

**A return-policy contract for a loss-averse game considering inventory inaccuracy****Kuo-Hsien Wang (Corresponding Author)****Department of Business Administration, Takming University of Science and Technology,  
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Taiwan.****Che-Tsung Tung****Department of International Trade, Takming University of Science and Technology, Taipei, Taiwan.****Abstract**

This article addresses a loss-averse retailer and a risk-neutral manufacturer decentralized supply chain wherein the manufacturer offers an item to the retailer in a stochastic demand market, considering inventory inaccuracy that includes temporary and permanent shrinkages. The manufacturer attempts to offer a cheaper wholesale price and a buy-all-back commitment to the retailer in hopes of operating the chain as a risk-neutral centralized chain. The objective is to jointly negotiate the wholesale price and buyback price to coordinate the chain, during which we find that once the two prices are negotiated, the manufacturer's profit remains independent of the level of loss aversion if the demand follows a uniform distribution, and the retailer's profit would not benefit from a high loss aversion level. Also, compared to the temporary shrinkage, the permanent shrinkage significantly compromises the return contract efficiency. Many managerial insights are observed hereafter.

*Keywords:* Return-policy; Loss aversion; Newsvendor; Inventory inaccuracy; Supply chain

**1. Introduction**

It is well known that the accuracy of an inventory information deeply influences a supply chain profit efficiency, as stated by Hollinger and Adam (2010) who pointed out that inventory shrinkages accounted for 1.44% of total annual sales in USA, and retailers lost more than \$33 billion in 2009 as a result of inventory errors. Before that, Atali et al. (2006) categorized inventory errors as temporary shrinkage, permanent shrinkage and transaction error. The temporary shrinkage could refer to product misplacement that affects available inventory but would return for salvage at the end of the selling season; the permanent shrinkage could refer to product theft that affects available inventory and would not be returned; and the transaction error refers to scanning problem that only affects inventory records but would not affect the physical inventory. Xu et al. (2012) thus examined the three inventory errors in the framework of a supply chain, and then investigated how RFID technology reducing inventory errors would economically benefit the supply chain.

In general, most existing newsvendor problems are risk-neutral assumed. For example, Tsay (1999) studied a quantity flexibility contract in a newsvendor supply chain, Yao et al (2005) dealt with demand uncertainty and return policies for style-good retailing competition, Bose and Anand (2007) contributed a practical finding to return policies with exogenous pricing, Yao et al. (2008) analyzed the impact of price-sensitivity factors on a supply chain, and Chen (2011) tackled a wholesale-price-discount contract in the context of a newsvendor setting, all of which assumed a risk-neutral supply chain.

Other than the risk-neutral newsvendor, a loss-averse one is now widely adopted when describing newsvendor behaviors because it is closer to human nature. The loss aversion, originated from Kahneman and Tversky (1979), indicates that people are more averse to losses than attracted by same-sized gains. Subsequently, Schweitzer and Cachon (2003) contended that an optimal order quantity of a loss-averse newsvendor is less than that of a risk-neutral newsvendor as shortage cost is negligible. Wang and Webster (2007) then extended the loss-averse newsvendor to a supply chain setting where a risk-neutral manufacturer offers an item to a loss-averse retailer. Wang and Webster (2009) further found that, if shortage cost is relatively high, a loss-averse newsvendor may play more order than a risk-neutral newsvendor does. Wang (2010) modified a single loss-averse retailer to multiple loss-averse retailers who then compete for inventory from a risk-neutral manufacturer. Recently, Liu et al. (2013) studied a newsvendor game in which two substitutable items are sold by two different loss-averse retailers who face stochastic customer demand and deterministic product substitution.

A price-only contract is widely considered as a basic, simple trade-off in the existing literature. In such an agreement, a manufacturer offers no incentive to retailer(s), and the retailer(s) then will take all responsibilities for excess inventory at the end of the selling period. However, researchers, such as Larivie and Porteus(2001), Cachon(2003) and Bernstein and Federgruen (2005), proved that the price-only contract fails to coordinate a supply chain. Conversely, a return-policy contract mitigating the risk of over-stocking because of market demand uncertainty is a commitment made by a manufacturer to accept his partner's unsold products, according to Padmanabhan and Png(1995). Pasternack(1985) who was the first to analyze manufacturer-retailer channel coordination via return policies for seasonal items, contended that return policies could be used as an instrument for supply chain coordination. Since then, a number of related articles have been published. Emmons and Gilbert (1998) investigated the role of return policies in pricing and inventory decisions for catalogue goods. Lau et al.(2000) investigated the problem of demand uncertainty and return policy for a seasonal product.

Unlike the mentioned articles, this paper considers a loss-averse retailer, a risk-neutral manufacturer decentralized supply chain, simultaneously taking into account the effects of the loss aversion and the contemporary and permanent inventory shrinkages. The manufacturer offers a return policy to the retailer so as to operate the game as a risk-neutral centralized chain.

The remainder of this paper is organized as follows. Assumptions and notation are provided in Section 2 where related newsvendor models will be proposed and analyzed. Numerical examples are conducted in Section 3, along with managerial insights. A summary of the paper and potential directions for further explorations are presented in Section 4 to end the study.

## 2. The models

The scenario investigated in the study is as follows. A risk-neutral manufacturer supplies an item to a loss-averse retailer in a stochastic demand market with a demand variable  $x$  defined on  $[0, M]$ , of which  $f(\square)$  is the possibility density function and  $F(\square)$  is the cumulative distribution function. The manufacturer would not only lower the wholesale price, but also accept all unsold products back with a price  $b$  at the end of the selling period. Assume that  $Q$  is the order quantity,  $r$  is the unit retail price,  $w$  is the unit wholesale price,  $c$  is the unit production cost, and  $s$  is the unit salvage value for leftover inventory. It is reasonable to assume that  $r > w > c > s$ . Let  $\alpha \in [0, 1)$  be a ratio representing the temporary inventory shrinkage that would return for salvage,  $\beta \in [0, 1)$  be a ratio representing the permanent inventory shrinkage that would not be returned, and  $\alpha + \beta \in [0, 1)$ . Also, define  $\delta = 1 - \alpha - \beta$ ; thus,  $(1 - \beta)Q$  is the inventory in store, including the misplaced inventory  $\alpha Q$  and the available inventory  $\delta Q$  on shelf for sale.

### 2.1 The price-only contract

In the price only-contract, the manufacturer provides no incentives to the retailer who then takes all responsibility of the leftover inventory. Thus, according to Schweitzer and Cachon (2000), we assume the following piecewise-linear loss aversion utility function  $U(W)$  for the retailer.

$$U(W) = \begin{cases} W - W_0 & W \geq W_0 \\ \lambda(W - W_0) & W \leq W_0 \end{cases} \quad (1)$$

where  $W_0$  is the retailer's reference wealth level at the beginning of the selling period,  $W$  is the retailer's final wealth level after the selling period, and  $\lambda > 1$  is the retailer's loss aversion level. Without loss of generality,  $W_0 = 0$  is normalized.

Accordingly, the loss-averse retailer's expected utility function  $E[U(\pi_r(Q))]$  and the risk-neutral manufacturer's expected profit  $E[\pi_m(w)]$  in the price-only contract are calculated by

$$\begin{aligned} E[U(\pi_r(Q))] &= \int_0^{\delta Q} (rx + s((1 - \beta)Q - x) - wQ) f(x) dx \\ &+ \int_{\delta Q}^M (r\delta Q + s\alpha Q - wQ) f(x) dx \\ &+ (\lambda - 1) \int_0^{\delta Q} (rx + s((1 - \beta)Q - x) - wQ) f(x) dx \\ &= -wQ + \int_0^{\delta Q} ((r - s)x + s(1 - \beta)Q) f(x) dx \end{aligned}$$

$$\begin{aligned}
& + \int_{\delta Q}^M (r\delta + s\alpha)Qf(x)dx \\
& + (\lambda - 1) \int_0^{AQ} ((r-s)x - (w-s(1-\beta))Q)f(x)dx \quad (2)
\end{aligned}$$

$$E[\pi_m(w)] = (w-c)Q \quad (3)$$

where  $A = \frac{w-s(1-\beta)}{r-s}$  and  $AQ$  is explained as the retailer's break-even quantity.

From Eq. (2),  $A < \delta$  is needed to ensure a well-defined  $E[U(\pi_r(Q))]$ , and this leads to  $r\delta + s\alpha > w$  which not only implies that only a positive unit profit can prompt the retailer to join the game, it also gives a tolerable maximal temporary shrinkage  $\bar{\alpha} = \frac{r-w}{r-s}$  and a tolerable maximal permanent shrinkage  $\bar{\beta} = \frac{r-w}{r}$ ; clearly,  $\bar{\alpha} > \bar{\beta}$ . Therefore, the permanent shrinkage is more intolerant in compared with the temporary shrinkage.

To maximize  $E[U(\pi_r(Q))]$ , we take the first- and second-order derivatives with respect to  $Q$  as follows.

$$\frac{dE[U(\pi_r(Q))]}{dQ} = r\delta + s\alpha - w - (r-s)\delta F(\delta Q) - (\lambda-1)(w-s(1-\beta))F(AQ) \quad (4)$$

$$\frac{d^2E[U(\pi_r(Q))]}{dQ^2} = -(r-s)\delta^2 f(\delta Q) - (\lambda-1)\frac{(w-s(1-\beta))^2}{r-s} f(AQ) < 0 \quad (5)$$

Based on Eq. (4), let

$$G(Q) = r\delta + s\alpha - w - (r-s)\delta F(\delta Q) - (\lambda-1)(w-s(1-\beta))F(AQ), \text{ then we have}$$

$\lim_{Q \rightarrow 0^+} G(Q) = r\delta + s\alpha - w$  and  $\lim_{Q \rightarrow \infty} G(Q) = -\lambda(w-s(1-\beta)) < 0$ . According to Eq. (5),  $G'(Q) < 0$  and consequently  $G(Q)$  is decreasing in  $Q$ ; therefore, the solution of  $G(Q) = 0$  uniquely exists if  $\lim_{Q \rightarrow 0^+} G(Q) = r\delta + s\alpha - w > 0$ . Thus, the following is obtained.

**Proposition 1** From the standpoint of the retailer's expected utility function,

- (1)  $E[U(\pi_r(Q))]$  is concave in  $Q$ , and
- (2) the optimal order quantity, denoted by  $Q^*$ , uniquely exists if  $r\delta + s\alpha > w$  and satisfies the following equation.

$$r\delta + s\alpha - w - (r-s)\delta F(\delta Q^*) - (\lambda-1)(w-s(1-\beta))F(AQ^*) = 0 \quad (6)$$

Consequently, optimization problem in the price-only contract becomes to find the wholesale price,  $w^*$ , maximizing  $E[\pi_m(w)]$  subject to Eq. (6), which satisfies

$$\left\{ \begin{array}{l} r\delta + s\alpha - w^* - (r-s)\delta F(\delta Q^*) - (\lambda-1)(w^* - s(1-\beta))F(A^*Q^*) = 0 \\ Q^* = (w^* - c) \frac{1 + (\lambda-1)(F(A^*Q^*) + \frac{w^* - s(1-\beta)}{r-s} Q^* f(A^*Q^*))}{(r-s)\delta^2 f(\delta Q^*) + (\lambda-1) \frac{(w^* - s(1-\beta))^2}{r-s} f(A^*Q^*)} \\ A^* = \frac{w^* - s(1-\beta)}{r-s} \end{array} \right. \quad (7)$$

And the optimal retailer's, optimal manufacturer's and the channel profits are denoted by  $\pi_r^*$ ,  $\pi_m^*$  and  $\pi^*$ , respectively.

## 2.2 The risk-neutral centralized supply chain

In this situation the manufacturer himself sells the item, whose expected profit,  $E^c$ , is then obtained by replacing  $w=c$  and  $\lambda=1$  in Eq. (2)

$$E^c = -cQ + \int_0^{\delta Q} ((r-s)x + s(1-\beta)Q) f(x) dx + \int_{\delta Q}^M (r\delta + s\alpha) Q f(x) dx \quad (8)$$

And the optimal order quantity,  $Q^c$ , in the chain satisfies

$$Q^c = \frac{1}{\delta} F^{-1}\left(\frac{r\delta + s\alpha - c}{\delta(r-s)}\right) \quad (9)$$

## 2.3 The return-policy contract

In the return-policy contract, the manufacturer provides a cheaper wholesale price and accepts all leftover inventories. For convenience, the salvage value  $s$  is also assumed. Thus, the retailer profit would be, according to Eq. (2),

$$\begin{aligned} \pi_r = & -wQ^c + \int_0^{\delta Q^c} ((r-b)x + b(1-\beta)Q^c) f(x) dx \\ & + \int_{\delta Q^c}^M (r\delta + b\alpha) Q^c f(x) dx \\ & + (\lambda-1) \int_0^{AQ^c} ((r-b)x - (w-b(1-\beta))Q^c) f(x) dx \quad (10) \end{aligned}$$

where  $A = \frac{w-b(1-\beta)}{r-b}$ . Clearly,  $R = \int_0^{\delta Q^c} ((1-\beta)Q^c - x)f(x)dx + \int_{\delta Q^c}^M \alpha Q^c f(x)dx$

is the total unsold products that will return to the manufacturer; therefore, the manufacturer's profit is given by

$$\pi_m = (w-c)Q^c - Rb \quad (11)$$

Obviously,  $\pi_r > \pi_r^*$  and  $\pi_m > \pi_m^*$  are needed to coordinate the chain, according to which the following examples are undertaken with the assumption of the demand  $X \square U[0,100]$ .

### 3. Numerical examples

Table 1  $r=8, c=3, s=1, \alpha=0.1, \beta=0.1, Q^c=78.13, b=2.5, w=4.3$

$\lambda$	$w^*$	$Q^*$	$\pi_r^*$	$\pi_m^*$	$\pi^*$	$\pi_r$	$\pi_m$	$\pi$	$\rho$
1.2	4.68	37.18	33.79	62.55	96.34	71.51	60.55	132.01	37.07
1.4	4.63	35.52	33.26	57.78	91.05	66.84	60.55	127.39	39.92
1.6	4.58	34.63	32.66	53.78	86.44	62.18	60.55	122.73	41.98
1.8	4.54	32.69	32.03	50.36	82.38	57.52	60.55	118.06	43.32
2.0	4.51	31.46	31.37	47.38	78.76	52.85	60.55	113.40	43.99
2.2	4.48	30.34	30.71	44.77	75.49	48.19	60.55	108.74	44.04
2.4	4.44	29.31	30.07	42.46	72.53	43.53	60.55	104.07	43.05
2.6	4.42	28.35	29.42	40.39	69.82	38.86	60.55	99.41	42.38
2.8	4.40	27.47	28.80	38.53	67.33	34.20	60.55	94.75	40.72
3.0	4.38	26.62	28.19	36.85	65.04	29.54	60.55	90.08	38.51

Table 2  $r=8, c=3, s=1, \lambda=2.0, b=1.5, w=4.0$ 

$(\alpha, \beta)$	$w^*$	$Q^*$	$\pi_r^*$	$\pi_m^*$	$\pi^*$	$Q^c$	$\pi_r$	$\pi_m$	$\pi$	$\rho$
(0.0,0.0)	5.12	30.57	44.05	64.78	108.83	71.43	95.37	58.67	154.04	41.55
(0.1,0.0)	4.83	31.78	39.23	58.21	97.44	75.84	75.00	60.40	135.40	38.96
(0.2,0.0)	4.54	32.77	33.69	50.59	84.29	80.36	51.61	61.99	113.60	34.78
(0.0,0.1)	4.79	30.71	36.95	55.08	92.03	74.07	62.95	62.96	125.92	36.82
(0.0,0.2)	4.47	30.18	29.16	44.30	73.46	75.89	27.61	66.68	94.28	28.35

In Tables 1 and 2, we define  $\rho \% = \frac{\pi - \pi^*}{\pi^*}$  to explore the profit efficiency of the return-policy contract. Also, let the negotiated prices are  $b=2.5, w=4.3$  in Table 1 and  $b=1.5, w=4.0$  in Table 2, both of which are confirmed in line with the chain coordination. According to Eq. (9), the optimal order quantity in the risk-neutral centralized supply chain  $Q^c=78.13$  is beforehand obtained in Table 1. Note that the italics in Tables 1 and 2 imply the negotiated prices are not coordinating the chain and need to be renegotiated.

Table 1 investigates how the retailer's loss aversion level will impact on the chain profit performances, from where we have that apart from  $\pi_m$ , all optimal values decrease with  $\lambda$ ; still, there exists at least a 37.07% of chain profit increment in the return contract. It is a breathtaking finding that the manufacturer's profit is independent of the level of loss aversion if the demand follows a uniform distribution, which highly recommends the manufacturer should operate the game as a risk-neutral centralized game not only for better profit but also for neglect toward the loss aversion. The return contract's profit efficiency, however, is not increasing in  $\lambda$  all the way, which peaks at  $\lambda=2.2$  with  $\rho=44.04$ . Apparently, the retailer profit does not benefit from a high loss aversion level, and this identifies the return contract could actually mitigate the retailer's fears of losses.

Table 2 intends to realize how the two shrinkages will affect the profit performances. It reveals that the permanent shrinkage damages the return contract efficiency more than the temporary shrinkage does, showing only a 28.35 of chain profit increment as  $(\alpha, \beta) = (0, 0.2)$  whereas a 34.78 as  $(\alpha, \beta) = (0.2, 0)$ , which is consistent with the reality that unsold products due to the temporary shrinkage would be salvaged. No surprisingly, both members' profits in the price-only contract are negatively impacted by the two shrinkages; but surprisingly, the manufacturer's profit in the return contract benefits from them as a result of an increasing optimal order quantity in the risk-neutralized centralized.

#### 4. Conclusion

This article dealt with a loss-averse retailer and a risk-neutral manufacturer decentralized supply chain, considering the temporary and permanent shrinkages, in which the manufacturer offers a cheaper wholesale prices and a buyback commitment to encourage the retailer operating the game as a risk-

neutral centralized game. The contributions of that are as follows. (1) High level of loss aversion undermines both members' profits in the price-only contract, but it would not negatively affect the manufacturer' profit in the return contract if demand follows a uniform distribution. (2) Although high level of loss aversion protects retailer from losses, it meanwhile slashes the retailer's profit if the return contract is in place. (3) High degree of the temporary shrinkage and/or permanent shrinkage damages the retailer' profit in both price-only and return-policy contracts; the manufacturer's profit, however, benefits from them in the return contract.

In course of analyzing the return contract, we failed to theoretically present the ranges of both wholesale price and buyback price for both members to negotiate, and this would be worth future explorations. Also, extending our proposed models to a loss-averse retailer and a risk-neutral manufacturer with multiple newsvendor-type items is a potential direction.

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