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# DOMINATION IN FUZZY DIGRAPHS

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**ABSTRACT:**

***The purpose of this paper is to introduce the concept of fuzzy kernel of the fuzzy digraph also the concept of fuzzy absorbant of the fuzzy digraph. The theorems based on fuzzy kernel and fuzzy absorbant of the fuzzy digraphs are proved.***

***Keywords: The Fuzzy digraph-the fuzzy outset of a node and fuzzy inset of a node in a fuzzy digraph-the fuzzy indegree and fuzzy outdegree of a node in a fuzzy digraph -the fuzzy independence number of a fuzzy digraph-the fuzzy absorbant set of the fuzzy digraph-the fuzzy kernel of the fuzzy digraph.***

## 1. INTRODUCTION:

Rosenfeld [9] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A Somasundram and S. Somasundram [2] discussed domination in fuzzy graph. They defined domination using effective edges in fuzzy graph. Nagoor Gani and Chandrasekaran [1] discussed domination in fuzzy graph using strong arc.

Recently, G. Nirmala & M. Vijay [7] & G. Nirmala, K Dhanbal [8] & G. Nirmala & M. Sheela [6] in their studies discussed the various concepts on related with fuzzy graph along with their properties. Proceeding in the same chain, the domination number of the fuzzy digraph, the fuzzy kernel of the fuzzy digraph and the fuzzy absorbant of the fuzzy digraph are discussed in this paper. We also introduce some new concepts as fuzzy Euler digraph, fuzzy balanced digraph and fuzzy regular digraph.

### Preliminaries1.1

- A fuzzy subset of a nonempty set V is a mapping $\sigma: V \rightarrow [0,1]$
- A fuzzy relation on V is a fuzzy subset of $V \times V$.
- A fuzzy graph $G = (\sigma, \mu)$ is a pair of function $\sigma: V\rightarrow[0,1]$ and $\mu: V\times V\rightarrow[0,1]$ where $\mu(u, v) \leq \sigma(u) \Lambda\, \sigma(v)$ for $u, v \in V$.
- The underlying crisp graph of $G= (\sigma, \mu)$ is denoted by $G^*=(V,E)$ where $V=\{u \in V : \sigma(u)>0\}$ and $E=\{(u, v) \in V \times V : (u, v)>0\}$.
- The order p and size q of the fuzzy graph $G = (\sigma, \mu)$ are defined by
$$p= \sum_{v \in V}\sigma(v) \text{ and } q= \sum_{(u, v)\in E}\mu(u,v).$$
- Let G be a fuzzy graph on V and $S\subseteq V$, then the fuzzy cardinality of S is defined to be $\sum_{v \in S}\sigma(v)$.
- The strength of the connectedness between two nodes u , v in a fuzzy graph G is $\mu^{\infty}(u, v)=\sup\{\mu^{k}(u,v):k=1,2,3\ldots..\}$,where $\mu^{k}(u,v)=\sup\{\mu(u,u_1)\Lambda\, \mu(u_1,u_2)\ldots\ldots\ldots\Lambda\, \mu(u_{k-1},v)\}$.
- An arc (u, v) is said to be a strong arc or strong edge, if $(u, v) \geq \mu^{\infty}(u,v)$ and the node v is said to be a strong neighbor of u.

## 2.FUZZY DIGRAPHS

**Definition2.1**

A fuzzy digraph GD= (σD,μD) is a pair of function σD :V→[0,1] and μD : V×V→[0,1] where μD (u,v)≤ σD (u) Λ σD (v) for u,v ϵ V and μD is a set of fuzzy directed edges are called fuzzy arcs.

**Definition2.2**

An fuzzy arc μ(u,v) is said to be directed from σ(u) to σ(v) in which case σ(u) is said to be a predecessor of σ(v), σ(v) is a successor of σ(u) and σ(u)
dominates σ(v) . In this case we also use the notational equivalence μ(u,v)≡σ(u)→σ(v).

**Definition2.3**

In a fuzzy digraph GD a closed fuzzy directed walk which traverses every directed edges of GD exactly once is called a fuzzy directed Euler line. A fuzzy digraph containing a fuzzy directed Euler line is called an fuzzy Euler digraph

**Definition2.4**

The outset of σ(u) is the set σO (u) ={ σ(v): μ(u,v) ϵ μD }
The inset of σ(u) is the set σI (u)={ σ(w): μ(w,u) ϵ μD } we also define σO [u] = σO (u) ∪ {σ(u)} and σI [u] = σI (u) ∪ {σ(u)}

**Definition2.5**

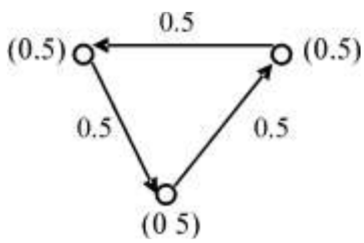
The outdegree of σ(u) is od σ(u)= │σO (u)│ and the indegree of σ(u)is inσ(u)=│ σI (u)│

**Definition2.6**

A fuzzy digraph is said to be fuzzy balanced digraph if for every node σ(vi) the indegree equals the out-degree

**Definition2.7**

A fuzzy balanced digraph is said to be fuzzy regular digraph if every node has the same in-degree and out-degree as every other node.




**Definition2.8**

Two nodes in a fuzzy digraphs GD are said to be fuzzy independent if there is no strong arc between them.

**Definition2.9**

A subset S of V is said to be fuzzy independent set of GD if every two nodes of S are fuzzy independent.

**Definition2.10**

The fuzzy independence number β0(GD) is the maximum cardinality of an independent set in GD.

**Definition2.11**

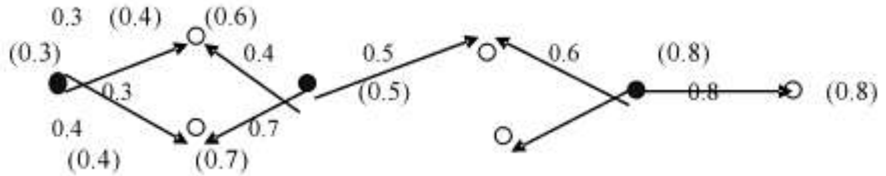
A set S ⊆ V is called fuzzy absorbant if for every σ(v) ЄV-S, there exist a σ(u) ϵ S which is a successor of σ(v) that is σ(v) →σ(u) is an arc in μD

**Definition2.12**

A subset S ⊆ V is a fuzzy dominating set of GD if every node σ(v) ЄV-S is dominated by at least one node σ(u) ϵ S.

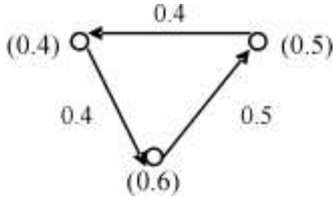
**Definition2.13**

The fuzzy domination number γ(GD) of a fuzzy digraph GD is the minimum cardinality of a fuzzy dominating set in GD



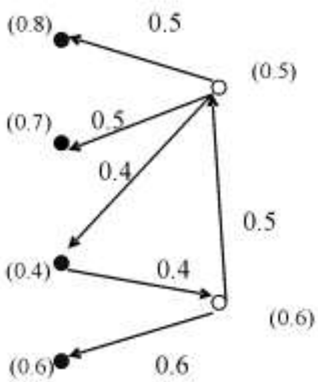

**Fuzzy Dominating set in Fuzzy Digraph**

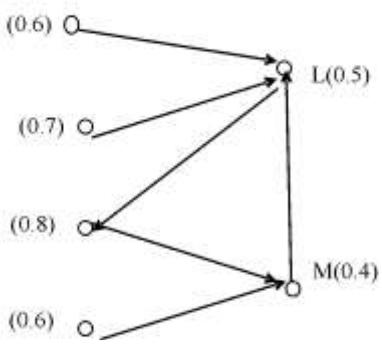
**Definition2.14**
A set S $\subseteq$V is a fuzzy kernel if it is both fuzzy independent and fuzzy absorbent.
**Definition2.15**
A set S $\subseteq$ V is called a fuzzy solution of a fuzzy digraph GD if S is both fuzzy independent and fuzzy dominating.
**Definition2.16**
A fuzzy tournament on n nodes is a fuzzy digraph GD = (σD,μD) in which for every pair σD(u),σD(v) of nodes,either μD (u,v) ϵ μD or μD (v,u) ϵ μD
but not both.




**Fuzzy Tournament on three nodes**

**Definition2.17**
A fuzzy symmetric digraph is a fuzzy digraph GD = (σD,μD) in which μD (u,v) ϵ μD implies μD (v,u) ϵ μD . fuzzy symmetric digraphs correspond one-to-one with fuzzy undirected graphs.
**Definition2.18**
A fuzzy digraph GD = (σD,μD) is fuzzy transitive if whenever μD (u,v) and μD (v,w) ϵ μD then μD (u,w) ϵ μD .
**Theorem 2.1**
If S is a fuzzy kernel of a fuzzy digraph GD = (σD,μD) ,then S is both fuzzy maximal independent and fuzzy minimal absorbant.
**Proof :**



**Fuzzy digraph with fuzzy kernel and without fuzzy kernels**

Let S be a fuzzy kernel of a fuzzy digraph GD . assume that S is not fuzzy maximal independent. Then there exist a node σ(u) ЄV-S which is not joined by an fuzzy arc to any node in S. but this implies that S is not fuzzy absorbant since there is no node in S which is a successor of σ(u); this contradicts the assumption that S is fuzzy kernel. Similarly, assume that S is not a fuzzy minimal absorbant set. Hence, there is a node σ(u) ЄS for which S-{ σ(u) } is fuzzy absorbent. But if S-{ σ(u) } is fuzzy absorbant ,then there must be a node in S-{ σ(u) } which is a successor ofσ(u). but this implies that S is not fuzzy independent, again contradicting the assumption that S is a fuzzy kernel.

**Theorem 2.2**

If GD is a fuzzy symmetric digraph, then GD has a fuzzy kernel and $S \subseteq V$ is a fuzzy kernel if and only if S is a fuzzy maximal independent set.

**Proof:**

Every fuzzy digraph has at least one fuzzy maximal independent set. Let $S \subseteq V$ be any fuzzy maximal independent set in a fuzzy symmetric digraph GD and let σ(u)V-S. since S is fuzzy maximal independent, there must be at least one fuzzy arc between σ(u)and node in S .but since GD is fuzzy symmetric, there must then be an fuzzy arc from σ(u)to a node in S, that is, S is fuzzy absorbent. Therefore GD has a fuzzy kernel S. Conversely,let S be a fuzzy kernel of GD then by theorem2.1 GD must be fuzzy maximal independent.

**CONCLUSION:**

In this paper we define the concepts of fuzzy digraphs, fuzzy symmetric digraph . Next we introduce the fuzzy tournament and further we proved the theorems based on fuzzy kernel of a fuzzy digraph.

**REFERENCES:**


1. A.Nagoor Gani, and V.T.Chandrasekaran, “Domination in Fuzzy Graph”, Advances in Fuzzy Sets and Systems, 1 (1) (2006), 17-26.
2. A.Somasundaram, S.Somasundaram, “Domination in Fuzzy Graphs-1”, Elsevier science, 19 (1998) 787-791.
3. B.Bollabas, E.J.Cockayne, “Graph Theoretic Parameters Concerning Domination, Independence, and Irredundance”, Journal of Graph Theory, Vol-3 (1979) 241-249.
4. B.R.Allan and R.Laskar “On domination and Independent domination numbers of a graph”, Discrete Mathematics, 23, (1978) 73-76.
5. E.J.Cockayne, O.Favaron, C.Payan, and Thomason, “A.G. Contribution to The Theory of Domination Independence and Irredundance in Graph”, Discrete Mathematics 33 (1981) 249-258.
6. G.Nirmala , M.Sheela, “Global and Factor Domination in Fuzzy Graph”, International Journal of scientific and Research Publications, Volume2,Issue6, June2012 ISSN 2250-3153
7. G. Nirmala, M. Vijaya “Strong Fuzzy Graph on Composition, Tensor & Normal Products” Aryabhatta J. of Maths & Info. Vol. 4 (2) [2012] pp. 343-350.
8. G. Nirmala & K. Dhamabal “Algorithm for Construction of self complementary Kv1, v2 Fuzzy Graph” Aryabhatta J. of Mathematics & Informatics Vol. 5 [1] [2013] pp 93-100.
9. K.R.Bhutani, and A.Rosenfeld, “Strong arcs in fuzzy graphs”, Information Sciences, 152 (2003), 319-322.
10. O.Faravan, “Stability, Domination and Irredundance in Graph”, Journal of Graph Theory, Vol-10 (1986) 429-438.
11. T.Haynes, S.T.Hedetniemi., P.J.Slater, “Fundamentals of Domination in Graph”, Marcel Deckker, New York, 1998.