_4 are vanishing but at $t = c_1$, the flow vector u_1 is zero and heat conduction vectors q_1, q_4 are infinite. At $k = 0$, the tilt angle $\sinh \lambda$, the flow vectors u_1 and heat conduction vectors q_1, q_4 are zero. The flow vectors u_1 and heat conduction vectors q_1 vanish for $c_2 = 0$.

The scalar expansion, shear scalar and rotation tensor are

$$\theta = (1+2k)\left(\frac{2}{2-k}\right)^{1/2}\frac{1}{(c_1-t)}, \quad (26)$$

$$\sigma^2 = \frac{2(1-k)^2}{3(t-c_1)^2}, \quad (27)$$

$$\omega_{14} = c_2\left(\frac{1-k}{2-k}\right)\sqrt{\frac{k}{2-k}}. \quad (28)$$

When $t=\infty$, the scalar expansion and shear scalar are vanishing but at $t=c_1$, the scalar expansion and shear scalar are infinite. For $k=2$, the scalar expansion and rotation tensor are infinite. The shear scalar and rotation tensor are zero at $k=1$ and the scalar expansion is vanish for $k=-\frac{1}{2}$. The models are nonrotating for $k=1$ or $c_2=0$, nonexpanding at $k=-\frac{1}{2}$ and nonshearing when $k=1$.

The spatial volume and the rate of expansion H_i in the direction of *x*, *y*, *z*-axis are

$$V = c_2 c_3^2 (t-c_1)^{1+2k},$$

$$H_1 = \frac{2}{t-c_1}, \; H_2 = H_3 = \frac{2k}{t-c_1}. \quad (29)$$

The spatial volume is constant for $k=-\frac{1}{2}$ and vanish at $t=c_1$ .When $t=c_1$, the rate of expansion H_i in the direction of *x*, *y*, *z*-axis are infinite. For $k=0$, the rate of expansion H_i in the direction of *y*, *z*-axis are zero. At $t=\infty$, the spatial volume is infinite and the rate of expansion H_i in the direction of *x*, *y*, *z*-axis is vanishing.

The density parameter and anisotropy parameter as

$$\Omega = -\frac{(1+8v)k^2}{12(1+4v)(1+2v)(1+2k)^2},$$

$$\Delta = \frac{k^2+2}{3(1+2k)^2}. \quad (30)$$

The density parameter and anisotropy parameter are constant. When $v=-\frac{1}{2}$ or $v=-\frac{1}{4}$, the density parameter is infinite but at $v=-1/8$ or $k=0$, the density parameter is vanish. The density parameter and anisotropy parameter are infinite for $k=-\frac{1}{2}$.

Conclusion

We have presented tilted cosmological models in $f(R,T)$ theory of gravitation without taking any relation between density ρ and pressure p .The model are expanding, shearing and rotating universe. The model starts with big bang at $t=c_1$ and the expansion in the model decreases as time increases and the expansion in the model stop at $t=\infty$.There is a singularity in the model at $t=c_1$ This singularity is pan cake type (MacCallum [52]). For large value of t, the pressure p and density ρ are vanishing but at $t=c_1$, the pressure p and density ρ are infinite. The density ρ is zero at $v=-\frac{1}{8}$. When $v=-\frac{1}{2}$ or $v=-\frac{1}{4}$, the pressure p and density ρ are infinite. At $k=0$, the pressure p and density ρ are zero therefore the model is empty space. Tiled angle λ and flow vectors u_4 are constant. When $k=2$, the tilt angle λ, the flow vectors u_1, u_4 and heat conduction vectors q_1, q_4 are infinite. For large value of t, the flow vector u_1 is infinite and heat conduction vectors q_1, q_4 are vanishing but at $t=c_1$, the flow vector u_1 is zero and heat conduction vectors q_1, q_4 are infinite. At $k=0$, the tilt angle $\sinh\lambda$, the flow vectors u_1 and heat conduction vectors q_1, q_4 are zero. The flow vectors u_1 and heat conduction vectors q_1 vanish for $c_2=0$. When $t=\infty$, the scalar expansion and shear scalar are vanishing but at $t=c_1$, the scalar expansion and shear scalar are infinite. For $k=2$, the scalar expansion and rotation tensor are infinite. The shear scalar and rotation tensor are zero at $k=1$ and the scalar expansion is vanish for $k=-\frac{1}{2}$. The models are nonrotating for $k=1$ or $c_2=0$, nonexpanding at

$k=-\frac{1}{2}$ and nonshearing when $k=1$. The spatial volume is constant for $k=-\frac{1}{2}$ and vanish at $t=c_1$. When $t=c_1$, the rate of expansion H_i in the direction of *x*, *y*, *z*-axis is infinite. For $k=0$, the rate of expansion H_i in the direction of *y*, *z*-axis are zero. At $t=\infty$, the spatial volume is infinite and the rate of expansion H_i in the direction of *x*, *y*, *z*-axis is vanishing. The density parameter and anisotropy parameter are constant. When $v=-\frac{1}{2} or\, v=-\frac{1}{4}$, the density parameter is infinite but at $v=-\frac{1}{8}$ *or* $k=0$, the density parameter is vanish. The density parameter and anisotropy parameter are infinite for $k=-\frac{1}{2}$.

Since $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right)^2 \neq 0$ the model does not approach isotropy for large value of *t*.

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