

# AVAILABILITY ANALYSIS OF SHEET FORMATION SYSTEM OF UTENSILS MANUFACTURING PLANT

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## ABSTRACT

*In this paper, the availability analysis of sheet formation system of utensils manufacturing unit under certain assumptions have been studied. The sheet formation systems consists of five sub systems i.e. cutting; furnace, hot rolling, roller furnisher & cold rolling are connected in series. The differential difference equations have been developed by using simple probabilistic approach & Markov Birth-Death process. Then these equations have been solved recursively and reduced to steady state condition to find out long run availability of sheet formation system. The effects of various parameters on availability have also been studied.*

**Keywords:** - Availability, Markov Process, Manufacturing system, Transition Diagram, Sheet Formation

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## 1. INTRODUCTION

Manufacturing processes involve a continuous flow of raw materials through a series of sequential operations which transform the raw materials into finished goods for increasing the productivity, availability of the subsystem must be maintained at higher level. Practically various systems / subsystems are subjected to random failures due to poor design, lack of operative skills and wrong manufacturing techniques etc. causing heavy production losses. The failed systems can be brought back to the working state in minimum down time through effective maintenance planning and control. The factory operating conditions and repair strategic plans play important role in maintaining the system failure free for maximum duration. This can be accomplished only through quantitative analysis of each working subsystem of the plant concerned. The system performance can be quantified in terms of the availability if the real system is modeled mathematically and analyzed in actual operating conditions. Further reliability & availability analysis can be beneficial for the industry in terms of higher productivity & lower maintenance cost. Singh and Mahajan [4] describe the Reliable operation of sheet formation system in the utensils industry. They analyzed a sheet formation system of a utensils manufacturing plant for its availability. Goel & Singh [5] discussed the availability Analysis of Butter Manufacturing System in a Dairy Plant. Padmavati, Rizwan, Gulshan et al (13) explored the reliability analysis of an evaporator of a desalination plant through semi-Markov process & regenerative point technique. Tewari et al. [7] studied Mathematical modeling and behavioral analysis of refining system using genetic algorithm. Khanduja, Tewari & Kumar [9] discussed the availability analysis of bleaching system in a paper plant. Khanduja, Tewari & Chauhan [10] discussed the Decision support system and performance evaluation of a digesting system of a paper plant. Kuldeep Kumar et. al. [12] studied the Mathematical Modeling and Analysis of Stainless Steel Utensil Manufacturing Unit using Fuzzy Reliability.

Various mathematical models can be developed for analysing the system performance. Among these methods, Markov Birth Death Process is proposed to be used for the present study on a sheet formation system of a Utensils manufacturing plant.

## 2. SYSTEM DESCRIPTION

The sheet formation system of utensils manufacturing plant consist of five subsystems

- 1.) Cutting system: - Here flat steel sheets are cut them into pieces as per the requirements.
- 2.) Furnace: - The pieces of steel sheets are then put into the furnace for heating at the temperature  $900^{\circ}$  c.
- 3.) Hot rolling machine: - the heated steel sheets are pressed to make them as thick as per requirements.
- 4.) Roller furnisher: - here the red hot pressed steel sheets are softened & removing of the irregularities in the steel sheets is takes place.
- 5.) Cold rolling machines: - here we get the smooth and fine sheets for manufacturing the utensils.

## 3. ASSUMPTIONS:

1. There is no simultaneous failure among the sub systems.
2. Sub system 'A' never fails.
3. Failure/repair rates are constant over time and statistically independent.
4. Sufficient repair facilities are provided.
5. A repaired unit is good as new, performance wise for a specified duration

## 4. NOTATIONS:

The following notations are addressed for the purpose of mathematical analysis of the sheet formation system:

1. Cutter (A) - Two Units, one working and other in standby.
2. Furnace Machine (B) - One Unit subjected to major failure only.
3. Hot Rolling Machine (C) - Two units working in parallel subjected to minor and major failures.
4. Roller Furnisher machine (D) - One Unit subjected to major failure only
5. Cold Rolling Machine (E) - Two units working in parallel subjected to minor and major failures.

$\bar{C}, \bar{E}$  = Indicates that the sub system C and D work in reduced capacity.

$\lambda_i$  = Respective mean constant failure rate of sub system B, C, D, E,  $\bar{C}$ ,  $\bar{E}$  ( $i=1, 2 \dots 6$ )

$\mu_i$  = Respective mean constant repair rates of subsystem B, C, D, E,  $\bar{C}$ ,  $\bar{E}$  ( $i=1, 2 \dots 6$ )

$P_i(t)$  = Probability when the system is in i-th state at time t.

## 5. SYSTEM MODELING:

Simple probabilistic approach gives the difference differential equations, associated with the transition diagram (figure1) of the sheet formation system.

$$P'_0(t) + \alpha_1 P_0(t) = \mu_1 P_4(t) + \mu_3 P_5(t) + \mu_2 P_1(t) + \mu_4 P_2(t) \quad - (1)$$

Where ( $\alpha_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ )

$$P'_1(t) + \alpha_2 P_1(t) = \mu_1 P_6(t) + \mu_2 P_7(t) + \mu_3 P_8(t) + \lambda_2 P_0(t) + \mu_4 P_3(t) \quad - (2)$$

Where ( $\alpha_2 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \mu_2$ )

$$P'_2(t) + \alpha_3 P_2(t) = \mu_1 P_9(t) + \mu_3 P_{10}(t) + \mu_4 P_{11}(t) + \mu_2 P_3(t) + \lambda_4 P_0(t) \quad - (3)$$

Where ( $\alpha_3 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \mu_4$ )

$$P'_3(t) + \alpha_4 P_3(t) = \mu_1 P_{12}(t) + \mu_2 P_{13}(t) + \mu_3 P_{14}(t) + \mu_4 P_{15}(t) + \lambda_4 P_1(t) + \lambda_2 P_2(t) \quad - (4)$$

Where ( $\alpha_4 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \mu_2 + \mu_4$ )

$$P'_4(t) + \mu_1 P_4(t) = \lambda_1 P_0(t) \quad - (5)$$

$$P'_5(t) + \mu_3 P_5(t) = \lambda_3 P_0(t) \quad - (6)$$

$$P'_{5+i}(t) + \mu_i P_{5+i}(t) = \lambda_i P_1(t), \quad i=1, 2, 3 \quad - (7)$$

$$P'_9(t) + \mu_1 P_9(t) = \lambda_1 P_2(t) \quad - (8)$$

$$P'_{10}(t) + \mu_3 P_{10}(t) = \lambda_3 P_2(t) \quad - (9)$$

$$P'_{11}(t) + \mu_4 P_{11}(t) = \lambda_4 P_2(t) \quad - (10)$$

$$P'_{11+i}(t) + \mu_i P_{11+i}(t) = \lambda_i P_3(t), \quad i=1,2,3 \quad - (11)$$

$$P'_{15}(t) + \mu_4 P_{15}(t) = \lambda_4 P_3(t) \quad - (12)$$

With initial conditions at time  $t = 0$

$$P_i(t) = 1 \text{ for } i = 0$$

$$= 0 \text{ for } i \neq 0$$

**Solution of equations:**

The steady state behavior of the system can be analyzed by setting the condition  $t \rightarrow \infty$ ,  $d/dt \rightarrow 0$  for equations(1)-(9)and solving them recursively;

After applying steady state condition, we get

$$\alpha_1 P_0 = \mu_1 P_4 + \mu_3 P_5 + \mu_2 P_1 + \mu_4 P_2$$

$$\alpha_2 P_1 = \mu_1 P_6 + \mu_2 P_7 + \mu_3 P_8 + \lambda_2 P_0 + \mu_4 P_3$$

$$\alpha_3 P_2 = \mu_1 P_9 + \mu_3 P_{10} + \mu_4 P_{11} + \lambda_4 P_0 + \mu_2 P_3$$

$$\alpha_4 P_3 = \mu_1 P_{12} + \mu_2 P_{13} + \mu_3 P_{14} + \lambda_4 P_1 + \mu_4 P_{15} + \lambda_2 P_2$$

$$\mu_1 P_4 = \lambda_1 P_0$$

$$\mu_3 P_5 = \lambda_3 P_0$$

$$\mu_i P_{5+i} = \lambda_i P_1, \quad i = 1, 2, 3$$

$$\mu_1 P_9 = \lambda_1 P_2$$

$$\mu_3 P_{10} = \lambda_3 P_2$$

$$\mu_4 P_{11} = \lambda_4 P_2$$

$$\mu_i P_{11+i} = \lambda_i P_3 \quad i = 1, 2, 3$$

$$\mu_4 P_{15} = \lambda_4 P_3$$

Solving these equations Recursively, we get

$P_1 = \alpha_7 P_0$	$P_2 = \alpha_6 P_0$	$P_3 = \alpha_5 P_0$
$P_4 = B_1 P_0$	$P_5 = B_3 P_0$	$P_6 = B_1 \alpha_7 P_0$
$P_7 = B_2 \alpha_7 P_0$	$P_8 = B_3 \alpha_7 P_0$	$P_9 = B_1 \alpha_6 P_0$
$P_{10} = B_3 \alpha_6 P_0$	$P_{11} = B_4 \alpha_6 P_0$	$P_{12} = B_1 \alpha_5 P_0$
$P_{13} = B_2 \alpha_5 P_0$	$P_{14} = B_3 \alpha_5 P_0$	$P_{15} = B_4 \alpha_5 P_0$

Where

$$B_i = \lambda_i / \mu_i \quad i = 1, 2, 3, 4 \quad \&$$

$$\alpha_5 = \left\{ \frac{\lambda_2 \lambda_4}{\mu_2 \mu_4} \left( \frac{\lambda_2 + \lambda_4 + \mu_2 + \mu_4}{\lambda_2 - \lambda_4 + \mu_2 + \mu_4} \right) \right\}$$

$$\alpha_6 = \left( \frac{\lambda_4 + \mu_2 \alpha_5}{\lambda_2 + \mu_4} \right)$$

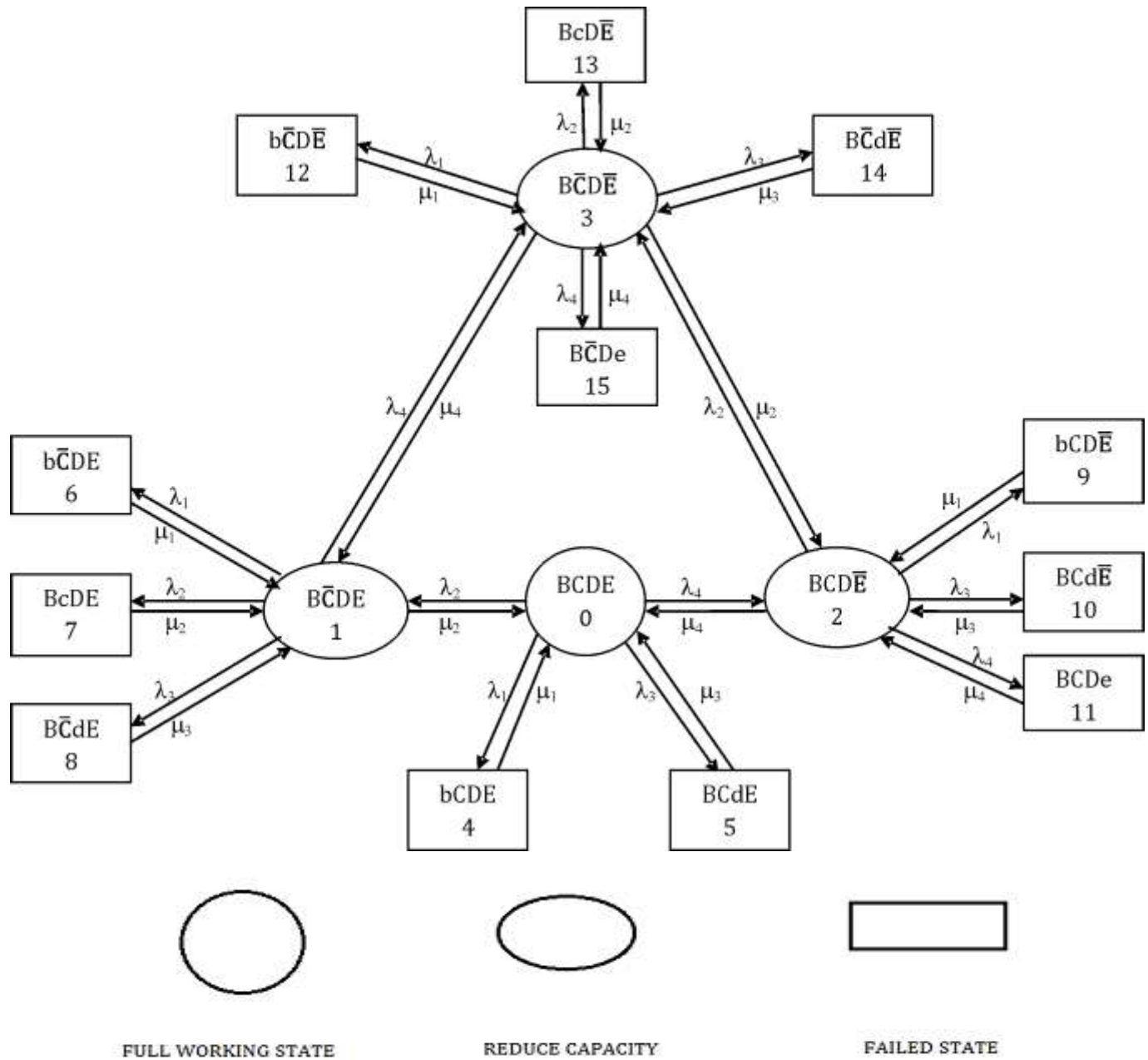
$$\alpha_7 = \left( \frac{\lambda_2}{\mu_2} + \frac{\lambda_4}{\lambda_2} + \frac{\mu_4}{\mu_2} \alpha_6 \right)$$

Transition diagram:

Total States = 15

Working States = 4

Failed States = 11



STATE TRANSITION DIAGRAM OF SHEET FORMATION SYSTEM

Figure 1.

Using Normalizing condition i.e. sum of all the state probabilities is equal to one

Probability of working states + Probability of failed States = 1

$$\sum_{i=0}^{i=15} P_i = 1$$

$$P_0 = [(1+\alpha_7+\alpha_6+\alpha_5) (1+\lambda_1/\mu_1+\lambda_3/\mu_3) +\lambda_2/\mu_2 (\alpha_5+\alpha_7) +\lambda_4/\mu_4(\alpha_5+\alpha_6)]^{-1} \tag{10}$$

$$\begin{aligned} \text{Availability (A}_v\text{)} &= \text{Summation of all working states} \\ &= \sum_{i=0}^3 P_i \\ &= P_0+P_1+P_2+P_3 \\ &= P_0 (1+\alpha_7+\alpha_6+\alpha_5) \end{aligned} \tag{11}$$

**6. AVAILABILITY ANALYSIS OF SHEET FORMATION SYSTEM:**

The availability or performance of the sheet formation system in a utensils manufacturing plant is mainly affected by the failure and repair rates of each subsystem. These system parameters ensure the high availability of the Sheet formation system. This performance evaluating model includes all possible states of nature, that is, future events ( $\lambda$ ) and the identification of all the courses of action, that is, repair priorities ( $\mu$ ). Table 1, 2, 3, 4 represent the availability matrices for various subsystems of the sheet formation system. These matrices simply reveal the various availability levels for different combinations of failure events and repair priorities. On the basis of analysis, one may select the best possible combination ( $\lambda, \mu$ ), that is, optimal maintenance strategies.

Table 1 and graph 1 show the effect of failure and repair rates of Furnace on the availability of sheet formation system. It is observed that for some known values of failure / repair rates of Hot rolling, Roller furnisher, Cold rolling subsystem ( $\lambda_2= 0.001, \mu_2= 0.1, \lambda_3= 0.005, \mu_3= 0.1, \lambda_4= 0.001, \mu_4= 0.1$ ), as failure rate of Furnace increases from 0.005 (once in 200 hrs) to 0.025 (once in 40 hrs), the system availability decreases by 17.26%. Similarly as repair rate of Pressing machine increases from 0.1(once in 10 hrs) to 0.5(once in 2 hrs), the system availability increases by 3.39%.

**Table 1: Availability matrix for the Furnace subsystem of the Sheet formation system**

$\lambda_1 \backslash \mu_1$	0.005	0.010	0.015	0.020	0.025
0.1	0.90495638	0.86578165	0.82985786	0.79679647	0.76626847
0.2	0.92590395	0.90495638	0.88493568	0.86578165	0.84743922
0.3	0.93310365	0.91881450	0.90495638	0.89151009	0.87845752
0.4	0.93674566	0.92590395	0.91531033	0.90495638	0.89483406
0.5	0.93894454	0.93021038	0.92163721	0.91322063	0.90495638

GRAPH - 1

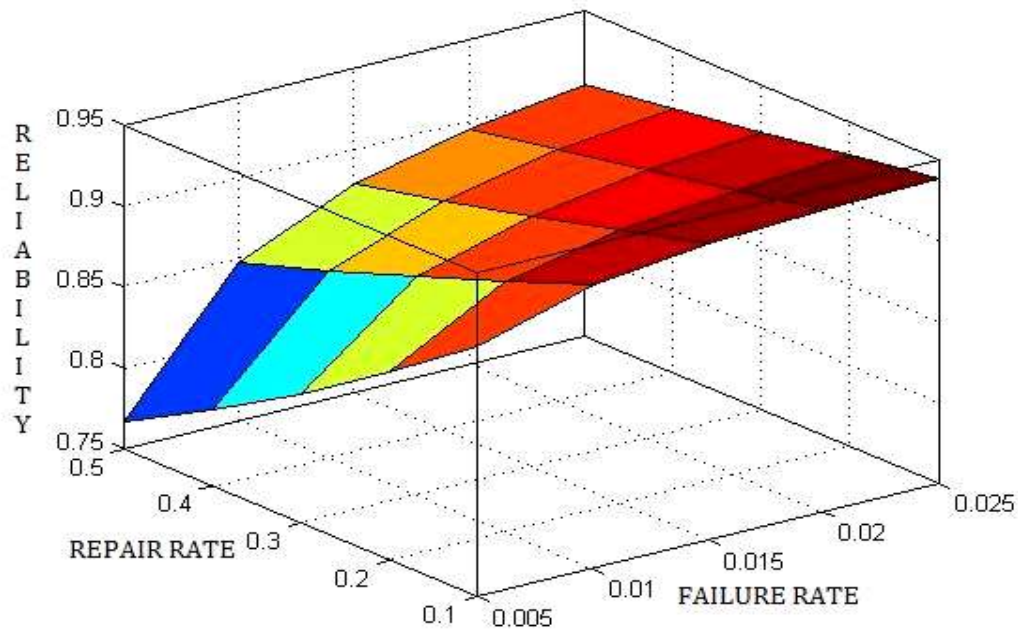


Table 2 and Graph 2 reveal the effect of failure and repair rates of Hot rolling subsystem on the availability of sheet formation system .It is observed that for some known values of failure / repair rates of Furnace, Roller furnisher & Cold rolling ( $\lambda_1 = 0.005$ ,  $\mu_1 = 0.1$ ,  $\lambda_3 = 0.005$ ,  $\mu_3 = 0.1$ ,  $\lambda_4 = 0.001$ ,  $\mu_4 = 0.1$ ), as failure rate of Hot rolling increases from 0.001 (once in 100 hrs) to 0.005(once in 200 hrs), the system availability decreases by 0.38% Similarly as repair rate of Hot Rolling increases from 0.1(once in 10 hrs) to 0.5(once in 2 hrs), the system availability increases by 0.32 %.

**Table 2: Availability matrix for the Hot Rolling subsystem of the Sheet formation system**

$\lambda_2$ \ $\mu_2$	0.001	0.002	0.003	0.004	0.005
0.1	0.90495638	0.90352242	0.90264478	0.90189826	0.90114431
0.2	0.90699852	0.90628928	0.90589420	0.90560075	0.90533864
0.3	0.90768126	0.90720717	0.90695223	0.90677404	0.90662535
0.4	0.90802301	0.90766528	0.90747643	0.90734890	0.90724697
0.5	0.90822818	0.90793987	0.90778938	0.90768999	0.90761287

GRAPH - 2

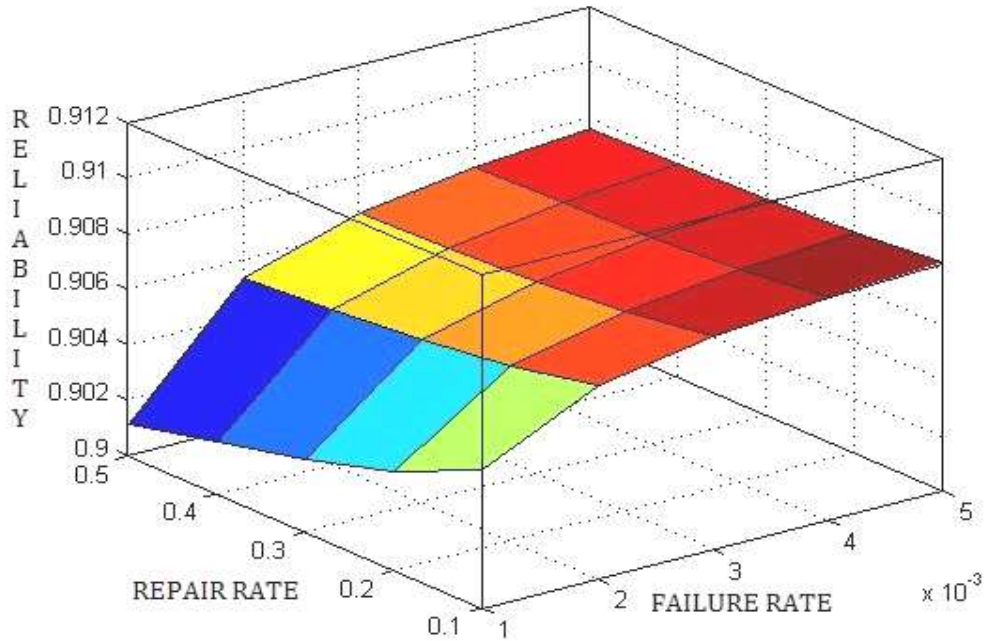


Table 3 and graph 3 reveal the effect of failure and repair rates of roller furnisher subsystem on the availability of sheet formation system .It is observed that for some known values of failure / repair rates of furnace, hot rolling & cold rolling ( $\lambda_1 = 0.005$ ,  $\mu_1 = 0.1$ ,  $\lambda_2 = 0.003$ ,  $\mu_2 = 0.1$ ,  $\lambda_4 = 0.002$ ,  $\mu_4 = 0.1$ ), as failure rate of roller furnisher increases from 0.005 (once in 200 hrs) to 0.025(once in 40 hrs), the system availability decreases by 13.70 %. Similarly as repair rate of Roller Furnisher increases from 0.1(once in 10 hrs) to 0.5(once in 2 hrs), the system availability increases by 3.36 %.

**Table 3: Availability matrix for the Roller furnisher subsystem of the Sheet formation system**

$\lambda_3$ \ $\mu_3$	0.005	0.010	0.015	0.020	0.025
0.1	0.89910758	0.86042677	0.82493688	0.79225871	0.76207084
0.2	0.91978216	0.89910758	0.87934200	0.86042677	0.84230815
0.3	0.92688660	0.91278579	0.89910758	0.88583326	0.87294519
0.4	0.93048014	0.91978216	0.90932737	0.89910758	0.88911495
0.5	0.93264967	0.92403170	0.91557152	0.90726487	0.89910758

**GRAPH – 3**

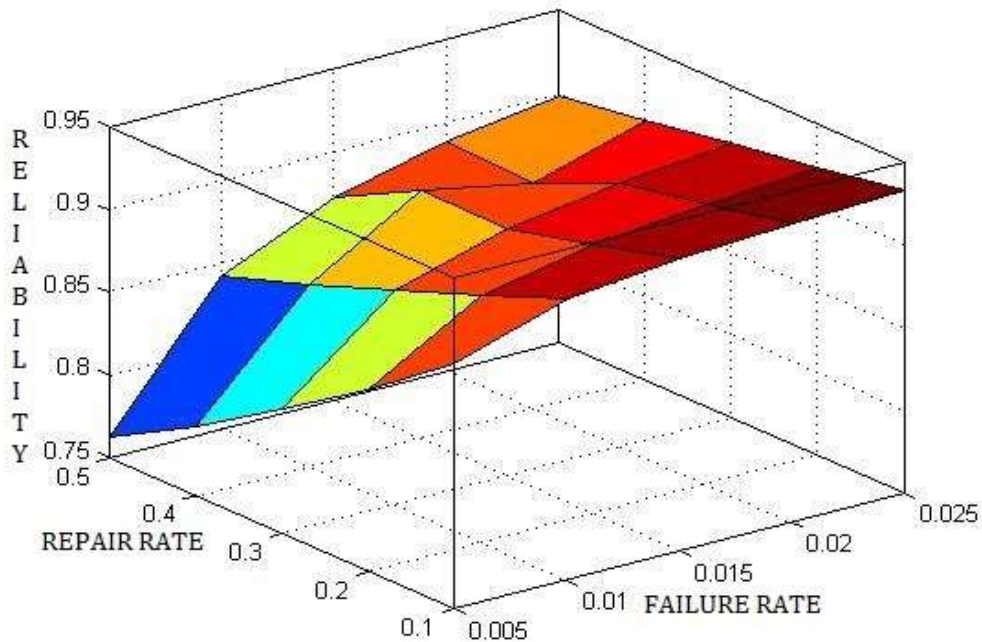


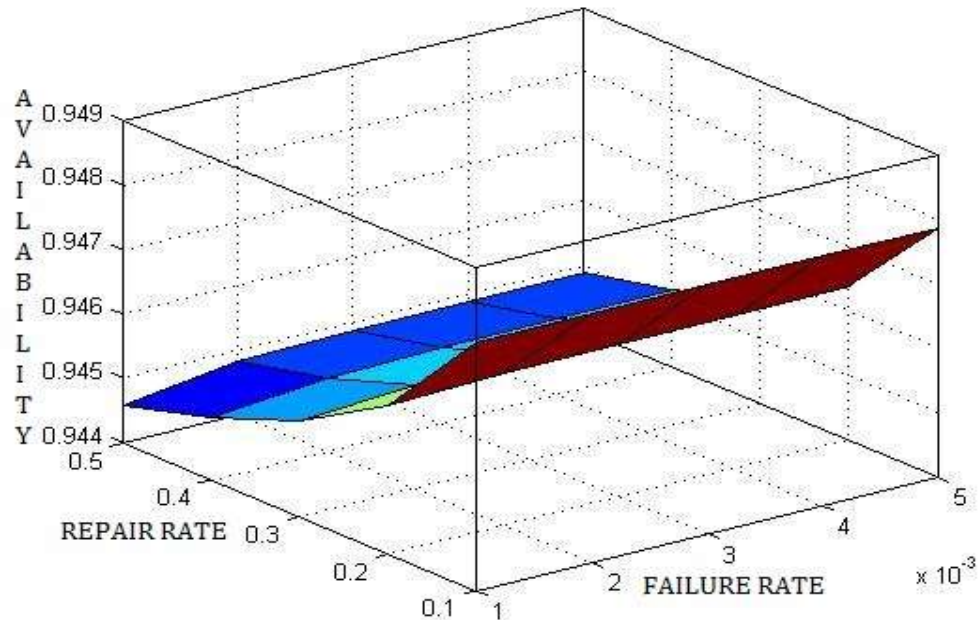
Table 4 and graph 4 reveal the effect of failure and repair rates of Cold Rolling subsystem on the availability of sheet formation system .It is observed that for some known values of failure / repair rates of furnace, hot rolling & roller furnisher ( $\lambda_1 = 0.002$ ,  $\mu_1 = 0.1$ ,  $\lambda_2 = 0.001$ ,  $\mu_2 = 0.1$ ,  $\lambda_3 = 0.003$ ,  $\mu_3 = 0.1$ ), as failure rate of Buffing increases from 0.001 (once in 100 hrs) to 0.005(once in 200 hrs), the system availability decreases by 0.33 %. Similarly as repair rate of Cold Rolling increases from 0.1(once in 10 hrs) to 0.5(once in 2 hrs), the system availability increases by 0.0028 %.

**Table 4: Availability matrix for the Cold Rolling subsystem of the Sheet formation system**

$\lambda_3 \backslash \mu_3$	0.001	0.002	0.003	0.004	0.005
0.1	0.94784427	0.94630139	0.94548485	0.94495890	0.94457824
0.2	0.94786707	0.94637183	0.94561188	0.94514654	0.94482864
0.3	0.94786966	0.94638200	0.94563179	0.94517721	0.94487063
0.4	0.94787001	0.94638457	0.94563752	0.94518654	0.94488379
0.5	0.94787192	0.94638531	0.94563959	0.94519020	0.94488919



GRAPH - 4



**CONCLUDING REMARKS:-**

Above Tables & Graphs shows that failure rate of all the sub systems i.e. furnace ( $\lambda_1$ ), hot rolling ( $\lambda_2$ ), roller furnisher ( $\lambda_3$ ) & Cold rolling ( $\lambda_4$ ) decreases the availability rapidly. The repair rates of the sub systems i.e. furnace ( $\mu_1$ ), hot rolling ( $\mu_2$ ), roller furnisher ( $\mu_3$ ) & Cold rolling ( $\mu_4$ ) have considerable effects on availability. It can thus be concluded that this performance-evaluating model is effectively used for the analysis of availability of various subsystems of the sheet formation system in a utensils manufacturing plant. It also shows the relationship between various failure and repair rates ( $\lambda, \mu$ ) for each subsystem that is, sheet formation system in a utensils manufacturing plant. It also provides the various availability levels ( $A_v$ ) for different combinations of failure and repair rates for each subsystem. One may select the best possible combination of failure events and repair priorities for each subsystem.

Table 5 as shown below. If we go beyond these optimum values of the repair rates, as shown in tables 1, 2 & 3 there is hardly any increase in the availability levels. Therefore, we have selected these optimum values for the highest possible availability levels (0.93894454, 0.90822818, 0.93264967 and 0.94787192). So the findings of this paper are discussed with the concerned utensils manufacturing plant management. Such results are found highly beneficial to the plant management for analysis of availability and hence to decide about the repair priorities of various subsystems of sheet formation system in a utensils manufacturing plant to enhance the system's performance.

**Table 5: Optimal Values of failure and repair rates**

Sr.No.	Failure Rates ( $\lambda_i$ )	Repair Rates ( $\mu_i$ )	Maximum Availability Level ( $A_v$ )
1.	$\lambda_1 = 0.005$	$\mu_1 = 0.5$	0.93894454
2.	$\lambda_2 = 0.001$	$\mu_2 = 0.5$	0.90822818
3.	$\lambda_3 = 0.005$	$\mu_3 = 0.5$	0.93264967
4.	$\lambda_4 = 0.001$	$\mu_4 = 0.5$	0.94787192

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