HALF-SYMMETRIC HSU-CONNECTIONS

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ABSTRACT. Half-symmetric F-connections have been defined and studied by some authors. Hsu-structure manifolds also play an important role in the theory of structures on manifolds. The aim of the present paper is to study some properties of Hsu-connections in a differentiable manifold.

1. *Preliminaries*

Let M^n be n-dimensional differentiable manifold of class C. Suppose there exist a tensor field F(0) of type (1,1) on the manifold M^n satisfying

$$F^2 = {}^r I \qquad (1.1)$$

where

$$= F(X), \qquad (1.2)$$

X is arbitrary vector field and any real or complex number. Then we say that the manifold M^n admits a Hsu-structure.

An affine connection D on the manifold M^n will be called a Hsu-connection if it satisfies

$$(D_X F)(Y) = 0 \qquad (1.3)$$

or equivalently

$$D_X =. \qquad (1.4)$$

If $S(X, Y)$ is a torsion tensor of D, we have

$$S(X, Y) = D_X Y - D_Y X - [X, Y]. \qquad (1.5)$$

Thus if a, b are real numbers,

$$S(ax + by) = abS(X, Y). \qquad (1.6)$$

Also $S(X, Y)$ is skew symmetric i.e.,

$$S(X, Y) = -S(Y, X). \qquad (1.7)$$

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Let us call the Hsu-connection D on the manifold M^n half-symmetric if $S(X, Y)$ satisfies

$$S(X, Y) = S(,) + \ + . \qquad (1.8)$$

2. *Some results*

In this section, we shall prove some theorems on Hsu-connections.

Theorem 2.1. *Lei D be an arbitrary affine connection on the manifold M^n. Then the connection B given by*

$$BxY = ({}^r\, D_x Y +) + (D_x +)$$
$$+ ({}^r +) + (D_x +) \qquad (2.1)$$

where , , , are C functions, is *also Hsu-connection on the manifold M^n.*

Proof: In view of (2.1), we have

$$B_X = ({}^r\, D_x +) + (D_x +)$$

$+ ({}^{r} +) + (+).$

By virtue of (1.1), the above equation takes the form

$$B_X = ({}^{r}\, D_x +) + ({}^{r}D_x Y +)$$
$$+ ({}^{r} +) + (+). \qquad (2.2)$$

Barring both sides of (2.1) and using (1.1), we have

$$= \ ({}^{r} \ + {}^{r}D_x) + (+ {}^{r}D_x Y)$$
$$+ ({}^{r} \ + {}^{r}) + (+ {}^{r}) \qquad (2.3)$$

In view of equations (2.2) and (2.3), it follows that

$$=$$

Hence B is a Hsu-connection on the manifold M^n.

Theorem 2.2. *Let D be an affine connection on the Hsu-structure manifold M^n. If S(X, Y) is a torsion tensor of D, the connection B given by*

$$B_X Y \quad = D x Y + ({}^{r}S(X,Y) +)$$
$$+ ({}^{r}S(,Y) +)$$
$$+ (S(X,) +)$$
$$+ (S(,) +) \qquad (2.4)$$

is a Hsu-connection on the manifold M^n.

Proof: Replacing Y by in (2.4) and using equations (1.1) and (1.6), we get

$$B_X \quad = Dx + ({}^{r}S(X,) + {}^{r})$$
$$+ ({}^{r}S(,) + {}^{r})$$
$$+ ({}^{r}\, S(X, \mathrm{Y}) +)$$
$$+ ({}^{r}S(, \mathrm{Y}) +) \qquad (2.5)$$

Barring both sides of (2.4) and using equations (1.1) and (1.4), we get

$$= Dx + ({}^{r} + {}^{r}\, S(X,))$$
$$+ ({}^{r} + {}^{r}S(,))$$
$$+ (+ {}^{r}S(X, \mathrm{Y}))$$
$$+ () + ({}^{r}S(, \mathrm{Y})) \qquad (2.6)$$

In view of equations (2.5 and (2.6), it follows that

$$B_X =$$

Hence B is a Hsu-connection on the manifold M^n.

Theorem 2.3 *Let D be an affine connection on the Hsu-structure manifold M^n. If S(X, Y) is a torsion tensor of D, the connection B given by*

$$B_X Y \quad = D_x Y + ({}^{r}S(X, Y) +)$$
$$+ ({}^{r}S(,Y) +)$$
$$+ ((X,) +)$$
$$+ ({}^{r}S(,) +) \qquad (2.7)$$

where , , , , are C functions, is a Hsu-connection on the manifold M^n.

Proof: The proof follows easily by virtue of theorem 2.2 and the fact that D is a Hsu-connection on the manifold M^n.

If N(X, Y) is the Nijenhuis tensor formed with F, then we have

$$\mathrm{N}(\mathrm{X}, \mathrm{Y}) = [,] - \ - + \qquad (2.8)$$

for arbitrary vector fields X, Y in the manifold M^n.

If N(X, Y) = 0, the structure is called integrable. Therefore we have the following:

Theorem 2.4. *Let D be a Hsu-connection on the manifold M^n with S(X, Y) as its torsion tensor, then the structure shall be integrable if and only if*

$$\mathrm{S}(,) + r\mathrm{S}(\mathrm{X}, \mathrm{Y}) = \ + \qquad (2.9)$$

Proof: The proof follows easily by using definitions of the torsion tensor $S(X, Y)$, Nijenhuis tensor $N(X, Y)$ and the properties of Hsu-connection D on the manifold M^n.

3. *Half-symmetric Hsu-connections*

Theorem 3.1. *Let D be a half-symmetric Hsu-connection on the manifold M^n. Then the connection B given by (2.1) is also half-symmetric provided that:*

(3.1)

Proof: In view of (2.1), we have

$B_X Y = (^r D_x Y +) + (^r D_x +)$

$+ (^r +) + (^r +).$

By virtue of (1.1) and (1.4), the above equation takes the form

$B_x Y = 2^r D_x Y + 2 + 2^r +$ (3.2)

If $s(X, Y)$ is the torsion tensor of the connection B,

$s(X, Y) = B_X Y - B_Y X - [X, Y].$ (3.3)

Substituting the value of $B_X Y$ etc. in (3.3) and on simplification, we get

$s(X, Y) = 2^r \{S(X, Y) + [X, Y]\} + 2\{+ \}$

$+ 2^r \{S(, Y)+[, Y]\}+ 2\{+ \}$

$– [X, Y]$

i.e.,

$s(X, Y) = 2^r S(X, Y) + 2 + 2^r S(, Y)$

$+ 2 + (2r – 1) [(X, Y] + 2$

$+ 2^r(, Y) + 2.$ (3.4)

Since D is half-symmetric, hence in view of (1.8) and (3.4) we have

$S(,) + + = 2^r S(X, Y) + 2$

$+ 2^r S(, Y) + + 2$

$+ (2^r – 1)\{+ + \}$

$+ 2\{ + + \}$

$+ 2^r \{+ + \}$

$+ 2 \{++ \}.$ (3.5)

If – –= $[X, Y]$, the equation (3.5) takes the form

$s(,) + + = 2^r S(X, Y) + 2$

$+ 2^r S(, Y) + 2$

$+ (2^r – 1)[X, Y] + 2$

$+ 2^r [, Y] + 2.$

Hence,

$s[, Y] + + = s(X, Y),$

Thus B is half-symmetric connection on M^n.

Theorem 3.2 *If D is a half-symmetric Hsu-connection on the manifold M^n. Then the connection B given by (2.4) is also half-symmetric provided that:*

(3.6)

Proof: In view of (2.4), we have

$BxY = (^r S(X, Y))$

$+ (^r S(, Y) +)$

$+ (S(X,) +)$

$+ (S(,) +).$

Let $s(X, Y)$ be the torsion tensor of the connections B. Putting the values of $B_X Y$ etc. in the equation (3.3) and applying the fact that

$S(X, Y)$ is skew-symmetric, we get

$$s(X, Y) = \{2^r S(X, Y) + \ - \ )$$
$$+ \ \{^r S(, Y) - {}^r S(, X) + 2\}$$
$$+ \{S(X,) - S(Y,) + 2\}$$
$$+ \{2S(,) + - \ - [X, Y]\}. \quad (3.7)$$

In view of (3.6), the equation (3.7) takes the form

$$s(X, Y) = 2^r S(X, Y) + 2 + 2$$
$$+ 2S(,) - [X, Y] \qquad (3.8)$$

Since D is half-symmetric, therefore in view of equations (1.8) and (3.8), we have

$$s(,) + \ + \ = \ 2^r S(X, Y) + 2$$
$$+ 2 + 2S(,)$$
$$- \{[,] + + \}. \qquad (3.9)$$

In view of (3.6), the equation (3.9) takes the form

$$s(,) + \ + \ = s(X, Y).$$

Thus B is half-symmetric connection on M^n.

REFERENCES

[1] **Duggal, K.L.,** *On differentiabie structures defined by algebraic equations,* Tensor, N.S., 22, 238-42 (1971).

[2] **Friedmann, A., and Schouten, J.A.,** *Uber die geometric der halbsym-metriscken iibertragung,* Math. Zeitschr, 21, 211-223 (1924).

[3] **Hamoui,** A., *Half-symmetric F-connections,* Journal of the Tensor Society of India, 2,21-29(1984).

[4] **Hayden, H.** A., *Subspaces of space with torsion,* Proc. Lou. Math. Soc., **34,** 27-50 (1932).

[5] **Hicks,** N.J., *Notes on differential Geometry,* D. Van Nostrand Company. Inc. Princeton, New York, (1965).

[6] **Mishra, R.S.,** *Structures on a differentiabie manifold and their applications,* Chandrama Prakashan, 50- A, Balrampur House, Allahabad, India (1984).

[7] Nivas, R. and Khan, N.I., *On manifolds admitting Hsu-metric structure,* Acta Cincia India, Vol. XXX M, No. 3, 645, 639-649 (2004).

[8] **Quan, P,H.,** *Introduction a la Geometric Desvarietes Differentiates,* Dunod, Paris, (1969).

[9] **Singh,** A., *On horizontal and complete lifts from a manifold with f*{9, - 1)- *Hsu-structure to its co-tangent bundle,* Journal of Rajasthan Academy of Physical Sciences, Vol. 5, no. 3, 275-282(2006).

[10] **Singh, A.,** *On CR-structures and F-$tructures satisfying* $F^{2K}+^{P} + F^{p} = 0$, Int. J. Contemp. Math. Sciences, Vol. 4, no. 21, 1029-1035(2009).

[11] **Yano, K.,** *On a structure defined by a tensor field f*(0) *of type* (1, 1) *satisfying* $f^{3} + f = 0$, Tensor, N. S., 14, 99-109 (1963).