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OPTIMUM ORDERING INTERVAL FOR DETERIORATING ITEMS WITH SELLING PRICE DEPENDENT DEMAND AND RANDOM DETERIORATION

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ABSTRACT:
In this paper we have developed an inventory model for obtaining optimum cycle length by minimizing the total cost per unit time under usual assumptions. The demand is taken as a function of selling price and a special form of two parameter Weibull function considered by Covert and Philip (6) is taken as deterioration rate. The results obtained in this paper reduces to well known result by choosing appropriate values of the parameter.

INTRODUCTION: -

Inventory models for determining the optimal stocking policies for items deteriorating with time have engaged attention of researchers in recent years. A good number of authors have taken different rate of deterioration in their analysis such as constant, linear function of time and two parameter weibull function etc. as the utility of goods does not remain constant over time.

In recent years, mathematical models, heave been used in different areas in real life problems, particularly for controlling inventory. One the important concern of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system is minimum. This is some, what more important, when the inventory under go decay or deterioration. When the items of the commodity are kept in the stock and an inventory for fulfilling the future demand, there may be the deterioration of items in the inventory system, which may occur due to one or many factors i.e. storage condition weather condition or due to the commodity.

Nahmias (2) reviewed the ordering policy for perishable inventories, covering both fixed life and random life models. Hollier and Mak (4) explored the demand rate to be decreasing negative exponentially without shortages. Cohen (7) considered joint pricing and ordering policy for exponentially decaying inventory with known demand considering constant deterioration rate. Mukherjee (3) developed an inventory model for optimum ordering interval for time varying decay rate of inventory. Hari Kishan etal (8) studied an inventory model with variable demand rate and random deterioration and shortages. Chung etal (5) developed a solution procedure to determine joint pricing and replenishment policy for deteriorating inventory systems with declining market.

ASSUMPTIONS AND NOTATIONS: -

(i) The demand rate d(p) is known, where ‘p’ is the selling price per unit and posses a negative derivatives throughout its domain.

(ii) A variable function $\theta(t,\alpha)$ of the on hand inventory deteriorates per unit time.
In the present model, the deterioration function $\theta(t,\alpha)$ is assumed in the form $\theta(t,\alpha) = \theta_0(\alpha)\, t \quad 0 < \theta_0(\alpha) << 1$, $t > 0$ which is a special form of two parameter weibull function considered by Covert and Philip (6) the function is some functions of the random variable α which lunges over a space Γ and in which a probability density function $p(\alpha)$ is defined such that $\int_\Gamma p(\alpha)d\alpha = 1$.

(iii) I(t) is the on hand inventory at any time ‘t’.

(iv) $I_W(t)$ is the on hand inventory at any time ‘t’.

(v) Z(t) is the stock loss due to decay at time ‘t’.

(vi) There is no replacement or repair of the decayed units during the period under consideration.
(vii) C is the unit purchase cost, h is the holding cost per order and constant during scheduling period T.
(viii) Shortages are not allowed.
(ix) T is the scheduling period.

MODEL DOVELOPMENT AND ANALYSIS : -

The differential equation describing the behaviour of the system is given by

$$\frac{dI(t)}{dt} = -\theta_0(t,\alpha)I(t) - d(p) \tag{1}$$

$$I(t)e^{\int \theta_0(t,\alpha)t^{dt}} = -d(p)\int e^{(\theta_0(\alpha)t)}dt + B$$

B is constant of integration

$$I(t)\exp.\left(\frac{\theta_0(\alpha)t^2}{2}\right) = -d(p)\int e^{\frac{\theta_0(\alpha)t^2}{2}}dt + B_1$$

$$= -d(p)\left[t + \frac{\theta_0(\alpha)t^3}{6}\right] + B_1$$

$$I(t) = -d(p)\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right)\left[t + \frac{\theta_0(\alpha)t^3}{6}\right] + B_1\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right)$$

At *t = 0, we get* B_1 = I(0)

$$I(t) = -d(p)\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right)\left[t + \frac{\theta_0(\alpha)t^3}{6}\right] + I(0)\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right) \tag{2}$$

Inventory without decay at time 't', the differential equation of the system will be $\frac{d}{dt}I_w(t) = -d(p)$

$$\Rightarrow I_w(t) = -d(p)t + B_2$$

Where B_2 is the constant of integration At $t = 0, B_2 = I(0)$.

$$\Rightarrow I_w(t) = -d(p)t + I(0) \tag{3}$$

also $$I(O) = I(t)\exp.\left(\frac{\theta_0(\alpha)t^2}{2}\right) + d(p)\left(t\frac{\theta_0(\alpha)t}{6}\right) \tag{4}$$

Stock loss due to decay Z(t) at time t is given by:

$$z(t) = I_w(t) - I(t)$$

$$= I(0) - d(p)t - I(t) \tag{5}$$

Substituting value of I(O) from (5.4.4), in (5.4.5) We get

$$z(t) = I(t)\exp.\left(\frac{\theta_0(\alpha)t^2}{2}\right) + d(p)\left(t + \frac{\theta_0(\alpha)t^3}{6}\right) - d(p)t - I(t)$$

$$= I(t)\frac{\theta_0(\alpha)t^2}{2} + d(p)\left(t + \frac{\theta_0(\alpha)t^3}{6}\right) - d(p)t \tag{6}$$

If T is the cycle length (ordering interval), the order quantity, Q_T required to satisfy the demand during a cycle of length T is equal to

$$Q_T = Z(T) + Td(p) \tag{7}$$

In case of instantaneous replenishment

$$I(O) = Q_T \text{ and } I(T) = 0$$

Also from (6) we have

$$Z(T)=I(T)\frac{\theta_0(\alpha)t^2}{2}+d(p)\left(T+\frac{\theta_0(\alpha)T}{6}\right)-d(p)T \tag{8}$$

$$Q_T=d(p)\left(T+\frac{\theta_0(\alpha)T^3}{6}\right) \tag{9}$$

Form (2), we have,

$$I(t)=-d(p)\exp.\left(-\frac{\theta_0(\alpha)t^2}{2}\right)\left(t+\frac{\theta_0(\alpha)t}{6}\right)$$

$$+d(p)\frac{\theta_0(\alpha)t^3}{6}\exp.\left(-\frac{\theta_0(\alpha)t^2}{2}\right) \tag{10}$$

Cost per unit time C(T, P,α) is given by

C (T, P, α) = $\frac{K}{T}+\frac{CQ_T}{T}+\frac{h}{T}\int_O^T I(t)d$

$$=\frac{K}{T}+\frac{Cd(p)}{T}\left(T+\frac{\theta_0(\alpha)T^3}{6}\right)+\frac{h}{T}\int_0^T -d(p)\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right)\left(t+\frac{\theta_0(\alpha)t}{6}\right)$$

$$+d(p)\frac{\theta_0(\alpha)t^3}{6}\exp\left(-\frac{\theta_0(\alpha)t^2}{2}\right)$$

$$=\frac{K}{T}+\frac{Cd(p)}{T}\left(T+\frac{\theta_0(\alpha)T^3}{6}\right)+\frac{hd(p)}{T}\left(\frac{T^2}{2}+\frac{\theta_0(\alpha)T^4}{12}\right) \tag{11}$$

neglecting higher power of $\theta_0(\alpha)$

Hence mean cost $\langle C(T,p,\alpha)\rangle$ is obtained by the integral

$$\langle C(T,p)\rangle=\int_\Gamma C(T,p,\alpha)p(\alpha)d\alpha$$

$$\langle C(T,p)\rangle=\frac{K}{T}+Cd(p)\left(1+\frac{\theta_0(\alpha)T^2}{6}\right)+hd(p)\left(\frac{T}{2}+\frac{\theta_0(\alpha)T^3}{12}\right)$$

$$\Rightarrow\langle C(T,p)\rangle=\frac{K}{T}+Cd(p)+\frac{Cd(p)AT^2}{6}+\frac{hd(p)T}{2}+\frac{hd(p)AT^3}{12} \tag{12}$$

Where A $=\int_\Gamma \theta_0(\alpha)p(\alpha)d\alpha$

The minimum mean cost $\langle C(T,p)\rangle$ for fixed prime p is minimized i.e.

$$\frac{\partial\langle C(T,p)\rangle}{\partial T}=0$$

$$-\frac{K}{T^2}+\frac{Cd(p)AT}{3}+\frac{hd(p)}{2}+\frac{hd(p)AT^2}{4}=0$$

$$-12K+4Cd(p)AT^3+6hd(p)T^2+3hd(p)AT^4=0 \tag{13}$$

Equation (13) can be solved numerically for T_p for given values of K, C, h, dp, etc.

Also $\frac{\partial^2\langle C(T,p)\rangle}{\partial T^2}=\frac{2K}{T^3}+\frac{Cd(p)A}{3}+\frac{hd(p)A}{2}>0$

For optimum price decision for a fixed period length, we maximize profit rate $f(T,p)$

Where $f(T,p)=pd(p)-\langle C(T,p)\rangle$ by holding T fixed, find (14)

$$\frac{\partial f}{\partial p}f(T,p)=0$$

$$\frac{\partial f}{\partial p}=pd'(p)+d(p).1-\frac{\partial\langle C(T,p)\rangle}{\partial p}=0 \quad (15)$$

$$\frac{\partial\langle C(T,p)\rangle}{\partial p}=pd'(p).\left[C+\frac{ACT^2}{6}+\frac{hT}{2}+\frac{hAT^3}{12}\right]$$

$$\Rightarrow\frac{\partial f}{\partial p}=d(p)+d'(p)\left[p-c-\frac{ACT^2}{6}-\frac{hT}{2}-\frac{hAT^3}{12}\right]=0$$

$$-\frac{d(p)}{d'(p)}+C\left(1+\frac{AT^2}{6}\right)+\frac{hT}{2}+\frac{hAT^3}{12}=p \quad (16)$$

Also $\frac{\partial^2 f}{\partial p^2}>0.(As\ d'(p)\text{and } d''(p)<0)$

MODEL ANALYSIS SPECIAL CASE: -

If $\theta_0(\alpha)=0$ and $d(p)=R$ i.e. when there is no decay and demand rate is constant, then $\langle C(T)\rangle=\frac{K}{T}+CR+\frac{hRT}{2}$ which is standard result for non decay inventory.

CONCLUDING REMARKS: -

We made an attempt to solve an inventory model for deteriorating items and without shortages and performed the analysis on the effect of perishability with optimal pricing and ordering decisions. The demand is taken as a function of selling price. Special case is also provided for further study to incorporate any factual relation that may exists between time and price both in the demand rate function and taken suitable deterioration rate. The result of the model is important for formulating the decisions when the inventory deteriorates with time together with another parameter such as temperature humidity etc.

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