Wave propagation at the imperfect boundary between two fluid saturated incompressible porous solids

Neelam Kumari[1] and Vinod Kaliraman[2]

Department of Mathematics,
Chaudhary Devi Lal University, Sirsa-125055, India.

Abstract

The present investigation is concerned with the reflection and transmission phenomenon at an imperfectly bonded interface between two different fluid saturated porous half spaces. P-wave or SV-wave incidents on the interface. The amplitude ratios for various reflected and transmitted waves to that of incident wave are obtained and hence deduced for normal force stiffness, transverse force stiffness and for perfect bonding. After finding the amplitude ratios, they have been computed numerically for a specific model and results thus obtained are depicted graphically to understand the behaviour of amplitude ratios with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of the considered medium and also these are affected by the stiffness. Also a special case of empty porous solid is obtained and discussed from the present study accordingly.

Keywords: Porous solid, reflection, transmission, longitudinal wave, amplitude ratios, stiffness, empty porous solid.

1. Introduction

The mechanical behaviour of fluid saturated porous material especially liquid filled pores is difficult to describe with the help of classical theory. Due to the different motions of the solid and liquid phases and different material properties and the complicated structures of pores, the mechanical behaviour of a fluid saturated porous medium is very complex. So many researchers tried to overcome this difficulty from time to time.

Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed a theory for incompressible fluid saturated porous medium based on the work of Fillunger model (1913). For example, in the composition of soil both the solid constituents and liquid constituents are incompressible. Based on this theory, many researchers like de Boer and Liu(1994,1995), Kumar and Hundal (2003),de Boer and Didwania (2004), Tajuddin and Hussaini (2006),Kumar et.al.(2011) etc. studied some problems of wave propagation in fluid saturated porous media. Imperfect interface considered in this problem means that the stress components are continuous and small displacement field is not. The infinite values of interface parameters imply vanishing of displacement jumps and therefore correspond to perfect interface conditions. On the other hand, zero values of the interface parameters imply vanishing of the corresponding interface tractions which corresponds to complete debonding. Any finite values of the interface parameters define an imperfect interface. The values of the interface parameters depend upon the material properties of the medium i.e. microstructure as well as the bi-material properties. Recently, Chen et.al. (2004), Kumar and Rupender (2009) and Kumar and Chawala (2010) etc. used the imperfect conditions at an interface to study the various types of wave problems.

Using de Boer and Ehlers (1990) theory for fluid saturated porous medium, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SV-wave) at an imperfect interface between two different fluid saturated porous half spaces is investigated. A special case when fluid saturated porous half spaces reduce to empty porous solid half spaces has been deduced and discussed accordingly. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences.

2. Basic equations

In 1990, de Boer and Ehlers described the governing equations for the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces as

$$\nabla.(\eta^S \dot{\mathbf{u}}^S + \eta^F \dot{\mathbf{u}}^F) = 0, \tag{1}$$

$$(\lambda^S + \mu^S)\nabla(\nabla.\mathbf{u^S}) + \mu^S \nabla^2 \mathbf{u^S} - \eta^S \nabla p - \rho^S \ddot{\mathbf{u}}^S + S_v(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \tag{2}$$

$$\eta^F \nabla p + \rho^F \ddot{\mathbf{u}}^F + S_v(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \tag{3}$$

$$\mathbf{T_E^S} = 2\mu^S \mathbf{E^S} + \lambda^S (E^S.\mathbf{I})\mathbf{I}, \tag{4}$$

$$\mathbf{E^S} = \frac{1}{2}(\text{grad}\,\mathbf{u^S} + \text{grad}^T \mathbf{u^S}), \tag{5}$$

where $\mathbf{u}^i$, $\dot{\mathbf{u}}^i$, $\ddot{\mathbf{u}}^i$, $i = F, S$ denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid constituents, respectively. $\mathbf{T_E^S}$ is the effective stress in the solid phase and $\mathbf{E^S}$ is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{6}$$

In the case of isotropic permeability, to describe the coupled interaction between the solid and fluid, de Boer and Ehlers (1990) gave the tensor $\mathbf{S}_v$ as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K}\mathbf{I}, \tag{7}$$

where γ^{FR} is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and $\mathbf{I}$ stands for unit vector.

The displacement vector $\mathbf{u}^i$ $(i = F, S)$ can be assumed as

$$\mathbf{u}^i = (u^i, 0, w^i), \quad \text{where} \quad i = F, S, \tag{8}$$

and therefore equations (1)- (3) describing the equations of motion for fluid saturated incompressible porous medium in the component form can be written as

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{9}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{10}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{11}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{12}$$

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t}+\frac{\partial^2 w^S}{\partial z\,\partial t}\right]+\eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t}+\frac{\partial^2 w^F}{\partial z\,\partial t}\right]=0, \tag{13}$$

where

$$\theta^S=\frac{\partial(u^S)}{\partial x}+\frac{\partial(w^S)}{\partial z}. \tag{14}$$

and

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}. \tag{15}$$

With the help of Helmholtz decomposition of displacement vector, the displacement components u^i and w^i are related to the potential functions ϕ^i and ψ^i as given below

$$u^i=\frac{\partial\phi^i}{\partial x}+\frac{\partial\psi^i}{\partial z},\quad w^i=\frac{\partial\phi^i}{\partial z}-\frac{\partial\psi^i}{\partial x},\qquad i=F,S. \tag{16}$$

Using, (16) in eqs. (9)- (13), we obtain the following equations:

$$\nabla^2\phi^S-\frac{1}{C^2}\frac{\partial^2\phi^S}{\partial t^2}-\frac{S_v}{(\lambda^S+2\mu^S)(\eta^F)^2}\frac{\partial\phi^S}{\partial t}=0, \tag{17}$$

$$\phi^F=-\frac{\eta^S}{\eta^F}\phi^S, \tag{18}$$

$$\mu^S\nabla^2\psi^S-\rho^S\frac{\partial^2\psi^S}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{19}$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{20}$$

$$(\eta^F)^2 p-\eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2}-S_v\frac{\partial\phi^S}{\partial t}=0, \tag{21}$$

where

$$C=\sqrt{\frac{(\eta^F)^2(\lambda^S+2\mu^S)}{(\eta^F)^2\rho^S+(\eta^S)^2\rho^F}}. \tag{22}$$

The normal and tangential stresses in the solid phase take the form,

$$t_{zz}{}^S=\lambda^S\left(\frac{\partial^2\phi^S}{\partial x^2}+\frac{\partial^2\phi^S}{\partial z^2}\right)+2\mu^S\left(\frac{\partial^2\phi^S}{\partial z^2}-\frac{\partial^2\psi^S}{\partial x\,\partial z}\right), \tag{23}$$

$$t_{zx}{}^S=\mu^S\left(2\frac{\partial^2\phi^S}{\partial x\,\partial z}+\frac{\partial^2\psi^S}{\partial z^2}-\frac{\partial^2\psi^S}{\partial x^2}\right). \tag{24}$$

Taking the time harmonic solution of the system of equations (17) - (21) as

$$(\phi^S,\phi^F,\psi^S,\psi^F,p)=\left(\phi_1{}^S,\phi_1{}^F,\psi_1{}^S,\psi_1{}^F,p_1\right)\exp(i\omega t), \tag{25}$$

where ω is the complex circular frequency.

Using equation (25) in equations (17)-(21), we get

$$\left[\nabla^2 + \frac{\omega^2}{C^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\right]\phi_1{}^S = 0, \tag{26}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]\psi_1{}^S = -i\omega S_v \psi_1{}^F, \tag{27}$$

$$[-\omega^2\rho^F + i\omega S_v]\psi_1{}^F - i\omega S_v\psi_1{}^S = 0, \tag{28}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2\phi_1{}^S - i\omega S_v\phi_1{}^S = 0, \tag{29}$$

$$\phi_1{}^F = -\frac{\eta^S}{\eta^F}\phi_1{}^S. \tag{30}$$

Equation (26) represents the propagation of a longitudinal wave with velocity V_1, where

$$V_1{}^2 = \frac{1}{G_1}, \tag{31}$$

and $$G_1 = \left[\frac{1}{C^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. \tag{32}$$

Using equations (27) and (28), we get

$$\left[\nabla^2 + \frac{\omega^2}{V_2{}^2}\right]\psi_1{}^S = 0. \tag{33}$$

Equation (33) describes the propagation of transverse wave with velocity V_2, which is given by

$$V_2{}^2 = \frac{1}{G_2},$$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v{}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}, \tag{34}$$

3. Formulation of the problem and its solution. Consider a fluid saturated porous half space medium M_2 $[z < 0]$ lying over another fluid saturated porous medium M_1 $[z > 0]$ (see figure1).The interface between two half spaces is considered an imperfect boundary and taking the z-axis pointing into lower half-space. A longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through the medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M_1 and two transmitted waves P-wave and SV-wave in medium M_2. The Geometry of the problem conforms the two dimensional problem.

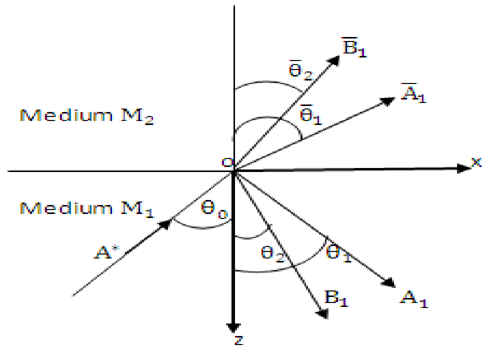


Fig.1 Geometry of the problem

In medium M_1

The potential functions satisfying the equations (17)-(21) can be taken as

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01}\ \exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1\exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}], \tag{35}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01}\ \exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + B_1\exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}], \tag{36}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S\omega_1{}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2\rho^F}, \tag{37}$$

and A_{01} and B_{01} are amplitudes of the incident P-wave and SV-wave, respectively. A_1 , B_1 are amplitudes of the reflected P-wave and SV-wave respectively.

In medium M_2

The potential functions satisfying the equations (17)-(21) can be taken as follow

$$\left\{\bar{\phi}^S, \bar{\phi}^F, \bar{p}\right\} = \{1, \bar{m}_1, \bar{m}_2\}\left[\bar{A}_1\exp\left\{i\bar{k}_1\left(x\sin\bar{\theta}_1 + z\cos\bar{\theta}_1\right) + i\bar{\omega}_1 t\right\}\right], \tag{38}$$

$$\left\{\bar{\psi}^S, \bar{\psi}^F\right\} = \{1, \bar{m}_3\}\left[\bar{B}_1\exp\left\{i\bar{k}_2\left(x\sin\bar{\theta}_2 + z\cos\bar{\theta}_2\right) + i\bar{\omega}_2 t\right\}\right], \tag{39}$$

where $\bar{k}_1$ and $\bar{k}_2$ are wave numbers of transmitted P-wave and SV-wave, respectively. $\bar{A}_1$ and $\bar{B}_1$ are amplitudes of transmitted P-wave and transmitted SV-wave.

and

$$\bar{m}_1 = -\frac{\bar{\eta}^S}{\bar{\eta}^F}, \quad \bar{m}_2 = -\left[\frac{\bar{\eta}^S\bar{\omega}_1{}^2\bar{\rho}^F - i\bar{\omega}_1\overline{S_v}}{\left(\bar{\eta}^F\right)^2}\right], \quad \bar{m}_3 = \frac{i\bar{\omega}_2\overline{S_v}}{i\bar{\omega}_2\overline{S_v} - \bar{\omega}_2{}^2\bar{\rho}^F}, \tag{40}$$

4. Boundary conditions. The appropriate boundary conditions at the interface z=0 are the continuity of displacement and stresses. These boundary conditions can be expressed in the mathematical form as:

$$t_{zz}{}^S - p = \overline{t_{zz}}^S - \bar{p}, \quad t_{zx}{}^S = \overline{t_{zx}}^S,$$

$$\overline{t_{zz}}^{S} - \overline{p} = K_n\left(w^S - \overline{w}^S\right), \quad \overline{t_{zx}}^{S} = K_t\left(u^S - \overline{u}^S\right), \tag{41}$$

In order to satisfy the boundary conditions, the extension of the Snell's law gives

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\overline{\theta}_1}{\overline{V}_1} = \frac{\sin\overline{\theta}_2}{\overline{V}_2}, \tag{42}$$

Also

$$k_1 V_1 = k_2 V_2 = \overline{k}_1 \overline{V}_1 = \overline{k}_2 \overline{V}_2 = \omega, \quad \text{at } z = 0, \tag{43}$$

where $\overline{V}_1$ and $\overline{V}_2$ are the velocities of the transmitted P-wave and transmitted SV –wave respectively and can be obtained in the similar way as V_1 and V_2 are obtained.

For P-wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{44}$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{45}$$

For incident longitudinal wave at the interface z=0, putting $B_{01} = 0$ in equation (36) and for incident transverse wave putting $A_{01} = 0$ in equation (35). Substituting the expressions of potentials given by (35)-(36) and(38)-(39) in equations (16),(23)-(24) and the use of equations (41)-(45), gives a system of four non homogeneous which can be written as

$$\sum_{j=0}^{4} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4) \tag{46}$$

where

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{A_2}{A^*}, \quad Z_3 = \frac{\overline{A}_1}{A^*}, \qquad Z_4 = \frac{\overline{B}_1}{A^*} \tag{47}$$

Also a_{ij} in non dimensional form can be written as

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S {k_1}^2}, \qquad a_{12} = -2\sin\theta_2\cos\theta_2 \frac{{k_2}^2}{{k_1}^2}, \qquad a_{13} = \frac{-{\overline{k}_1}^2}{{k_1}^2 \mu^S}\left[\overline{\lambda}^S + 2\overline{\mu}^S\cos^2\overline{\theta}_1 + \frac{\overline{m}_2}{{\overline{k}_1}^2}\right],$$

$$a_{14} = -\frac{\overline{\mu}^S {\overline{k}_2}^2}{{k_1}^2 \mu^S}\sin 2\overline{\theta}_2, \quad a_{21} = 2\sin\theta_1\cos\theta_1, \quad a_{22} = \frac{{k_2}^2}{{k_1}^2}(\cos^2\theta_2 - \sin^2\theta_2), \quad a_{23} = \frac{\overline{\mu}^S {\overline{k}_1}^2}{{k_1}^2 \mu^S}\sin 2\overline{\theta}_1,$$

$$a_{24} = -\frac{\overline{\mu}^S {\overline{k}_2}^2}{{k_1}^2 \mu^S}\cos 2\overline{\theta}_2, \quad a_{31} = i\sin\theta_1, \quad a_{32} = \frac{i\, k_2\cos\theta_2}{k_1}, \quad a_{33} = -\frac{i\,\overline{k}_1}{k_1}\sin\overline{\theta}_1 - \frac{\overline{\mu}^S {\overline{k}_1}^2 \sin 2\overline{\theta}_1}{K_t\, k_1}$$

$$a_{34} = \frac{i\,\overline{k}_2\cos\overline{\theta}_2}{k_1} + \frac{\overline{\mu}^S {\overline{k}_2}^2 \cos 2\overline{\theta}_2}{K_t\, k_1}, \quad a_{41} = i\cos\theta_1, \quad a_{42} = -\frac{i\, k_2\sin\theta_2}{k_1},$$

$$a_{43} = \frac{\overline{i\,\bar{k}_1\cos\bar{\theta}_1}}{k_1} + \frac{\bar{k}_1^{\;2}\left[\bar{\lambda}^S + 2\bar{\mu}^S\cos^2\bar{\theta}_1 + \frac{\overline{m}_2}{\bar{k}_1^{\;2}}\right]}{K_n\, k_1}, \qquad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1} + \frac{\bar{\mu}^S\bar{k}_2^{\;2}\sin2\bar{\theta}_2}{K_n\, k_1}, \tag{48}$$

For incident longitudinal wave (P-wave)

$$A^* = A_{01},\; Y_1 = -a_{11},\; Y_2 = a_{21}, \qquad Y_3 = -a_{31},\; Y_4 = a_{41}, \tag{49}$$

For incident transverse wave:

$$A^* = B_{01},\; Y_1 = a_{12},\; Y_2 = -a_{22},\; Y_3 = a_{32},\; Y_4 = -a_{42}, \tag{50}$$

5. Particular cases:

Case I: Normal force stiffness $(K_n \neq 0, K_t \rightarrow \infty)$

In this case, we obtain a system of four non homogeneous equations similar to those given by equation (48) with the changed a_{ij} as follows

$$a_{33} = -\frac{i\,\bar{k}_1}{k_1}\sin\bar{\theta}_1\,, \qquad a_{34} = \frac{i\,\bar{k}_2\cos\bar{\theta}_2}{k_1}, \tag{51}$$

Case II: Transverse force stiffness $(K_t \neq 0, K_n \rightarrow \infty)$

In this case also, a system of four non homogeneous equations as those given by equation (48) is obtained but with some a_{ij} changed as

$$a_{43} = \frac{\overline{i\,\bar{k}_1\cos\bar{\theta}_1}}{k_1}, \qquad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1}, \tag{52}$$

Case III: Welded contact $(K_n \rightarrow \infty, K_t \rightarrow \infty)$

Again in this case, a system of four non homogeneous equations is obtained as those given by equation (48) with some a_{ij} changed as

$$a_{33} = -\frac{i\,\bar{k}_1}{k_1}\sin\bar{\theta}_1, \qquad a_{34} = \frac{i\,\bar{k}_2\cos\bar{\theta}_2}{k_1}, \qquad a_{43} = \frac{\overline{i\,\bar{k}_1\cos\bar{\theta}_1}}{k_1}, \qquad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1}, \tag{53}$$

Special case:-

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \tag{54}$$

and the coefficients a_{11}, a_{13} and a_{43} in (48) changes to

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1, \qquad a_{13} = \frac{-\bar{k}_1^{\;2}}{k_1^{\;2}\mu^S}\left[\bar{\lambda}^S + 2\bar{\mu}^S\cos^2\bar{\theta}_1\right],$$

$$a_{43} = \frac{\overline{i\,k}_1 \cos\bar{\theta}_1}{k_1} + \frac{\bar{k}_1^{\,2}\left[\bar{\lambda}^S + 2\bar{\mu}^S \cos^2\bar{\theta}_1\right]}{K_n\, k_1} \tag{55}$$

and the remaining coefficients in (48) remain same. In this situation the problem reduces to the problem of empty porous solid half space over empty porous solid half space.

6. Numerical results and discussion

After obtaining the theoretical results in above sections, we have computed them numerically by taking the following values of relevant elastic parameters to study in more detail the behaviour of various amplitude ratios.

In medium M_1, the physical parameters for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^s = 0.67, \quad \eta^F = 0.33, \quad \rho^s = 1.34\ \mathrm{Mg/m^3}, \quad \rho^F = 0.33\ \mathrm{Mg/m^3}, \quad \lambda^s = 5.5833\ \mathrm{MN/m^2},$$

$$K^F = 0.01\mathrm{m/s}, \quad \gamma^{FR} = 10.00\mathrm{KN/m^3}, \quad \mu^s = 8.3750\mathrm{N/m^2}, \quad \omega^* = 10/\mathrm{s}, \tag{56}$$

In medium M_2, the physical parameters are

$$\bar{\eta}^s = 0.6, \quad \bar{\eta}^F = 0.4, \quad \bar{\rho}^s = 2.0\ \mathrm{Mg/m^3}, \quad \bar{\rho}^F = 0.01\ \mathrm{Mg/m^3}, \quad \bar{\lambda}^s = 4.2368\ \mathrm{MN/m^2},$$

$$\bar{K}^F = 0.02\ \mathrm{m/s}, \quad \bar{\gamma}^{FR} = 9.00\ \mathrm{KN/m^3}, \quad \bar{\mu}^s = 3.3272\mathrm{N/m^2}, \quad \omega^* = 10/\mathrm{s}, \tag{57}$$

Also, we take $K_n = 0.5, \quad K_t = 0.25,$ (58)

Using MATLAB, a computer programme has been developed and modulus of amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ for various reflected and transmitted waves have been computed. $|Z_1|$ and $|Z_2|$ represent the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also, $|Z_3|$ and $|Z_4|$ represent the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively. Dashed dotted line represents the variations of the amplitude ratios for imperfect boundary, dotted line correspond to transverse force stiffness, dashed line for normal force stiffness and solid line for welded contact in all the figures (2)-(9) w.r.t. angle of incidence of the incident P or SV-wave. Also when medium M_1 reduces to empty porous solid, the variations of the amplitude ratios for this case are represented by figures (10)-(17). In these figures EGEN denotes the curve for imperfect boundary, ENFS denote normal force stiffness, and ETFS denote transverse force stiffness, EWD for welded contact. The variations in all the figures are shown for the range $0^0 \le \theta \le 90^0$.

Incident P-wave

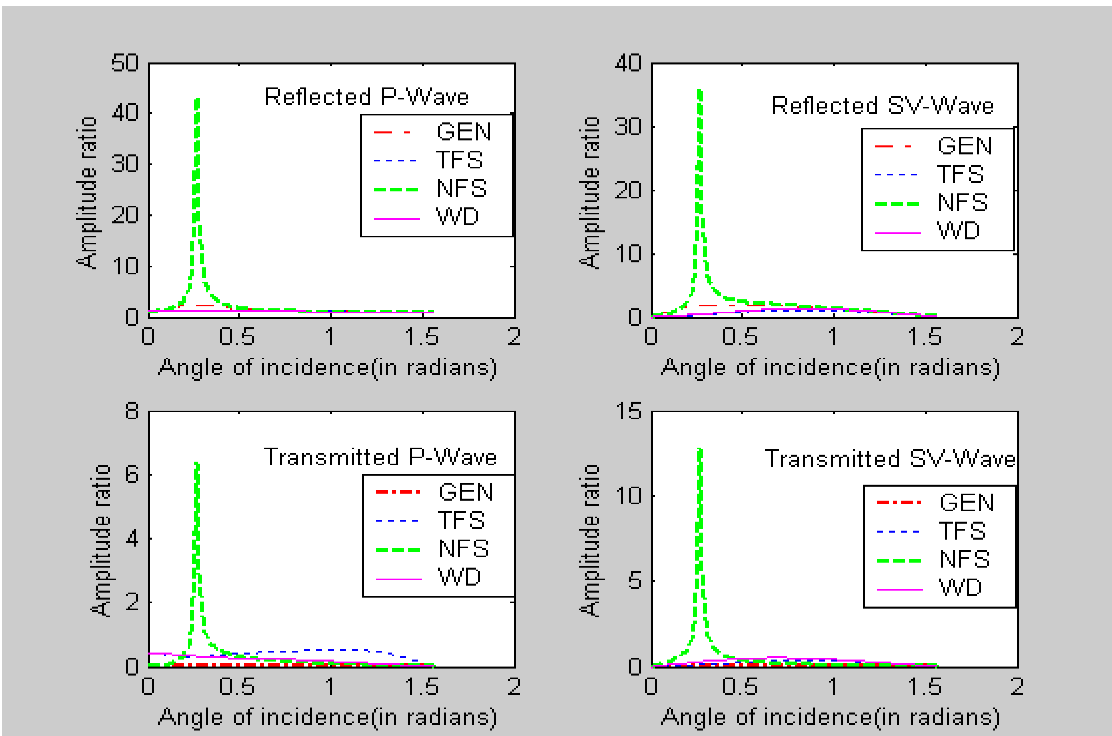
Figures (2)-(5) show the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave. The behaviour of all these distribution curves is similar i.e. increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence. Figures (10)-(13) show the variations of the amplitude ratios $|Z_i|$ with angle of incidence of the incident P wave in case of empty porous solid. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (2)-(5) and (10)-(13).

Incident SV-wave

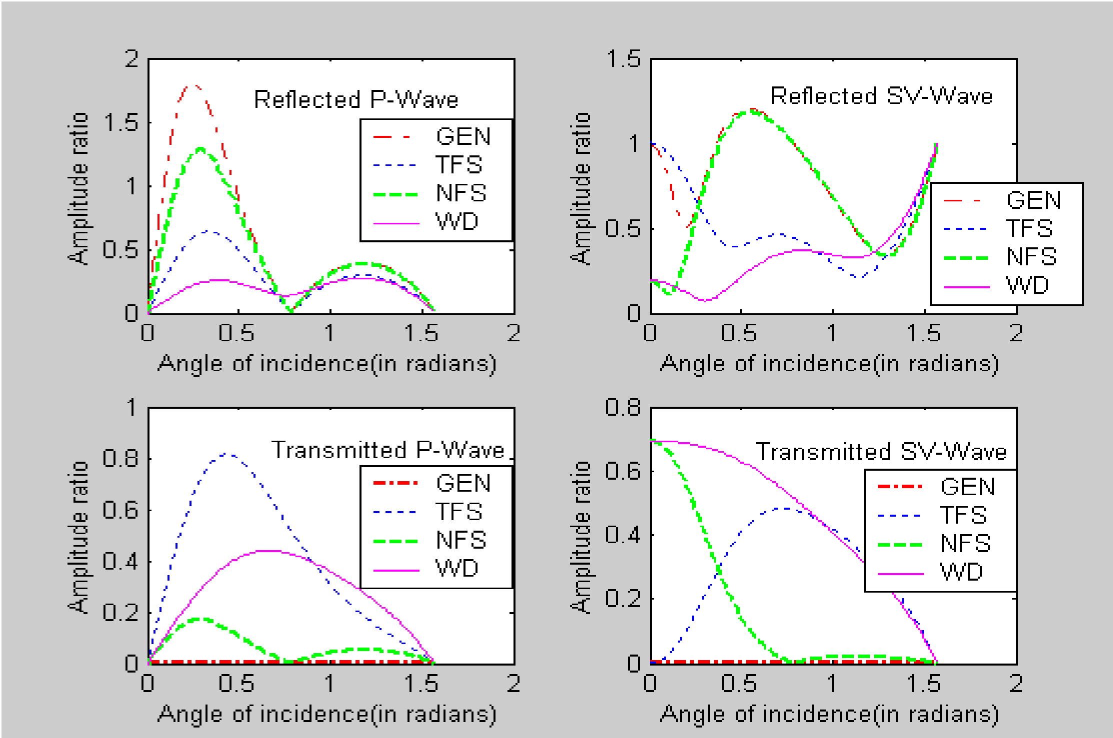
Figures (6)-(9) show the variations of the amplitude ratios for reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of the incident SV-wave whereas figures (14)-(17) represent the case of empty porous solid. The behaviour of all these curves in figures (6)-(9) and (14)-(17) is same i.e. they oscillates. In figures (6)-(9), the amplitude ratios for reflected P-wave and for reflected SV-wave, the values for imperfect boundary (GEN) are greater than other stiffness cases whereas in case of transmitted P-wave and transmitted SV-wave, the values for imperfect boundary (GEN) are smaller than the values of other stiffness cases. In figures (14)-(17), the values for reflected P-wave are greater than other stiffness cases and the values for imperfect boundary (EGEN) for transmitted P-wave and transmitted SV-wave are smaller than other stiffness cases.

7. Conclusion

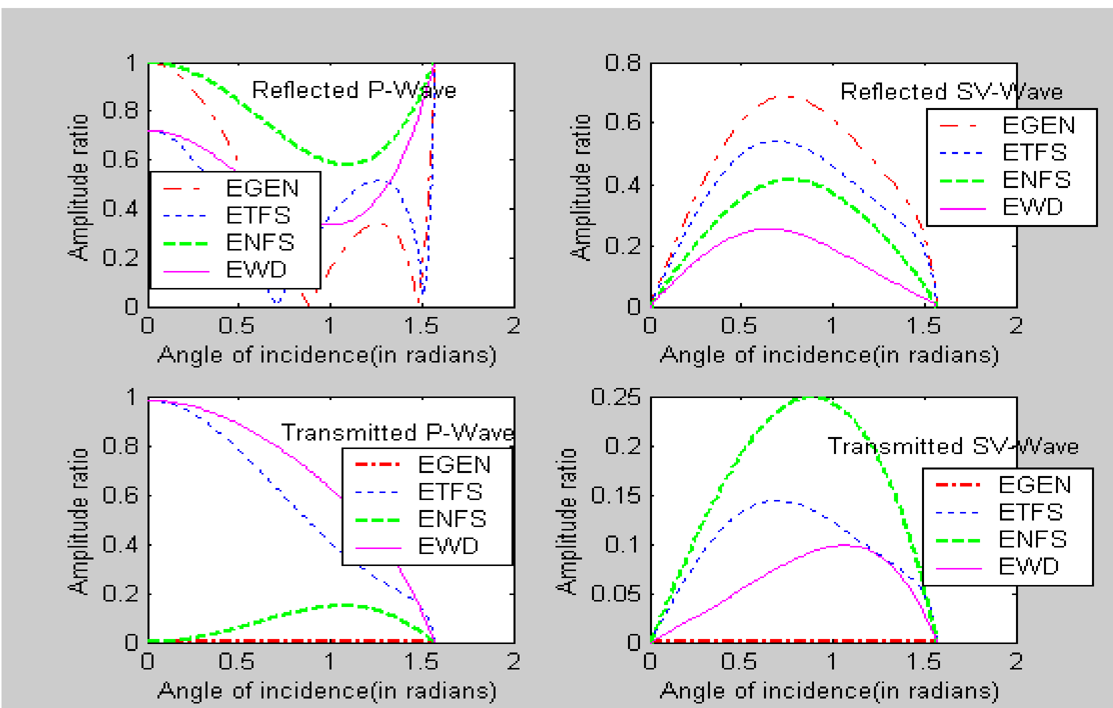
Reflection and transmission phenomenon of incident elastic waves at an imperfect interface between viscoelastic solid half space and fluid saturated porous half space has been studied when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios. Effect of stiffness is observed on amplitude ratios. The research work is supposed to be useful in further studies; both theoretical and observational of wave propagation in more realistic models of fluid saturated porous solid present in the earth's interior. The problems may be of use in engineering, seismology and geophysics etc.



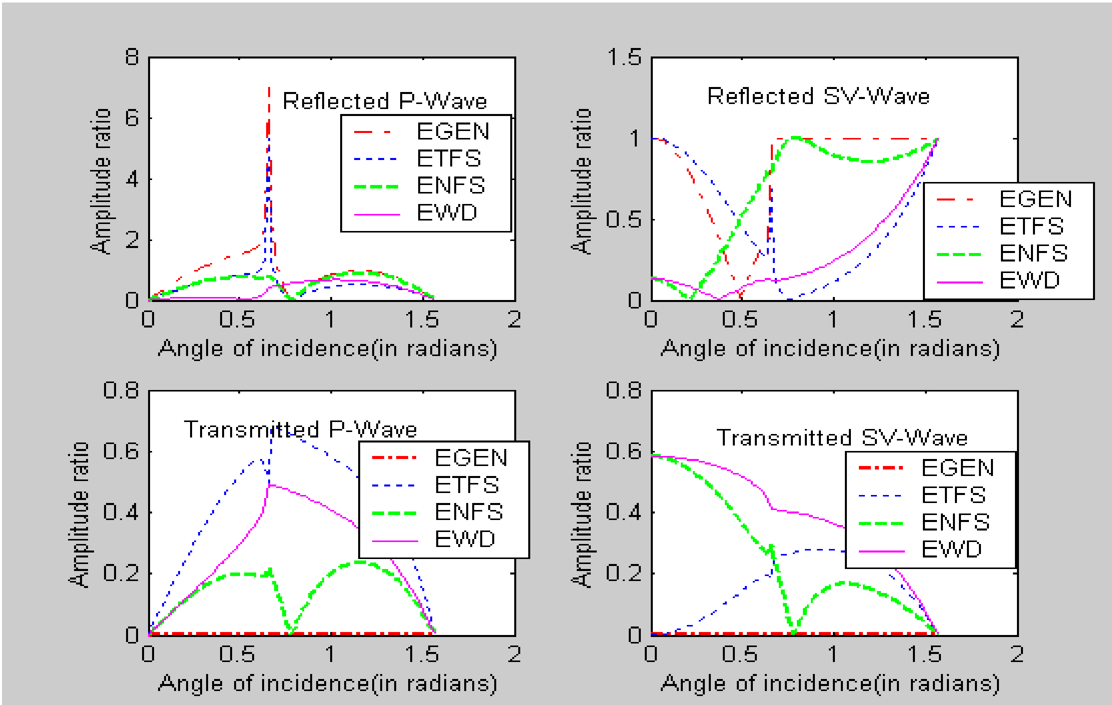
Figures 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave.



Figures 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



Figures 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solid.



Figures 14-17. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave in case of empty porous solid.

8. References

[1] Chen W.Q., Cai J.B., Ye G.R., Wang Y.F., *Exact three-dimensional solutions of laminated orthotropic piezoelectric rectangular plates featuring interlaminar bonding imperfections modeled by a general spring layer*, International Journal of Solid and Structures,41,5247-5263,2004.

[2] Bowen, R.M., *Incompressible porous media models by use of the theory of mixtures*, International Journal of Engineering Science, 18(9), 1129-1148, 1980.

[3] de Boer, and Didwania, A. K., *Two phase flow and capillarity phenomenon in porous solid- A Continuum Thermomechanical Approach*, Transport in Porous Media (TIPM), 56, 137-170, 2004.

[4] de Boer, R. and Ehlers, W., *The development of the concept of effective stress*, Acta Mechanica A 83, 77-92,1990.

[5] de Boer, R. and. Ehlers, W ,*Uplift, friction and capillarity-three fundamental effects for liquid- saturated porous solids*, Int. J, Solid Structures B, 26,43-57,1990.

[6] de Boer, R., Ehlers, W. and Liu, Z., *One dimensional transient wave propagation in fluid saturated incompressible porous media*, Arch. App. Mech. 63, 59-72, 1993.

[7] de Boer, R. and Liu, Z., *Plane waves in a semi-infinite fluid saturated porous medium*, Transport in Porous Media, 16 (2), 147-173, 1994.

[8] de Boer, R. and Liu, Z. , *Propagation of acceleration waves in incompressible liquid – saturated porous solid*, Transport in porous Media (TIPM), 21, 163-173, 1995.

[9] Fillunger, P.; *Der Auftrieb in Talsperren. Osterr., Wochenschrift fur den offentl. Baudienst*, 19, 532 - 556, 1913.

[10] Kumar, R. and Hundal, B.S., *Wave propagation in a fluid saturated incompressible porous medium*, Indian J. Pure and Applied Math.4, 51-65,2003.

[11] Kumar,R.,Miglani,A. and Kumar,S., *Reflection and Transmission of plane waves between two different fluid saturated porous half spaces*, Bull. Pol. Ac., Tech., 227-234, 59(2), 2011.

[12] Kumar,R, and Rupender , *Propagation of plane waves at the imperfect boundary of elastic and electro-microstretch generalized thermoelastic solid*, Applied Mathematics and Mechanics,30(11)1445-1454,2009.

[13] Kumar,R, and Chawla V., *Effect of rotation and stiffness on surface wave propagation in elastic layer lying over a gereralized thermodiffusive elastic half-space with imperfect boundary*, Journal of solid mechanics,2(1),28-42,2010.

[14] Tajuddin, M. and Hussaini, S.J., *Reflection of plane waves at boundaries of a liquid filled poroelastic half-space*, J. Applied Geophysics 58,59-86,2006.