"BAYESIAN DECISION THEORY IN MARKETING RESEARCH" - AN ANALYSIS OF BAYES THEOREM IN MARKETING DECISION MAKING

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Abstract: This article is an attempt to explain the rudiments of the Bayesian approach and its potential applicability to marketing decisions. First the major aspects of the theory will be discussed in terms of simple illustrations. Second, an illustrative decision problem associated with the marketing area will be outlined and the use of Bayesian theory in its resolution described. Finally the advantages and limitations of the Bayesian approach in dealing with the complexities of real-world problems will be discussed and some speculative comments offered regarding the future use of these techniques in business. A list of source readings is provided at the end of this article for the reader who wishes to explore the subject in more detail.

Keywords:*Payoffs, Probability, Bayesian approach, Prior analysis*, *Preposterous analysis, Posterior Analysis*

Introduction

Decision making means selection of one course of action from various alternative courses of action, one of which is judged to be the best under given circumstances according to the decision maker. The decision maker may be an individual or group or a company etc. Statistical decision theory consists of a large number of quantitative techniques which helps in analyzing a decision situation and enable us to arrive at a strategy which is the best under given circumstances of the case.

Achieving successful innovation requires a deep understanding of what is meaningful to customers and the marketplace. By approaching research in a way that actually improves creativity rather than hinders it, companies can successfully produce more effective products and brands. Market research methods and tools used for product development vary according to the product type, the extent of incremental change from other products, the investment and risk factors, and the costs of seeding the new product in the market place. For measuring the customer perception of pricing and value, the quantitative research tools are preferred over qualitative tools as they provide more precise results. The quantitative approach is generally used to study the pricing research problems like customers' willingness to pay, price sensitivity, perceived value and the impact of perceived value of different customer groups.

The advertising research can help describe shifts in the marketplace and effectiveness of promotions. Advertising research is conducted to improve the efficacy of advertising. It may focus on a specific ad or campaign, or may be directed at a more general understanding of how advertising works or how consumers use the information in advertising. It can entail a variety of research approaches, including psychological, sociological, economic and other perspectives.

Motivational research is a systematic analysis of the motives behind customer decisions, used especially by advertisers and marketers to assess attitudes towards products and services. It seeks to thoroughly understand the soft unspoken motives and beliefs held by customers and prospects in regard to a brand or product category. Distribution research is conducted for tracking changes in channel preferences among consumers and businesses and providing tools to assist with local market planning. This article takes the point of view how Bayesian decision theory helps the marketing manager to take effective decisions during product, pricing, advertising, motivational and distribution researches.

During recent years some interesting techniques have been developed for coping systematically with decision making under uncertainty, and hence with decisions required in marketing.

Many theories of making rational choices under uncertainty have been formulated. Scholars from a variety of disciplines, e.g. Mathematics, Statistics, Economics and Psychology, have addressed themselves to the problem. And it is not surprising that the term rationality itself hardly enjoys universal acceptance among decision theorists.

Rather than try to cover all the proposals dealing with the subject, this article emphasizes only one. But this particular approach, known as Bayesian decision theory, is unusually complete and relevant to the real-world problems of the marketing executives. The origins of the approach go back to the work of 18th century English clergyman, the Reverend Thomas Bayes, who developed a theorem for using observed evidence to modify prior judgments concerning different possible “causes” of the evidence. The modern form of Bayesian analysis has been developed during the last decade and reflects the combined effects of many distinguished contributors.

Literature Review

Decisions are made based upon the information available about the occurrences of events as well as the decision situations. The types of decision-making environments are

Decision making under certainty

In this type the decision maker has the complete knowledge of consequence of every decision choice with certainty.

Decision making under risk

In this situation the decision maker has less than the knowledge under certainty about the consequences of every decision choice. In other words, there is more than one state of nature. It is assumed that the decision maker has the knowledge about the probability with which each state of nature will occur.

Decision making under uncertainty

In this case the decision maker is unable to specify the probabilities with which the various states of nature will occur. Thus decisions under uncertainty are taken with even less information than decisions under risk. Expected Monetary Value (EMV) Criterion

The expected monetary value (EMV) is the expected value of the conditional payoff for a given course of action. It is obtained for each action by considering various act event combinations.

Expected Opportunity Loss (EOL) Criterion

An alternative approach to maximizing monetary value (EMV) is to minimize expected opportunity loss (EOL) .The expected opportunity loss or expected value of regrets represents the amount by which maximum possible profit will be reduced under various possible stock auctions.

Expected Value of Perfect Information (EVPI)

After making prior analysis the decision maker must decide either to collect additional information regarding the states of nature or to take the action as suggested by the prior analysis. Prior distribution is not always a perfect predictor regarding the states of nature.

This is more so in business decision problems. However, if somehow, the decision maker finds a perfect predictor, he would prefer action based on perfect predictor for it would enable him to maximize his profits or minimize his losses. The highest expected profits resulting in the presence of perfect predictor is called the expected payoffs of perfect information (EPPI). EPPI is often called the expected value of payoff under certainty. The perfect prediction reduces the opportunity losses due to uncertainty to zero. The difference between EPPI and EMV is called the expected value of perfect information (EVPI).It represents the maximum amount of money which a decision -maker could spend to obtain additional information regarding the states of nature. It may be noted that EVPI is always equal to the EOL of selecting the optimum action under uncertainty. The identity EMV + EOL = EPPI follows from the result EVPI = EOL and EVPI = EPPI - EMV.

Objective

The objective of this paper is to explore the possibility of applying Bayesian decision theory to marketing research.

Bayesian approach

The Bayesian approach has three types of analysis:

1. Prior analysis
2. Preposterous analysis
3. Posterior analysis.

Prior analysis means that the decision-maker uses the available information and his subjective estimates of probabilities as the basis for making business decisions. If the decision-maker wants to improve or to revise his prior probabilities, he needs a market survey in order to obtain the new information from his target market. However, in order to undertake such a market investigation, he must pay the survey expenses. The question is whether the survey cost is greater or less than the value of the additional information obtained. To make such a decision, he needs a preposterous analysis. Preposterous analysis employs Bayes' theorem to revise the decision maker's prior probability distribution, given certain facts about the ad-additional data. If and only if the value of the additional information exceeds the total survey cost, the market research is justified. After the market survey has been undertaken, the decision maker will use the revised probabilities as the basis for selecting the best strategy. This decision making process is termed posterior analysis.

Prior analysis

Mr. Mohan the Marketing Planning Manager of computer and Allied Co. Ltd. is given the assignment of recommending whether his firm should produce a new type of a pocket calculator. His boss feels that a chance of 60:40 exists that the calculator will be successful. The payoff table in lacs of rupees is as follows:

Strategy	N_1 (Success)	N_2 (Failure)
S_1: Produce	150	-100
S_2: Do not produce	0	0

Two market survey firms have submitted bids to do the field work of interviewing the target audience. The Alpha company is known to conduct highly reliable surveys (but at a rather high cost), whereas Beta company's reliability is somewhat lower but so is its cost. Pertinent conditional probabilities of survey results Z_1 (indicating N_1) and Z_2 (indicating N_2) are shown below:

Firm	Conditional Probabilities	Cost in Rs
Alpha Company	$P(Z_1/N_2)=P(Z_2/N_2)=0.9$ $P(Z_1/N_2)=P(Z_2/N_1)=0.1$	50000
Beta Company	$P(Z_1/N_1)=P(Z_2/N_2)=0.7$ $P(Z_1/N_2)=P(Z_2/N_1)=0.3$	30000

Mr. Mohan has the following options: conduct no surveys, buy Alpha survey or buy Beta survey.
We need to

(i) Evaluate the expected payoff of each option, and suggest which option Mr. Mohan should follow.
(ii) Evaluate the expected value of perfect information.

On the basis of available information, Mr. Mohan has to make a decision between the following:

1. To produce the new type of a pocket calculator.
2. Not to produce the new type of a pocket calculator.

Mr. Mohan estimates that there will be only two states of nature:

1. The market situation is a success.
2. The market situation is a failure.

$$\begin{matrix} & N1 & N2 \\ S1 & \\ S2 & \end{matrix} \begin{pmatrix} 150 & -100 \\ 0 & 0 \end{pmatrix} \overset{P(N_j)}{\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}} = \overset{EP}{\begin{pmatrix} 50 \\ 0 \end{pmatrix}} max$$

S_1: Strategy I: Produce the new type of pocket calculator.
S_2: Strategy II: Do not produce the new type of pocket calculator.
N_1: Success
N_2: Failure
$P(N_j)$: Prior Probability
EP: Expected payoff.

According to the expected payoff Mr. Mohan should select S_1, e.g. produce the new type of a pocket calculator. In fact, the expected payoff can also be transferred into an expected opportunity loss (EOL) in the form of regret matrix:

EOL = Max $A_j - A_{jj}$
Max A_j= maximum value in the j'th column
Ajj — payoff value for the cell in the i'th row of the j'th column (i.e. the original matrix)

$$\begin{matrix} & N1 & N2 \\ S1 & \\ S2 & \end{matrix} \begin{pmatrix} 0 & 100 \\ 150 & 0 \end{pmatrix} \overset{P(N_j)}{\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}} = \overset{EOL}{\begin{pmatrix} 40 \\ 900 \end{pmatrix}} min$$

In comparing these two EOL's Mr. Mohan should also select S_1, for its EOL is the lowest (40). This minimum EOL is the inherent cost of uncertainty itself. The only way to reduce the cost of uncertainty is to obtain additional information.

Table 1 Conditional Probabilities

$P(Z_1/N_1) = 0.9$
$P(Z_2/N_2) = 0.9$
$P(Z_1/N_2) = 0.1$
$P(Z_2/N_1) = 0.1$

Table 1: Conditional Probabilities for Alpha Company

Survey Result Z_i	State of Nature (N_j) N_1	N_2
Z_1	0.9	0.1
Z_2	0.1	0.9

Where Z_i(i=1, 2) = survey results

N_j (j=1, 2) = states of nature

Next, Mohan computes the join probability by using the following formula:

$$P(Z_i \cap N_j) = P(Z_i / N_j)\ P(N_j)$$
$$P(Z_1 \cap N_1) = P(Z_1 / N_1)\ P(N_1) = 0.9 \times 0.6 = 0.54$$
$$P(Z_1 \cap N_2) = P(Z_1 / N_2)\ P(N_2) = 0.1 \times 0.4 = 0.04$$
$$P(Z_2 \cap N_1) = P(Z_2 / N_1)\ P(N_1) = 0.1 \times 0.6 = 0.06$$
$$P(Z_2 \cap N_2) = P(Z_2 / N_2)\ P(N_2) = 0.9 \times 0.4 = 0.36$$

Table 2: Joint probabilities for Alpha Company

N_j / Z_i	N_1	N_2	$P(Z_i)$
Z_1	0.54	0.04	0.58
Z_2	0.06	0.36	0.42
$P(N_j)$	0.6	0.4	1

The marginal or unconditional probabilities of the market survey results $P(Z_i)$ are calculated by summing over columns, so that the total equals 1 as stated in the above table. Next, Mohan uses the Bayes' formula to revise the prior probabilities. By using the Bayes' formula the company could eliminate the necessity for gathering masses of data over a long period of time. Bayes' formula is stated as follows:

$$P(N_j / Z_i) = \frac{P(Z_i / N_j).P(N_j)}{P(Z_i)}$$

$$\text{Therefore} \quad P(N_1 / Z_1) = \frac{P(Z_1 / N_1).P(N_1)}{P(Z_1)} = \frac{0.9 \times 0.6}{0.58} = \frac{0.54}{0.58} = 0.93$$

$$P(N_1 / Z_2) = \frac{P(Z_2 / N_1).P(N_1)}{P(Z_2)} = \frac{0.1 \times 0.6}{0.42} = \frac{0.06}{0.42} = 0.14$$

$$P(N_2 / Z_1) = \frac{P(Z_1 / N_2).P(N_2)}{P(Z_1)} = \frac{0.1 \times 0.4}{0.54} = \frac{0.04}{0.54} = 0.07$$

$$P(N_2 / Z_2) = \frac{P(Z_2 / N_2).P(N_2)}{P(Z_2)} = \frac{0.9 \times 0.4}{0.42} = \frac{0.36}{0.42} = 0.86$$

Table 3: Revised probabilities for Alpha Company

Nj / Z_i	N_1	N_2
Z_1	0.93	0.07
Z_2	0.14	0.86

In order to enable the multiplication across the j states of nature, the above revised probabilities must be transposed as follows:

$$\begin{array}{c} \\ N1 \\ N2 \end{array} \begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{pmatrix} 0.93 & 0.14 \\ 0.07 & 0.86 \end{pmatrix} \end{array}$$

In the original matrix, the rows represent the results of the survey. When transposed, the rows represent the states of nature and the columns represent the results of a survey. By transposing the revised probabilities, the company obtains the conditional probability matrix. Multiplying this conditional probability matrix by the original payoff matrix, Mohan then obtains the expected payoff matrix with revised probabilities (EPRP):

$$\begin{array}{c} \\ S1 \\ S2 \end{array} \begin{array}{c} \begin{array}{cc} N1 & N2 \end{array} \\ \begin{pmatrix} 150 & -100 \\ 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{pmatrix} 0.93 & 0.14 \\ 0.07 & 0.86 \end{pmatrix} \end{array} = \begin{array}{c} EPRP \\ \begin{pmatrix} 132.5 & -75 \\ 0 & 0 \end{pmatrix} \end{array}$$

The expected payoff for strategy 1 is 132.5 and the result 1 of the market survey Z_2 and 0 is the expected payoff of strategy 2 (S_2) and the result 1(Z_1). Since there are four expected payoffs of two different results, Mohan selects the maximum expected payoff of each result and multiplies this by marginal probabilities $P(Z_i)$ to obtain the expected payoff of the market survey (EPMS) or the expected value of the survey information (EVSI):

$$\begin{pmatrix} 132.5 & 0 \end{pmatrix} \begin{pmatrix} 0.58 \\ 0.42 \end{pmatrix} = 76.85$$

By deduction the market survey costs Rs 50,000 from the expected payoff of the market survey, the expected net payoff of the market survey (ENPMS) is obtained.

ENPMS = EPMS - Survey cost = 76.85 - 0.5 = 76.35

The difference between ENPMS and EP_{max} is expected net gain of research (ENCR). 76.35 - 50 = 26.35

Since ENMPS is greater than EP_{max}, Mohan should make a terminal decision that the market survey for producing a new type of pocket calculator is worthwhile. If not, the market survey would not be undertaken for purely economic reasons. In this case the company would use its prior probabilities as the basis for decision making.

For Beta Company

$P(Z_1/N_1) = 0.7$
$P(Z_2/N_2) = 0.7$
$P(Z_1/N_2) = 0.3$
$P(Z_2/N_1) = 0.3$

Table 4 Conditional probabilities for Beta Company

Survey Result Z_i	State of Nature (N_j) N_1	N_2
Z_1	0.7	0.3
Z_2	0.3	0.7

$$P(Z_i \cap N_j) = P(Z_i/N_j)\ P(N_j)$$
$$P(Z_1 \cap N_1) = P(Z_1/N_1)\ P(N_1) = 0.7 \times 0.6 = 0.42$$
$$P(Z_1 \cap N_2) = P(Z_1/N_2)\ P(N_2) = 0.3 \times 0.4 = 0.12$$
$$P(Z_2 \cap N_1) = P(Z_2/N_1)\ P(N_1) = 0.3 \times 0.6 = 0.18$$
$$P(Z_2 \cap N_2) = P(Z_2/N_2)\ P(N_2) = 0.7 \times 0.4 = 0.28$$

Table 5 Joint Probabilities for Beta Company

N_j / Z_i	N_1	N_2	$P(Z_i)$
Z_1	0.42	0.12	0.54
Z_2	0.18	0.28	0.46
$P(N_j)$	0.6	0.4	1

To calculate the revised probabilities for Beta Company

$$P(N_j/Z_i) = \frac{P(Z_i/N_j).P(N_j)}{P(Z_i)}$$

Therefore $P(N_1/Z_1) = \frac{P(Z_1/N_1).P(N_1)}{P(Z_1)} = \frac{0.7 \times 0.6}{0.54} = \frac{0.42}{0.54} = 0.78$

$$P(N_1/Z_2) = \frac{P(Z_2/N_1).P(N_1)}{P(Z_2)} = \frac{0.3 \times 0.6}{0.46} = \frac{0.18}{0.46} = 0.39$$

$$P(N_2/Z_1) = \frac{P(Z_1/N_2).P(N_2)}{P(Z_1)} = \frac{0.3\times 0.4}{0.54} = \frac{0.12}{0.54} = 0.22$$

$$P(N_2/Z_2) = \frac{P(Z_2/N_2).P(N_2)}{P(Z_2)} = \frac{0.7\times 0.4}{0.46} = \frac{0.28}{0.46} = 0.61$$

Table 6: Revised probabilities for Beta Company

N_j / Z_i	N_1	N_2
Z_1	0.78	0.22
Z_2	0.39	0.61

In order to enable the multiplication across the j states of nature, the above revised probabilities must be transposed as follows:

$$\begin{matrix} & N_1 & N_2 \\ S_1 & 150 & -100 \\ S_2 & 0 & 0 \end{matrix} \begin{pmatrix} Z_1 & Z_2 \\ 0.78 & 0.39 \\ 0.22 & 0.61 \end{pmatrix} = \overset{EPRP}{\begin{pmatrix} 95 & -2.5 \\ 0 & 0 \end{pmatrix}} \quad \begin{matrix} & Z_1 & Z_2 \\ N_1 & 0.78 & 0.39 \\ N_2 & 0.22 & 0.61 \end{matrix}$$

The expected payoff matrix with revised probabilities (EPRP):

The expected payoff for strategy 1 is 95 and the result 1 of the market survey Z_1, and 0 is the expected payoff of strategy 2(S_2) and the result 1(Z_1). Since there are four expected payoffs of two different results, Mohan selects the maximum expected payoff of each result and multiplies this by marginal probabilities $P(Z_i)$ to obtain the expected payoff of the market survey (EPMS) or the expected value of the survey information (EVSI):

$$\begin{pmatrix} 95 & 0 \end{pmatrix}\begin{pmatrix} 0.54 \\ 0.46 \end{pmatrix} = 51.3$$

By deduction the market survey costs Rs 30,000 from the expected payoff of the market survey, the expected net payoff of the market survey (ENPMS) is obtained.
ENPMS = EPMS – Survey cost = 51.3– 0.3 = 51
The difference between ENPMS and $EP_{max}$is expected net gain of research (ENCR).
ENCR = 51 – 50 = 1

Posterior Analysis

Suppose that, due to the favorable result of the pre-posterior analysis Mr. Mohan has made a decision that his firm should buy Alpha Survey. Using the random sampling method one hundred target audiences are selected as samples. The main question of the survey is to ask the respondents whether they are willing to buy the new pocket size calculators. Suppose five of the one hundred respondents indicate the preference for buying the new type of calculator. The estimated payoffs under three different states of nature and their probabilities are stated as follows:

$$\begin{array}{c} \\ S_1 \\ S_2 \end{array} \begin{array}{c} \begin{array}{ccc} N_1 & N_2 & N_3 \end{array} \\ \begin{pmatrix} 120 & 150 & 200 \\ -50 & 30 & 100 \end{pmatrix} \end{array} \begin{array}{c} P(N_j) \\ \begin{pmatrix} 0.2 \\ 0.5 \\ 0.3 \end{pmatrix} \end{array} = \begin{array}{c} EP \\ \begin{pmatrix} 159 \\ 35 \end{pmatrix} \end{array} max$$

N_1: Expected market share = 0.05
N_2: Expected market share = 0.10
N_3: Expected market share = 0.20
$P(N_j)$: Prior probability of N_j(j= 1, 2, 3)

The company analyzes this problem by using binomial probabilities. At first, the company calculates the revised probabilities by using Bayes formula and compiles a table with various probabilities as follows:

State of Nature N_j	Prior Probability $P(N_j)$	Conditional Probability P(r=5/ N_j ; n=100)	Joint Probability P(r=5/ N_j ; n=100). $P(N_j)$	Posterior Probability P(N_j /r=5 ; n=100)
0.05	0.2	0.18002	0.03600	0.675676
0.1	0.5	0.03387	0.01694	0.317943
0.2	0.3	0.00113	0.00034	0.006381
		Marginal Probability	= 0.05328	1.000000

Conditional probabilities can be obtained by using the cumulative binomial probability distribution. The computation of joint probability is based on the formula

$$P(Z_i \cap N_j) = P(r=5/N_j; n=100) \times P(N_j)$$

By using Bayes' formula the posterior probabilities are obtained.

$$P(N_j/r=5, n=100) = \frac{P(r=5/N_j; n=100) \times P(N_j)}{\sum_{j=1}^{3} P(r=5/N_j; n=100) \times P(N_j)}$$

$$P(N_1=0.05/r=5)=\frac{0.03600}{0.05328}=0.675676$$

$$P(N_2=0.10/r=5)=\frac{0.01694}{0.05328}=0.317943$$

$$P(N_3=0.20/r=5)=\frac{0.00034}{0.05328}=0.006381$$

Now the company is in the position to calculate the expected payoff with revised probabilities as stated in the following:

$$\begin{matrix} \\ S_1 \\ S_2 \end{matrix}\begin{matrix} N_1 & N_2 & N_3 \\ \end{matrix}\begin{pmatrix} 120 & 150 & 200 \\ -50 & 30 & 100 \end{pmatrix} \overset{P(N_j/r=5)}{\begin{pmatrix} 0.675676 \\ 0.317943 \\ 0.006381 \end{pmatrix}} = \overset{EPRP}{\begin{pmatrix} 130.05 \\ -23.6 \end{pmatrix}} max$$

The following are the findings of the study:

The expected net payoff of the market survey in the case of Alpha company = 76.35 lakhs

Expected net gain of research in this case = 76.35 -50 = 26.35 lakhs

The expected net payoff of the market survey in the case of Beta company = 51 lakhs

Expected net gain of research in this case = 51 - 50 = 1 lakhs

Expected profit without survey = 50 lakhs

Expected value of perfect information = 40 lakhs

The expected payoff with revised probabilities for first strategy = 130.05 lakhs

The expected payoff with revised probabilities for second strategy = - 23.6 lakhs

Recommendations

Mr. Mohan should buy Alpha survey because the expected net payoff of the market survey is maximum in this case and hence the expected net gain of research is also maximized. Since the expected payoff with revised probabilities for S_1 is higher than for S_2, the firm should produce the new type of pocket calculators.

Conclusion

This article has covered only a few of the many features of Bayesian analysis. We have tried to show via example some of the computational aspects of the Bayesian approach and how, given each possible state of nature, the analysis is affected by such considerations as (a) the economic consequences attached to alternative courses of action, (b) the prior judgments of the decision maker, and (c) the conditional probabilities of observing specific events. Our concluding comments pertain to some of the pragmatic problems encountered in implementing the Bayesian approach in real problem situations.

First, in realistic settings the breadth of the problem frequently necessitates use of a computer to derive expected payoffs. Second, sensitivity analysis offer a means to find out how critical is the choice of some best act remains “best” under a variety of different conditions. Implementation of the Bayesian approach may thus involve fairly elaborate models of the problem situation in order to reflect adequately the characteristics of the system.

As an assist to the decision maker's judgment, decision theory should begin to be able to make a significant contribution as more firms familiarize themselves with the techniques and start applying them to actual problems. As may have been inferred from this simplified exposition, Bayesian decision theory is not “anti-data” at all. Rather, it looks at advertising experimentation or data gathering in general in the cold light of its net gain-if any- toward reducing the cost of uncertainty. Finally, the approach provides a formal way to incorporate the information right into the decision problem rather than the less direct.

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