

GRILL ON GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce grill generalized topological spaces and to investigate the relationships between generalized topological spaces and grill generalized topological spaces. For establishment of their relationships, we define some closed sets in these spaces. Basic properties and characterization related to these sets are also discussed.

Keywords and phrases: generalized topological space, grill generalized topological space, \mathcal{G}_μ -closed set, μ^Φ -closed set, μ - \mathcal{G}_g -closed set.

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1 INTRODUCTION

The study of grill topological spaces[16] as like ideal topological spaces[9] has been started from 2007 although the study of grill [3,1,2,18] in topological spaces was started from 1947 at different point of view. Generalized closed sets[10] in topological space as well as in grill topological space[11] has been discussed at various research papers. We have introduced the generalized closed sets in grill generalized topological space (generalized topological space(GTS) [5,6] with grill), and characterized the same at different aspect. We also obtain the relations with earlier generalized closed sets in topological space, generalized topological space and grill generalized topological space etc.

2 PRELIMINARIES

Definition 2.1[3]. A nonempty collection \mathcal{G} of nonempty subsets of a topological space (X, τ) is called grill if

- i) $A \in \mathcal{G}$ and $\subseteq B \subseteq X \Rightarrow B \in \mathcal{G}$, and
- ii) $A, B \subseteq X$ and $A \cup B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$

If \mathcal{G} is grill on X , then (X, τ, \mathcal{G}) is called a grill topological space[16].

Definition 2.2[16]. Let (X, τ, \mathcal{G}) be a grill topological space. An operator $\Phi: \exp(X) \rightarrow \exp(X)$ is called a local function with respect to τ and \mathcal{G} is defined as follows: for $A \subseteq X$, $\Phi(A)(\mathcal{G}, \tau) = \Phi(A) = \{x \in X: U \cap A \in \mathcal{G} \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau: x \in U\}$ [16].

It is well known from [16], $A \cap \Phi(A) = \psi(A)$ is a Kuratowski closure operator [9].

Definition 2.3[16]. Corresponding to a grill on a topological space (X, τ) , there exists a unique topology $\tau_{\mathcal{G}}$ on X given by

$$\tau_{\mathcal{G}} = \{U \subseteq X: \psi(X \setminus A) = (X \setminus A)\}, \text{ where for any } A \subseteq X, \psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}\text{-cl}(A).$$

Definition 2.4. Let (X, τ, \mathcal{G}) be a grill topological space. A subset A of a grill topological space (X, τ, \mathcal{G}) is $\tau_{\mathcal{G}}$ -closed[16](resp. $\tau_{\mathcal{G}}$ -dense in itself[11], $\tau_{\mathcal{G}}$ -perfect), if $\psi(A) = A$ or equivalently if $\Phi(A) \subseteq A$ (resp. $A \subseteq \Phi(A)$, $A = \Phi(A)$).

Definition 2.5. Let (X, τ, \mathcal{G}) be a grill topological space. A subset A of a grill topological space (X, τ, \mathcal{G}) is g -closed with respect to the grill \mathcal{G} (briefly, \mathcal{G} - g -closed)[11] if $\Phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

A subset A of X is said to be \mathcal{G} - g -open if $X \setminus A$ is \mathcal{G} - g -closed.

Definition 2.6. Let (X, τ) be a topological space. A subset A of a space (X, τ) is said to be g -closed set[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Remark 2.1[11]. Every g -closed set is a \mathcal{G} - g -closed but not vice versa.

Remark 2.2[19]. Every closed set is g -closed.

Very interesting notion in literature has been introduced by Csaszar[4] on 1997. Using this notion topology has been reconstructed. The concept is:

A map $\tau: \exp(X) \rightarrow \exp(X)$ is possessing the property monotony (i.e. such that $A \subseteq B$ implies $\tau(A) \subseteq \tau(B)$). We denote by $\Gamma(X)$ the collections of all mapping having this property.

One of the consequence of the above notion is generalized topological space(GTS) [5,6], its formal definition is:

Definition 2.7. Let X be a non-empty set, and $\mu \subseteq \exp(X)$, μ is called a generalized topology (GTS) on X if $\emptyset \in \mu$ and the union of elements of μ belongs to μ .

The member of μ is called μ -open set and the complement of μ -open set is called μ -closed set. Again c_μ is the notation of μ -closure[5,6,13,14].

Definition 2.8[15]. Let (X, μ) be a generalized topological space. Then the generalized kernel of $A \subseteq X$ is denoted by $g\text{-ker}(A)$ and defined as $g\text{-ker}(A) = \bigcap \{G \in \mu: A \subseteq G\}$.

Lemma 2.1[15].]. Let (X, μ) be a generalized topological space and $A \subseteq X$. Then $g\text{-ker}(A) = \{x \in X: c_\mu(\{x\}) \cap A \neq \emptyset\}$.

If \mathcal{G} is a grill on X , then (X, μ, \mathcal{G}) is called a grill generalized topological space (GGTS).

3 GGTS

Definition 3.1. Let (X, μ, \mathcal{G}) be a GGTS. A mapping $()^{\Phi\mu}: \exp(X) \rightarrow \exp(X)$ is defined as follows :

$$(A)^{\Phi\mu} = (A)^{\Phi\mu}(\mathcal{G}, \mu) = \{x \in X: A \cap U \in \mathcal{G}\}, \text{ where } U \in \psi(x)[5].$$

The mapping is called the local function associated with the grill \mathcal{G} and generalized topology μ .

Properties:

Theorem 3.1. Let (X, μ, \mathcal{G}) be a GGTS. Then

- (1) $(\emptyset)^{\Phi\mu} = \emptyset$.
- (2) for $A, B \subseteq X$ and $A \subseteq B$, $(A)^{\Phi\mu} \subseteq (B)^{\Phi\mu}$.
- (3) $(A)^{\Phi\mu} \subseteq c_\mu(A)$.
- (4) $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu(A)$.
- (5) $(A)^{\Phi\mu}$ is a μ -closed set.
- (6) $((A)^{\Phi\mu})^{\Phi\mu} \subseteq (A)^{\Phi\mu}$.
- (7) for $\mathcal{G} \subseteq \mathcal{G}_1$ implies $(A)^{\Phi\mu}(\mathcal{G}_1) \supseteq (A)^{\Phi\mu}(\mathcal{G})$.
- (8) for $\in \mu$, $U \cap (U \cap A)^{\Phi\mu} \subseteq U \cap (A)^{\Phi\mu}$.
- (9) for $G \notin \mathcal{G}$, $(A \setminus G)^{\Phi\mu} = (A)^{\Phi\mu} = (A \cup G)^{\Phi\mu}$.

Proof. (1). It is obvious from definition.

(2). It is done by the fact, $A \cap G \in \mathcal{G}$ implies $B \cap G \in \mathcal{G}$.

- (3). Obvious from [5,13].
- (4). $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu(c_\mu(A)) = c_\mu(A)$ [5,13].
- (5). From [5], for $G \in \mu$ and $x \in G$, there exists $V \in \psi(x)$ such that $V \subseteq G$. Now if $A \cap G \notin \mathcal{G}$ then for $A \cap V \subseteq A \cap G$, $A \cap V \notin \mathcal{G}$. It follows that $X \setminus (A)^{\Phi\mu}$ is the union of μ -open sets. We know that the arbitrary union of μ -open sets is a μ -open set. So $X \setminus (A)^{\Phi\mu}$ is a μ -open set and hence $(A)^{\Phi\mu}$ is a μ -closed set.
- (6). From above, $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu((A)^{\Phi\mu}) = (A)^{\Phi\mu}$, since $(A)^{\Phi\mu}$ is a μ -closed set.
- (7). Obvious from $A \cap V \in \mathcal{G}$ implies $\cap V \in \mathcal{G}_1$.
- (8). Since $U \cap A \subseteq A$, then $(U \cap A)^{\Phi\mu} \subseteq (A)^{\Phi\mu}$ so $U \cap (U \cap A)^{\Phi\mu} \subseteq U \cap (A)^{\Phi\mu}$.
- (9). Let $x \in (A)^{\Phi\mu}$. If possible suppose that $x \notin (A \setminus G)^{\Phi\mu}$. Then there is a $V \in \psi(x)$, $V \cap (A \setminus G) \notin \mathcal{G}$. Therefore $(V \cap (A \setminus G)) \cup G \notin \mathcal{G}$, i.e., $G \cup (A \cap V) \notin \mathcal{G}$. Then $\cap A \notin \mathcal{G}$, a contradiction to the fact that $x \in (A)^{\Phi\mu}$. Hence, $(A \setminus G)^{\Phi\mu} = (A)^{\Phi\mu}$.

Proof of 2nd part is similar.

It is obvious from (2), $()^{\Phi\mu} \in \Gamma(X)$ [4].

Definition 3.2. Let (X, μ) be a GTS with a grill \mathcal{G} on X .

The set operator $c^{\Phi\mu}$ is called a generalized $\Phi\mu$ -closure and is defined as $(c)^{\Phi\mu}(A) = AU(A)^{\Phi\mu}$, for $A \subseteq X$. We will denote by $\mu^\Phi(\mu; \mathcal{G})$ the generalized structure, generated by $c^{\Phi\mu}$, that is, $\mu^\Phi(\mu; \mathcal{G}) = \{U \subseteq X: c^{\Phi\mu}(X \setminus U) = (X \setminus U)\}$. $\mu^\Phi(\mu; \mathcal{G})$ is called $\Phi\mu$ -generalized structure with respect to μ and \mathcal{G} (in short $\Phi\mu$ -generalized structure) which is finer than μ .

The element of $\mu^\Phi(\mu; \mathcal{G})$ are called μ^Φ -open and the complement of μ^Φ -open is called μ^Φ -closed.

Theorem 3.2. The set operator $c^{\Phi\mu}$ satisfy following conditions:

- (a) $A \subseteq c^{\Phi\mu}(A)$, for $A \subseteq X$.
- (b) $c^{\Phi\mu}(\emptyset) = \emptyset$ and $c^{\Phi\mu}(X) = X$.
- (c) $c^{\Phi\mu}(A) \subseteq c^{\Phi\mu}(B)$ if $A \subseteq B \subseteq X$.
- (d) $c^{\Phi\mu}(A) \cup c^{\Phi\mu}(B) \subseteq c^{\Phi\mu}(A \cup B)$.
- (e) $c^{\Phi\mu} \in \Gamma(X)$.

Proof: Proof is obvious from Theorem 3.1.

Definition 3.3. Let (X, μ) be a GTS. A subset A of X is said to be g_μ -closed set [12] if $c_\mu(A) \subseteq M$ whenever $A \subseteq M$ and $M \in \mu$.

Definition 3.4. A subset A of a GGTS (X, μ, \mathcal{G}) is μ^Φ -dense in itself (resp. μ^Φ -perfect) if $A \subseteq (A)^{\Phi\mu}$ (resp. $(A)^{\Phi\mu} = A$).

Definition 3.5. A subset A of a GGTS (X, μ, \mathcal{G}) is called μ - G -generalized closed (briefly, μ - G_g -closed) if $(A)^{\Phi\mu} \subseteq U$ whenever U is μ -open and $A \subseteq U$. A subset A of a GGTS (X, μ, \mathcal{G}) is called μ - G -generalized open (briefly, μ - G_g -open) if $X \setminus A$ is μ - G_g -closed.

Theorem 3.3. Let (X, μ, \mathcal{G}) be a GGTS. Every g_μ -closed set is μ - G_g -closed.

Proof: Let U any μ -open set containing A . Since A is g_μ -closed, then $c_\mu(A) \subseteq U$. By Theorem 3.1(3), we have $(A)^{\Phi\mu} \subseteq U$.

Remark 3.1. Let (X, τ) be a topological space. If we take $\mu = \tau$, then g_μ -closed set coincide with g -closed sets [7,8].

Proposition 3.1. Let (X, μ, \mathcal{G}) be a GGTS.

- (a) Every μ^Φ -perfect set is μ^Φ -dense in itself.
- (b) Every μ^Φ -perfect set is μ^Φ -closed.

Proof: The proof can be easily done.

Remark 3.2. Let (X, τ) be a topological space and \mathcal{G} be a grill on X . If we take $\mu = \tau$, then μ - G_g -closed (resp. μ^Φ -closed, μ^Φ -dense in itself) sets coincide with \mathcal{G} - g -closed [11] (resp. $\tau_{\mathcal{G}}$ -closed [16], $\tau_{\mathcal{G}}$ -dense in itself [11]).

Theorem 3.4. If (X, μ, \mathcal{G}) is a GGTS and $A \subseteq X$, then A is μ - G_g -closed if and only if $c^{\Phi\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ -open in X .

Proof: Since A is μ - G_g -closed, we have $(A)^{\Phi\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ -open in X . $c^{\Phi\mu}(A) = A \cup (A)^{\Phi\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ -open in X .

Converse part: Let $A \subseteq U$ and U be μ -open in X . By hypothesis $c^{\Phi\mu}(A) \subseteq U$. Since $c^{\Phi\mu}(A) = A \cup (A)^{\Phi\mu}$, we have $(A)^{\Phi\mu} \subseteq U$.

Theorem 3.5. Let (X, μ, \mathcal{G}) is a GGTS and $A \subseteq X$. Then the following are equivalent :

- (a) A is μ - G_g -closed.
- (b) $c^{\Phi\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ -open in X .
- (c) $c^{\Phi\mu}(A) \subseteq g\text{-ker}(A)$.
- (d) $c^{\Phi\mu}(A) \setminus A$ contains no nonempty μ -closed set.
- (e) $(A)^{\Phi\mu} \setminus A$ contains no nonempty μ -closed set.

Proof: (a) \Leftrightarrow (b). It follows from Theorem 3.4.

(b) \Rightarrow (c). Suppose $x \in c^{\Phi\mu}(A)$ and $x \notin g\text{-ker}(A)$. Then $c_\mu(\{x\}) \cap A = \emptyset$. Implies that $A \subseteq X \setminus (c_\mu(\{x\}))$.

Now from (b), $c^{\Phi\mu}(A) \subseteq X \setminus c_\mu(\{x\})$. This implies $c^{\Phi\mu}(A) \cap \{x\} = \emptyset$, a contradiction. Hence the result.

(c) \Rightarrow (d). Suppose $c^{\Phi\mu}(A) \setminus A$, F is μ -closed and $x \in F$. Since $F \subseteq (c^{\Phi\mu}(A)) \setminus A$, $F \cap A = \emptyset$. We have $c_\mu(\{x\}) \cap A = \emptyset$ because F is μ -closed and $x \in F$. From (c), this is a contradiction.

(d) \Rightarrow (e). This is obvious from the definition of $c^{\Phi\mu}(A)$.

(e) \Rightarrow (a). Let U be a μ -open subset containing A . Since $(A)^{\Phi\mu}$ is μ -closed by means of Theorem 3.1(5). Now $(A)^{\Phi\mu} \cap (X \setminus U) \subseteq (A)^{\Phi\mu} \setminus A$. Since intersection of two μ -closed sets is a μ -closed set, then $(A)^{\Phi\mu} \cap (X \setminus U)$ is a μ -closed set contained in $(A)^{\Phi\mu} \setminus A$. By assumption, $(A)^{\Phi\mu} \cap (X \setminus U) = \emptyset$. Hence, we have $(A)^{\Phi\mu} \subseteq U$.

Remark 3.3. Let (X, τ, \mathcal{G}) be a GGTS. If $\mu = \tau$ then the above theorem coincides with Theorem 2.7 in [11].

Proposition 3.2. Let (X, μ, \mathcal{G}) be a GGTS. Every μ^Φ -closed set is μ - G_g -closed.

Proof: Let A be a subset of X and A be μ^Φ -closed. Assume that $A \subseteq U$ and U is μ -open. Since A is μ^Φ -closed, we have $(A)^{\Phi\mu} \subseteq A$ and so A is μ - G_g -closed.

For the relationship related to several sets defined in the paper, we have the following diagram :

$$\mu^\Phi\text{-dense in itself} \Leftarrow \mu^\Phi\text{-perfect} \Rightarrow \mu^\Phi\text{-closed} \Rightarrow \mu\text{-}G_g\text{-closed} \Leftarrow g_\mu\text{-closed} \Leftarrow \mu\text{-closed}$$

The following examples show that the converse implications of the diagram are not satisfied.

Example 3.1.(a). Let $X = \{a, b, c\}$, $\mu = \{X, \emptyset, \{b\}, \{b, c\}\}$, $\mathcal{G} = \{\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, X\}$ and $A = \{b\}$. Here $(A)^{\Phi\mu} = \emptyset$ and $c_\mu(A) = X$. Thus, A is μ - G_g -closed. But A is not g_μ -closed.

(b) In (a), let $B = \{a, b\}$. Note that the only μ -open set containing B is X . $c_\mu(B) = X$ is also contained in X . Therefore B is g_μ -closed but not μ -closed.

(c) In (a), A is μ^Φ -closed but not μ^Φ -perfect.

(d) Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{b, c\}\}$, $\mathcal{G} = \{\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, X\}$ and $A = \{b\}$. Then $(A)^{\Phi\mu} = \emptyset$ which is also a subset of $\{b\}$ and $\{b, c\}$. So, A is μ - G_g -closed but not μ^Φ -closed.

(e) In (a), let $B = \{c\}$. Then $(B)^{\Phi\mu} = \{a, c\}$, so B is μ^Φ -dense in itself but not μ^Φ -perfect.

Definition 3.6[17]. A space (X, μ) is called μ - T_1 if any pair of distinct points x and y of X , there exists a μ -open set U of X containing x but not y and a μ -open set V of X containing y but not x .

It is obvious from definition that every singleton set is μ -closed if and only if the space is μ - T_1 .

Remark 3.4. Let (X, μ, \mathcal{G}) be a GGTS and $A \subseteq X$. If (X, μ) is a μ - T_1 space, then A is μ^Φ -closed if and only if A is μ - G_g -closed.

Theorem 3.6. Let (X, μ, \mathcal{G}) be a GGTS and $A \subseteq X$. If A is an μ - G_g -closed set, then the following are equivalent :

- (a) A is an μ^Φ -closed set.
- (b) $c^{\Phi\mu}(A) \setminus A$ is an μ -closed set.
- (c) $(A)^{\Phi\mu} \setminus A$ is an μ -closed set.

Proof: (a) \Rightarrow (b). If A is μ^Φ -closed, then $c^{\Phi\mu}(A) \setminus A = \emptyset$. $c^{\Phi\mu}(A) \setminus A$ is μ -closed.

(b) \Rightarrow (c). Since $c^{\Phi\mu}(A) \setminus A = (A)^{\Phi\mu} \setminus A$, it is clear.

(c) \Rightarrow (a). If $(A)^{\Phi\mu} \setminus A$ is μ -closed and A is μ - G_g -closed, from Theorem 3.5(e), $(A)^{\Phi\mu} \setminus A = \emptyset$ and so A is μ^Φ -closed.

Lemma 3.1. Let (X, μ, \mathcal{G}) be a GGTS and $A \subseteq X$. If A is μ^Φ -dense in itself, then $(A)^{\Phi\mu} = c_\mu((A)^{\Phi\mu}) = c_\mu(A) = c^{\Phi\mu}(A)$.

Proof: Let A be μ^Φ -dense in itself. Then we have $A \subseteq (A)^{\Phi\mu}$ and hence $c_\mu(A) \subseteq c_\mu((A)^{\Phi\mu})$. We know that $(A)^{\Phi\mu} = c_\mu((A)^{\Phi\mu}) \subseteq c_\mu(A)$ from Theorem 3.1(5). In this case $c_\mu(A) = c_\mu((A)^{\Phi\mu}) = (A)^{\Phi\mu}$. Since $(A)^{\Phi\mu} = c_\mu(A)$, we have $c^{\Phi\mu}(A) = c_\mu(A)$.

We obtained that every g_μ -closed set is μ - G_g -closed in Theorem 3.3 but not vice versa. For μ^Φ -dense in itself sets, g_μ -closedness and μ - G_g -closedness are equivalent.

Theorem 3.7. Let (X, μ, \mathcal{G}) be a GGTS and $A \subseteq X$. If A is μ^Φ -dense in itself and μ - G_g -closed, then A is g_μ -closed.

Proof. Assume A is μ^Φ -dense in itself and μ - G_g -closed on X . If U is an μ -open set containing A , then we have $(A)^{\Phi\mu} \subseteq U$. Since A is μ^Φ -dense in itself, Lemma 3.1 implies $c_\mu(A) \subseteq U$ and so A is g_μ -closed.

Theorem 3.8. Let (X, μ, \mathcal{G}) be a GGTS and $A \subseteq X$. If A is μ - G_g -closed and μ -open then A is μ^Φ -closed.

Proof: Let A be an μ -open. Since A is μ - G_g -closed, we have $(A)^{\Phi\mu} \subseteq A$. Hence A is μ^Φ -closed.

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