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IMPLICATION RELATIONS IN FUZZY PROPOSITIONS

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ABSTRACT:
In crisp logic, the truth values acquired by propositions or predicates are 2-valued, namely, True, False which may be equivalent to {0,1}. But in fuzzy the truth values are multi-valued such as absolutely true, partially true, very true, absolutely false, and so on and are numerically equivalent to [0-1]. This work gives an idea of the logic that put forward the inference rules - Modus Ponens, Modus Tollens, Chain Rule and IF THEN rules and its compositional rule of inference.
Key Words : ***Fuzzy rule, Tautology, Compound Proposition, Interpretations.***

1. INTRODUCTION

Logic is the science of reasoning. Mathematical logic has turned out to be a powerful computational paradigm. Not only does mathematical logic help in the description of events in the real world but has also turned out to be an effective tool for inferring or deducing information from a given set of facts. Just as mathematical sets have been classified into crisp sets and fuzzy sets, logic can also be broadly viewed as crisp logic and fuzzy logic. Crisp sets survive on a two state membership (0/1) and fuzzy sets on a multi-state membership [0-1], crisp logic is built on (True/False) and fuzzy logic on a multi-state truth value (True/False/Very True/Partially false etc...). Inference is a technique by which, given a set of facts or premises $F_1, F_2, \ldots, F_n$, a goal G is to be derived. Various Authors applied fuzzy logic in different ways in solving complex problems (Zadah (1992, 1978) Dubois etal (1996, 2000), Singh T.P. (2012), G. Nirmala etal. (2013).

2. PRELIMINARIES

Definition 2.1: Proposition

A proposition is a declarative sentence that is either True or False.

Examples:

1. Is the colour of milk is white? Ans: True
2. Is 2+5=8? Ans: False

Definition2.2. Compound Proposition:

The formation of new proposition from existing proposition using logical operators or connectives is called compound proposition.

NAME	NICK NAMES	SYMBOLS
Negation	NOT	$\neg$
Conjunction	AND	$\wedge$
Disjunction	OR	$\vee$
Exclusive OR	XOR	$\oplus$
Implication	Implies	$\rightarrow$

Bi-conditional	If and only if	$\leftrightarrow$

Definition2.3. Propositional Equivalence:
Propositional equivalence is to replace a statement with another same truth value.

Examples:
I have a notebook and I have a pen

p = I have a notebook
q = I have a pen

Propositional equivalence of p and q : $\neg(p\wedge q) =(\neg p\vee q\neg)$

Definition2.4. Tautology:
A compound proposition p is tautology if every truth assignment satisfies true. i.e All entries of its truth values are true.

Examples:
1. $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
2. $p \vee p \leftrightarrow p$
3. $p \vee \neg (p\wedge q)$

Definition 2.4. Fuzzy Connectives:
A new statement is formed in fuzzy sets using connectives such as $\wedge$,$\vee$,$\Rightarrow$,$\neg$ are called fuzzy connectives.

Definition 2.5.Rule of Inference:
In logic, a rule of inference is a logical form consisting of a function, which takes premises, analyzes their syntax and returns a conclusion.

3: 3.1 Propositional Logic Connectives
The symbols $\wedge$,$\vee$,$\Rightarrow$,== are binary operators requiring two propositions while $\neg$ is a unary operator requiring a single proposition .$\vee$ and $\wedge$ operations are referred as disjunction and conjunction respectively. In the case of $\Rightarrow$ operator, the proposition occurring before the signal is called antecedent and the one occurring after is called as consequent. The semantics or meaning of the
Logical connectives are explained using a truth table. A truth table comprises rows known as interpretations, each of which evaluates the logical formula for the given set of truth values.

p	q	p$\wedge$q	p$\vee$q	$\neg$p	p$\Rightarrow$q	p=q
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	F	F	F	T	T	T
F	T	F	T	T	T	F

A logical formula comprising n propositions will have 2^n interpretations in its truth table. A formula which has all its interpretations is true is known as tautology and the one which is false is called contradiction.
Using this tautology we derive the equivalence of $(p \Rightarrow q) = (\neg p \vee q)$

p	**q**	**A:p⇒q**	**¬p**	**B:¬p∨q**	**A=B**
T	T	T	F	T	T
T	F	F	F	F	T
F	F	T	T	T	T
T	T	T	T	T	T

Here the last column yields "True" for all interpretations, it is a tautology. The logical formula presented above is of practical importance of $(p \Rightarrow q)$ is equivalent to $(\neg p \vee q)$ a formula devoid of " ⇒" connective. This equivalence can be applied to eliminate " ⇒" in logical formulae.

3.2.1 Inference in propositional Logic

In propositional logic, three rules are widely used for inferring facts, namely

(i) Modus Ponens:

Given $p \Rightarrow q$ and p to be true, q is true.

$$\frac{\begin{array}{c} p \Rightarrow q \\ p \end{array}}{q}$$

Here the formula above the line are the premises and the one below is the goal which can be inferred from the premises.

(ii) Modus tollens

Given $p \Rightarrow q$ and $\neg q$ to be true, $\neg p$ is true.

$$\frac{\begin{array}{c} p \Rightarrow q \\ \neg q \end{array}}{\neg p}$$

(iii) Chain rule

Given $p \Rightarrow q$ and $q \Rightarrow r$ to be true, $p \Rightarrow r$ is true

$$\frac{\begin{array}{c} p \Rightarrow q \\ q \Rightarrow r \end{array}}{p \Rightarrow r}$$

The chain rule is the representation of transitivity relation with respect to '$\Rightarrow$' connective.

3.3 Fuzzy Connectives

Symbol	Connective	Usage	Definition
-	Negation	$\bar{p}$	$1 - T(p)$
$\vee$	Disjunction	$p \vee q$	$\max (T(p),T(q))$
$\wedge$	Conjunction	$p \wedge q$	$\min (T(p),T(q))$
$\Rightarrow$	Implication	$p \Rightarrow q$	$\bar{p} \vee q = \max (1 - T(p), T(q))$

IF x is A THEN y is B, and is equivalent to

$$R = (A*B) \cup (A*Y)$$

The membership function of R is given by

$$\mu_R(x,y) = \max (\min (\mu_A(x),\mu_B(y)),1- \mu_A(x))$$

For the compound implication IF x is A THEN y is B ELSE y is C the relation R is given as

$$R = (A*B) \cup (\bar{A}*C)$$

The membership function of R is given by

$$\mu_R(x,y) = \max (\min (\mu_A(x),\mu_B(y),\min (1- \mu_A(x),\mu_C(y)))$$

4. DETERMINATION OF THE IMPLICATION RELATION

Let X = {a, b, c, d}; Y = {1, 2, 3, 4}

A = {(a,0.1), (b,0.6), (c,0.3), (d,1)}
B = {(1,0.2), (2,0.5), (3,0.8), (4,0)}
C = {(1,0), (2,0.3), (3,0.4), (4,0.8)}

Determine the implication relation

(i) IF x is A THEN y is B
(ii) IF x is A THEN y is B ELSE y is C

Solution:

To find (i) we know that,

$$R = (A*B) \cup (A*Y)$$

Where,

$$\mu_R(x,y) = \max (\min (\mu_A(x),\mu_B(y)),1- \mu_A(x))$$

Here (i) The operator " $*$ " represents the minimum of two sets .

(ii)The operator " ∪ " represents the maximum of two sets.

A * B =

	1	2	3	4
a	0.1	0.1	0.1	0
b	0.2	0.5	0.6	0
c	0.2	0.3	0.3	0
d	0.2	0.5	0.8	0

$\bar{A}$ * Y =

	1	2	3	4
a	0.9	0.9	0.9	0.9
b	0.4	0.4	0.4	0.4
c	0.7	0.7	0.7	0.7
d	0	0	0	0

R =

	1	2	3	4
a	0.9	0.9	0.9	0.9
b	0.4	0.5	0.6	0
c	0.7	0.7	0.7	0.7
d	0.2	0.5	0.8	0

Which represent IF x is A THEN y is B

To find (ii) we know that,

$R = (A*B) \cup (\bar{A}*C)$

$\mu_R(x,y) = \max(\min(\mu_A(x),\mu_B(y),\min(1- \mu_A(x),\mu_C(y)))$

A * B =

	1	2	3	4
a	0.1	0.1	0.1	0
b	0.2	0.5	0.6	0
c	0.2	0.3	0.3	0
d	0.2	0.5	0.8	0

$\bar{A}$ * C =

	1	2	3	4
a	1	1	1	1
b	0.7	0.7	0.7	0.7
c	0.6	0.6	0.6	0.6
d	0.2	0.2	0.2	0.2

$R = \max((A*B), (\bar{A} * C))$ gives

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0.7 & 0.7 & 0.7 & 0.7 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.2 & 0.5 & 0.8 & 0 \end{pmatrix} \end{array}$$

The above R represents IF x is A THEN y is B ELSE y is C

CONCLUSION:

This work has emphasized the rules of inference such as Modus ponens, Modus tollens, Chain rule and the laws of propositional logic are applicable for inferring propositional and predicate logic. Here we have obtained an inferred conclusion by applying the compositional rule of inference to the fuzzy implication relation.

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