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FUZZY RELIABILITY ANALYSIS FOR THE EFFECT OF VASOPRESSIN BASED ON NORMAL DISTRIBUTION

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ABSTRACT:

The theoretical study of the effect of the administration of Vasopressin in abdominal vascular injury of animals was investigated. Formulae of fuzzy reliability functions, fuzzy normal distributions and its α -cut sets are presented. Using a Fuzzy reliability analysis based on normal distribution, we showed that vasopressin treatment ensured to permit at least short term survival of animals.

Keywords: Fuzzy reliability function, Fuzzy normal distribution, Vasopressin.

Mathematics Subject Classification 2000: *95V05, 28E10, 62Exx*

1. INTRODUCTION

The most frequently used functions in lifetime data analysis are the reliability or survival function. This function gives the probability of an item operating for a certain amount of time without failure. Many methods and models in classical reliability theory assume that all parameters of lifetime density function are precise. But in the real world, randomness and fuzziness are mixed up in the lifetime of the systems. In 1965, Zadeh [1] introduced fuzzy set theory. Subsequently, the theory and the mathematics of fuzzy sets were fleshed out and applied in many research fields [3]. The theory of Fuzzy reliability was proposed and developed by several authors [5], [4], [6].

The fuzzification process characterizes the imprecision of classes for phenomena that do not have sharply defined boundaries. The increase of mean arterial blood pressure during the administration of vasopressin was observed by Stadlbauer KH. et al. [7], Shelly MP. et al. [2], Krismer AC. et al. [8], and Tsuneyoshi I et al. [9].

In this paper we propose a fuzzy reliability analysis of the effect of Vasopressin on haemodynamic variables in an abdominal vascular injury with uncontrolled haemorrhagic shock in animals based on normal distribution.

Fuzzification converts the original values of the Phenomenon to the possibility that they belong to a defined set. The defined set can consist of being suitable or having the possibility of finding a specified manner. The original values of the phenomenon are reclassified on this membership continuum through predefined membership functions.

In the fuzzification process, the ideal definition for membership of the set is defined. Each value of the phenomenon more central to the core of the definition of the set will be assigned as 1. Those values that are definitely not part of the set are assigned as 0. Those values that fall between the two extremes fall in the transitional zone of the set, the boundary. As the values move away from the ideal or the center of the set, they are assigned a decreasing value on a continuous scale from 1 to 0. As the assigned value decreases, the original Phenomenon value has less possibility of being a member of the set.

The fuzzification value of 0.5 is the cross over point. Any fuzzy value greater than 0.5 implies that the original phenomenon value may be a member of the set. As the fuzzification values go below 0.5, it is likely that the original phenomenon's value may not be the part of the set.

2. FUZZY RELIABILITY FUNCTION

Reliability or survival function $S(t,\bar{\theta})$ is the probability a system survives beyond time t. Let the random variable X denote lifetime of a System components, also let X has a probability density function with fuzzy parameter.

$$f(x,\bar{\theta}) = \{ f(x)[\alpha],\ \mu_{f(x)} / f(x)[\alpha]$$
$$= [f_{\min}(x)[\alpha],\ f_{\max}(x)[\alpha]], \mu_{f(x)} = \alpha\}$$

$$f_{\min}(x)[\alpha] = \inf\{ f(x,\theta)[\alpha] / \theta \in \bar{\theta}[\alpha]\},$$

$$f_{\max}(x)[\alpha] = \sup\{ f(x,\theta)[\alpha] / \theta \in \bar{\theta}[\alpha]\},$$

The cumulative distribution function with fuzzy parameter of a random variable is defined as

$$F(x,\bar{\theta}) = \{F(x)[\alpha],\ \mu_{F(x)} / F(x)[\alpha]$$
$$= [F_{\min}(x)[\alpha], F_{\max}(x)[\alpha]], \mu_{F(x)} = \alpha\}$$

$$F_{\min}(x)[\alpha] = \inf\{F(x,\theta)[\alpha] / \theta \in \bar{\theta}[\alpha]\},$$

$$F_{\max}(x)[\alpha] = \sup\{F(x,\theta)[\alpha] / \theta \in \bar{\theta}[\alpha]\},$$

Let the lifetime random variable is a fuzzy random variable with probability density function $f(x,\bar{\theta})$, then fuzzy reliability function is defined as

$$S(t,\bar{\theta}) = \{S(t)[\alpha], \mu_{s(t)} / S(t)[\alpha]$$
$$= [S_{\min}(t)[\alpha], S_{\max}(t)[\alpha], \mu_{s(t)} = \alpha\},$$

$$S_{\min}(t)[\alpha] = 1 - \sup\{F(t,\theta)[\alpha] / \theta \in \bar{\theta}(\alpha)]$$

$$S_{\max}(t)[\alpha] = 1 - \inf\{F(t,\theta)[\alpha] / \theta \in \bar{\theta}(\alpha)]$$

Let $\bar{\theta}$ is a triangular fuzzy number ($\theta_1, \theta_2, \theta_3$) and then any given t_o; fuzzy reliability is a fuzzy number $[1 - F(t_0,\theta_3), 1 - F(t_0,\theta_2), 1 - F(t_0,\theta_1)]$ and membership function is defined as

$$\mu_{\tilde{s}(t_0)}(x) = \begin{cases} \dfrac{x - \{1 - F(t_0, \theta_3)\}}{F(t_0, \theta_3) - F(t_0, \theta_2),} & , 1 - F(t_0, \theta_3) \le x \le 1 - F(t_0, \theta_2) \\ \dfrac{1 - F(t_0, \theta_1)\} - x}{F(t_0, \theta_2) - F(t_0, \theta_1)} & , 1 - F(t_0, \theta_2) \le x \le 1 - F(t_0, \theta_1) \\ 0 & , otherwise \end{cases}$$

3. FUZZY NORMAL DISTRIBUTION

The normal density N(μ, σ^2) has density function f($x; \mu, \sigma^2$), $x \in R$, mean μ and variance σ^2. If the mean and variance are unknown we must estimate them from a random sample and we obtain fuzzy estimator $\overline{\mu}$ $for\, \mu$ and fuzzy estimator $\overline{\sigma}^2$ $for\, \sigma^2$.

So consider the fuzzy normal $N(\overline{\mu}, \overline{\sigma}^2)$ for fuzzy numbers $\overline{\mu}$ and $\overline{\sigma}^2 > 0$. We wish to complete the fuzzy probability of obtaining a value in the interval [c, d]. We write this fuzzy probability as $\overline{P}$ [c, d]. We may easily extend our results to $\overline{P}$ [E] for other subsets E of R.

For $\alpha \in [0, 1]$, $\mu \in \bar{\mu}[\alpha]$ and $\sigma^2 \in \overline{\sigma}^2[\alpha]$, let $Z_1 = \dfrac{(c-\mu)}{\sigma}$ and $Z_2 = \dfrac{(d-\mu)}{\sigma}$.

then,

$$\overline{P}[c,d][\alpha] = \{\int_{z_1}^{z_2} f(x;0,1)\,dx \,/\, \mu \in \overline{\mu}[\alpha], \sigma^2 \in \overline{\sigma}^2[\alpha]\} \quad ----> (1)$$ for $0 \le \alpha \le 1$

The above equation gets the α-cuts of $\overline{P}$ [c, d]. Also, in the equation (1), f (x; 0, 1) is the standard normal density with zero mean and unit variance.

Let $\overline{P}$ [c, d] [α] = ($P_1[\alpha]$, $P_2[\alpha]$). Then the minimum (maximum) of the expression on the right side of the above equation is

($P_1[\alpha]$; $P_2[\alpha]$) where

$$P_1[\alpha] = Max\ \{\int_c^d \overline{f}(x;0,1)\,dx \,/\, \mu \in \overline{\mu}[\alpha],\ \sigma \in \bar{\sigma}[\alpha]\}$$

$$P_2[\alpha] = Min\ \{\int_c^d \overline{f}(x;0,1)\,dx \,/\, \mu \in \overline{\mu}[\alpha],\ \sigma \in \bar{\sigma}[\alpha]\}$$

We first specify the α -cuts of $\overline{\sigma}$ from the α -cuts of $\overline{\sigma}^2$. If $\overline{\sigma}^2[\alpha] = [\sigma_1^2[\alpha], \sigma_2^2[\alpha]]$, then

$\bar{\sigma}[\alpha] = [\sqrt{\sigma_1^2[\alpha]}, \sqrt{\sigma_2^2[\alpha]}] = [\sigma_1[\alpha], \sigma_2[\alpha]]$ similarly the α -cuts of $\overline{\mu}$ is

$\bar{\mu}[\alpha] = [\mu_1[\alpha], \mu_2[\alpha]]$

Let the fuzzy reliability of normal distribution be $\overline{R}[\alpha]=[R_1[\alpha],R_2[\alpha]]$ where

$$R_1[\alpha]=1-\phi\left\{\frac{t-\mu_1[\alpha]}{\sigma_1[\alpha]}\right\}$$ and

$$R_2[\alpha]=1-\phi\left\{\frac{t-\mu_2[\alpha]}{\sigma_2[\alpha]}\right\}$$

4. APPLICATION

When mean arterial blood pressure was below 20 mm Hg, and heart rate had declined progressively, experimental therapy was initiated on the animals. At that point, animals were randomly assigned to receive vasopressin. Vasopressin treated animals were then given a continuous infusion as 0.08 U/kg/min vasopressin. After 30min of experimental therapy bleeding was controlled by surgical intervention, and further fluid resuscitation was performed. Thereafter, the animals were observed for an additional hour. Mean arterial blood pressure increased in both vasopressin-treated and fluid resuscitation animals from about 15mm Hg to about 55mm Hg within 5min, but afterward it decreased more rapidly in the fluid resuscitation group. Seven out of seven vasopressin treated animals survived, whereas six out of seven fluid resuscitations died. The effect of vasopressin with those of fluid resuscitation on haemodynamic variables and short term survival in an abdominal vascular injury model of uncontrolled haemorrhagic shock was observed.

Time(min)	BP(mm Hg)
0	95
10	45
20	35
30	33
40	32
50	30
60	19

Table: 1 Mean Arterial blood pressure before the administration of 0.08 U/ kg/ per min continuous infusion vasopressin.

Here μ = 41.29 are σ = 4.99 and assume t = 50

$$\overline{\mu}=[40,41.29,42]$$

$$\overline{\mu}(\alpha)=[40+1.29\alpha,42-0.71\alpha]$$

$$\sigma=[4.89,4.99,5.12]$$

$$\overline{\mu}(\alpha)=[4.89+0.1\alpha,5.12-0.13\alpha]$$

$$\overline{R}[\alpha]=[1-\phi[\frac{t-(40+1.29\alpha)}{4.89+0.1\alpha}],1-\phi\left[\frac{t-(42-0.71\alpha}{5.12-0.13\alpha}\right]]$$

α	$R_1[\alpha]$	$R_2[\alpha]$
0	0.5203	0.5591
0.1	0.5220	0.5570
0.2	0.5236	0.5550
0.3	0.5254	0.5530
0.4	0.5272	0.5511
0.5	0.5289	0.5492
0.6	0.5312	0.5473
0.7	0.5333	0.5455
0.8	0.5356	0.5438
0.9	0.5380	0.5421
1	0.5404	0.5404

Table : 2 Alpha – cuts of the fuzzy probability.

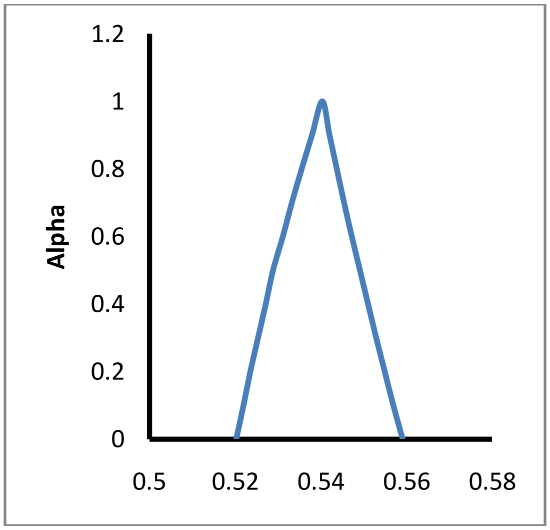

Fig:1 Fuzzy Probability

5. CONCLUSION

As the fuzzy value is greater than 0.5, the mean arterial blood pressure value be a member of the set. By the Fuzzy probability it was concluded that Vasopressin treatment ensured short term survival in the vascular injury model of uncontrolled haemorrhagic shock animals.

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